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Courts of Law and Unforeseen Contingencies[*]

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ABSTRACT.   We study a contracting model with unforeseen contingencies in which the court is an active player. Ex-ante, the contracting parties cannot include the risky unforeseen contingencies in the contract they draw up. Ex-post the court observes whether an unforeseen contingency occurred, and decides whether to *void* or *uphold* the contract. If the contract is voided by the court, the parties can renegotiate a new agreement ex-post.

There are two effects of a court that voids more contracts. The parties' incentives to undertake relationship-specific investment are reduced, while the parties enjoy greater insurance against the unforeseen contingencies which the ex-ante contract cannot take into account.

In this context, we are able to characterize fully the optimal decision rule for the court. The behavior of the optimal court is determined by the tradeoff between the need for incentives and the gains from insurance that voiding in some circumstances offers to the agents.

1. Introduction

1.1. Motivation

Courts regularly intervene in contracts at the behest of one of the contracting parties to void, or otherwise modify, an agreement the parties have agreed to. One justification for courts overriding voluntary agreements is to insure the parties against changes in the environment between the time the agreement was made and the time when it is to be consummated. Changes in the environment can lead to changes in the costs and benefits to the parties involved that expose them to risks which they prefer to avoid. The possibility of renegotiation protects the parties from carrying out Pareto-dominated transactions, but not from the fluctuations in utility that stem from the uncertainty in the underlying environment.

If the parties foresee all relevant contingencies and agree on the optimal transactions given them, these can be included in the contract, thus providing protection from these risks. Such foresight seems unreasonable, however. Regardless of the parties' experience and care in designing their contract, there will always be residual risk they face due to "unforeseen contingencies".

There is considerable ambiguity about the meaning of unforeseen contingencies, and it is worth discussing the term briefly before proceeding further. We take the position that parties can perfectly foresee the *possibility* of various contingencies, but are unable to foresee the circumstances in sufficient detail to include all relevant contingencies in their contract.[1]

Although contracting parties may be unable to identify all relevant contingencies ex-ante, it may be clear both to the parties and outsiders that the circumstances at the time the contract calls upon one of the parties to act differ materially from those envisioned at the time the contract was written. In this event, a court can make such a determination, and void the contract in order to provide insurance the parties arguably desire, but cannot effect on their own.

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[1]Al-Najjar, Anderlini, and Felli (2001) provide a formal model that fits this view of what an unforeseen contingency is.

A court that voids contracts in this way may provide desirable insurance, but not without cost. A central benefit of a contract is a guarantee that parties will receive a return for investments that have specific value in their relationship. Without a guarantee, an individual has a diminished incentive to invest, because he may obtain only a portion of the benefits stemming from the investment under an ex-post (re-)negotiated outcome. Courts that void contracts to provide insurance do so at the cost of reducing the ability to provide incentives for an efficient level of ex-ante investment.

We develop and analyze a model of a buyer and seller who contract in an environment that includes an active court whose role is to determine which contracts to void and which to uphold.[2] Our court will specify the rules for making these decisions, taking account of the contracting parties' incentives given these rules. We assume that the court maximizes ex-ante expected gains from contracting, and characterize the optimal policy, which is to void contracts in events which are ex-ante low probability, but which impose a high level of risk on the contracting parties.

Of course, in the simple set-up that we analyze the interests of all participants are aligned. Ex-ante, the objective function of the court is not in conflict with the expected utility of either of the trading parties. This, in turn, implies that the parties could attempt to replicate the behavior of the optimal court using private means. However, this will only be true in the simplified set-up that we deal with here. For example, if there is any asymmetric information between the contracting parties informational externalities would arise and this conclusion would no longer be true.[3]

1.2. Relation to the Literature

The seminal works on incomplete contracts by Grossman and Hart (Grossman and Hart 1986) and Hart and Moore (Hart and Moore 1990) took as given the existence

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[2]Throughout the rest of the paper we use the terms uphold and enforce (a contract) in a completely interchangeable way.

[3]In Anderlini, Felli, and Postlewaite (2001) we analyze a model that is parallel to the one we focus on here, but where there is an informational externality between different types of contracting parties.

of contingencies that may occur after the signing of a contract, but that cannot be described at the time the parties contract. The inability to describe all relevant contingencies, and make contract terms a function of them, affects agents' incentives. When contracts are incomplete, the contracting parties may find it optimal to renegotiate the terms of trade in the event certain contingencies arise. Agents whose investments are sunk at this time will not receive the full benefits of those investments; this holdup problem leads to inefficient initial investments. In summary, incomplete contracts may make it impossible to avoid inefficient outcomes.

A number of papers have shown that the *amount* of inefficiency, however, is not fixed. Grossman and Hart (1986) and Hart and Moore (1990) show that the ownership structure of physical assets can affect investment incentives, and hence, efficiency; Bernheim and Whinston (1998) show that if it is impossible to contract over some part of a relationship, it may be optimal to be less specific than is possible in other parts of that relationship; Aghion and Tirole (1997) and Rajan and Zingales (1998) show that the distribution of authority and power in a firm can affect efficiency when complete contracts are impossible.

Both the original work illustrating how incomplete contracts can precipitate inefficiency, and the subsequent work demonstrating how institutional design can ameliorate that inefficiency, essentially ignore the role of a court in adjudicating and enforcing contracts that are written.[4] The inefficiencies analyzed in the papers discussed above are diminished if a court can attempt to ameliorate them through various forms of intervention.[5] Stated more strongly, the work on incomplete contracts is "partial equilibrium," analyzing a subset of agents' behavior taking as fixed the behavior of agents outside the model (the courts), without investigating whether the assumed fixed behavior of the outside agents is in fact optimal. Maskin and Tirole (1999) make this point most forcefully by showing that in a standard incomplete contracting model, the existence of indescribable (unforeseen) contingencies does not affect the

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[4]A "minimal" court is assumed to exist to force parties of a contract to perform according to the contract as originally written.

[5]See Eggleston, Posner, and Zeckhauser (2000) for a discussion of the role of courts in interpreting and enforcing contracts.

set of payoff outcomes that can be achieved through contracting, if one allows a court with large discretionary authority. This is shown by exhibiting a mechanism capable of generating as equilibrium any payoffs that could be achieved with complete contracts. This mechanism does not mean that incompleteness of contracts is irrelevant, however. Maskin and Tirole invest courts with substantially more authority than we see in practice, and the plausibility of the mechanism they exhibit is open to discussion. (We return to this at length in Section 5 below.)

Our paper incorporates an active court whose authority lies between that in the traditional literature (simply enforcing contracts that are written) and that implicit in Maskin and Tirole. We provide a detailed specification of indescribable contingencies, including the information available to a court at the time performance is called for. The contracts that parties write differ from those they would write if courts did nothing more than passively enforce the contracts that are written. Despite the inclusion of a more active court, the basic message of the incomplete contract literature remains: contracts will still be incomplete, and the incompleteness causes inefficiency.

There is a relatively large literature on the effect of the rules courts use on the actions of those governed by the rules. For example, there has been substantial analysis comparing the incentive effects of strict liability with the incentive effects of a negligence rule in tort theory, and comparisons of different remedies for breach in contract theory.[6] Our analysis differs from this work in two ways. First, these literatures focus largely on particular rules that are used in practice, and compare the incentive effects of those rules in different environments. In contrast, we consider a much richer set of rules, with courts optimizing across that set; our framework admits more easily the formulation of alternative rules to those already in existence. The second difference is that earlier work is typically concerned with comparisons between qualitatively different rules, while our court must make quantitative decisions, such as the threshold for which unforeseen contingencies will change the court's decision of whether or not to void the contract.[7]

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[6]See Kaplow and Shavell (2000) for a discussion of these literatures.
[7]See Kaplow and Shavell (2000) and Kaplow (2000).

A major benefit of formally incorporating the court is that it allows a richer analysis of contracting. In addition, it provides the structure for a serious examination of what precisely a court might do. In this paper, we restrict attention to particularly simple rules a court can follow, namely to determine the circumstances under which a contract will be voided. As discussed above, Maskin and Tirole demonstrate that the indescribability of contingencies doesn't limit the outcomes achievable through general mechanism by constructing a game form which can implement any interesting outcome. The rules that we allow a court don't admit the kind of game forms they analyze, and we discuss several reasons for excluding them below in Section 5.

1.3.   Court Practices

We discussed in the previous subsection the relation of our work to previous literature. It is useful to also discuss the relation between our work and actual court practice before proceeding. Courts will, in fact, discharge a party's obligation to perform under a contract based on the emergence of risks that were not foreseen at the time the contract was entered into under some conditions. There are two primary categories under which this might happen. The first is impracticability of performance, that is, when unanticipated events subsequent to contracting have made the promised performance impossible. The second category is termed frustration of purpose. One view of frustration is that it will "... excuse performance where performance remains possible, but the value of the performance to at least one of the parties and the basic reason recognized by both parties for entering into the contract have been destroyed by a supervening and unforeseen event."[8]

The court intervention proposed in this paper that voids contracts under some circumstances is essentially the second category — frustration of purpose. It clearly is *not* impracticability, since ultimately the contracted transaction *is* consummated; the voiding of the contract serves only to relieve one or the other of the parties from

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[8]Spalding & Son, Incorporated v. The United States; 28 Fed. Cl. 242; 1993 Claims Lexis 39. See also Everett Plywood Corporation v. The United States; 227 Ct. Cl. 415; 651 F.2d 723; 1981 U.S. Ct. Cl. Lexis 278; 31 U.C.C. Rep. Serv. (Callaghan) 1234; 11 ELR 21026; 28 Cont. Cas. Fed. (CCH) P81,397.

abnormally high risk resulting from supervening events. Frustration of purpose has been applied in a manner very similar to that proposed in this paper. Small risks will not be cause for voiding the contract, but sufficiently large risks will be.

> Departures from what was expected are deemed part of the risks inherent in the promises exchanged. However, events not foreseen at the time of contracting may so fundamentally alter the nature of the bargain as to suggest that the risks associated with those events were not within the contemplation of the parties. In such situations the central inquiry is whether "the nonoccurrence of these events was a 'basic assumption on which the contract was made.'" ... Where the answer is yes, that is, where the contract reasonably cannot be interpreted as allocating to either party the risks caused by an extraordinary event, then further performance is excused.[9]

1.4. Outline

The plan of the rest of the paper is as follows. In Section 2 below we describe the model of buyers and sellers in full detail, and we comment on the assumptions we make. We characterize in Section 3 the equilibrium contract that the parties to the trade will choose for general court's decision rules while in Section 4 we present the main result of the paper: the characterization of the optimal decision rule for the court. Section 5 concludes the analysis by discussing the relationship between our model of a court and the model implicit in the approach of Maskin and Tirole (1999). For ease of exposition we have relegated all proofs to an Appendix.

2. The Model

As mentioned in the introduction, we are interested in courts that have a role in trading off parties' incentive to invest with their desire for insurance in the event of

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[9]Spalding & Son, Incorporated v. The United States, op cit.

unforeseen contingencies. To investigate this tradeoff, we consider a simple buyer and seller model.

For insurance to have any benefit, at least one of the parties must be risk averse; we assume a risk neutral buyer and risk averse seller. The buyer and seller trade a widget; the risk they face is that the cost and benefit of the widget are uncertain at the time they contract. The uncertainty about costs and benefits captures the idea that there is a "normal" cost and benefit, c_N and v_N, but that both parties are aware that there is a possibility that an unforeseen contingency could give rise to costs and benefits either taking on high values, c_H and v_H, or taking on low values, c_L and v_L.[10] For simplicity we assume that the gains from trade are constant, that is

$$\Delta = v_H - c_H = v_N - c_N = v_L - c_L.$$

This assumption is made for tractability. Our results would not qualitatively change if the costs and benefits were not perfectly correlated or if the magnitude of the gains from trade were variable. We assume that

$$c_H \geq c_N \geq c_L.$$

Hence, it is efficient to trade whether the costs and benefits are normal, high or low.

We assume that the buyer has all the bargaining power ex-ante when a contract is proposed. In other words, the equilibrium contract is the result of a take-it-or-leave-it offer from the buyer to the seller. Ex-post, in some instances, renegotiation will take place. We assume that the seller has all the bargaining power in the ex-post renegotiation: if renegotiation occurs, the seller makes a take-it-or-leave-it offer to the buyer. The assumption that both ex-ante and ex-post, one or the other of the parties has all the bargaining power is primarily for expositional ease; none of our results depends on bargaining power being absolute for one or the other. Our results would *not* hold, however, if either the buyer has all the bargaining power ex-post or

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[10]One might think of unexpected changes in the real value of the currency, for example.

the seller has all the bargaining power ex-ante.

A central concern of this paper is to model formally unforeseen contingencies. We do this by describing the set of all states of the world, and the distinction between "normal" states and "exceptional" states. Let the state space be the unit circle, denoted Ω, with an elementary state being ω. We denote the subset of these states that are the exceptional states by Θ, which we take to be an interval on the unit circle with length $\theta \leq 1/2$ and center x.

If the realized state ω is such that $\omega \notin \Theta$ then the cost of the widget to the seller is c_N while the value of the widget to the buyer is $v_N = c_N + \Delta$. For an exceptional state, $\omega \in \Theta$, the cost of the widget to the seller will be either c_L or c_H (and hence, from the assumption that the gains from trade are constant, the buyer's valuation is respectively $v_L = c_L + \Delta$ or $v_H = c_H + \Delta$).

For simplicity, we take x and ω to have uniform probability distributions on the unit circle and θ to have a uniform probability distribution on $[0, 1/2]$. Hence, θ is the probability that an exceptional cost and value will arise; this probability is itself the outcome of a random draw, and the expected value of θ is $\bar{\theta} = 1/4$. An interpretation of θ is that there is some chance that an event such as an earthquake will occur that will result in abnormal cost and benefit; the probability that this event will occur is θ, and nature decides how likely earthquakes are.[11]

When $\omega \in \Theta$, the state is not a normal state, and the cost and value of the widget differ from c_N and v_N. In this case, the cost will either be c_L or c_H; we represent the cost of the widget to the seller as a binary random variable κ which takes one of the two values c_L and c_H: $\kappa \in \{c_L, c_H\}$. We assume $\kappa = c_H$ with probability q_H and c_L with the complementary probability $1 - q_H$. We assume that whether the cost increases or decreases in an exceptional state is independent of θ and x, but that the size of the change in cost is not independent of θ. Specifically, we assume that the difference between c_L and c_N is a function of θ, $f(\theta)$. This will allow us to consider,

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[11]We take it as given that earthquakes are negative shocks for some contracting parties, but are positive shocks for, say, building contractors.

for example, exceptional events that have small probability but which lead to large deviations from normal cost.

We also assume that while costs could be abnormally high or abnormally low in exceptional states, the expected cost is c_N. This does not drive our results in any way, but again facilitates computations. If we denote by $g(\theta)$ the difference between c_H and c_N for a given θ, we have that

$$c_H = c_N + g(\theta), \qquad c_L = c_N - f(\theta) \tag{1}$$

where we assume that

$$q_H \; g(\theta) - (1 - q_H) \; f(\theta) = 0 \tag{2}$$

We also take $g(1/2) = f(1/2) = 0$ and $\lim_{\theta \to 0} g(\theta) = \lim_{\theta \to 0} f(\theta) = \infty$. Thus, for any given θ, the expected cost is the same as the normal cost, and for $\theta = 1/2$ there is no risk associated with whether costs increase or decrease. This risk increases (in the sense that the variance of costs increases) as θ decreases. Finally we make the following technical assumption that we will use below

$$\lim_{\theta \to 0} \theta \; g(\theta) = \lim_{\theta \to 0} - \theta^2 \; g'(\theta) > 0. \tag{3}$$

Once x and θ are drawn, Nature determines ω and, consequently, whether $\omega \in \Theta$, and lastly, κ.

We assume that the buyer's valuations of the widget and the seller's costs are not contractible: the parties do not have the terminology to describe these values.[12] The parties can only contract on events consisting of (subsets of) elementary states ω.

We also assume that Θ constitutes an unforeseen contingency for both parties to the contract. In particular, we assume that both the buyer and the seller, although

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[12]Of course, following Maskin and Tirole (1999), the parties could devise a message contingent mechanism that induce them to report the truth on their costs and values to the court when the time comes to enforce the contract. See the discussion in this section and in Section 5 below for why, given the type of court we model, there is no loss in generality in ruling out these type of mechanisms.

aware of the possibility that Θ may occur, do not know its position x, and they do not know the value of θ that determines the probability that $\omega \in \Theta$ and, finally, do not possess the necessary vocabulary to describe the potential values of θ in their contract. They can only rely on the court to be protected against this uncertainty (if this is what the court finds optimal to do).

We assume that the size θ and whether $\omega \in \Theta$ to be *verifiable* information, in the sense that this information is observable to the court. Therefore the court can condition its decisions on this information. Conversely, the buyer and the seller observe the realization of ω and κ only at the time trade is to take place, while the court never observes the realization of κ.

To summarize, the parties face a risk at the time they contract that the cost and value of the widget will be abnormally high or low at the time production and delivery are to take place. This risk can be avoided by not contracting ex-ante, and simply contracting after the state is realized. So that there is a benefit to contracting ex-ante, we assume that the buyer can undertake an ex-ante, non-contractible, investment $e \in [0, 1]$ at a cost $\psi(e)$, where we assume that ψ is convex, $\psi'(0) = 0$ and $\lim_{e \to 1} \psi'(e) = +\infty$. A buyer's investment of e increases the value to him of the widget of an amount eR. Consequently, if the buyer chooses the level of relationship-specific investment e his value of the widget is $eR + \Delta + c_i$, where $i \in \{L, N, H\}$.

Since the buyer is risk-neutral, he maximizes expected profit, minus the convex cost of investment as above. The risk-averse seller maximizes the expected value of a strictly increasing and concave function $V(\cdot)$ such that $V'(\cdot) > 0$, $V''(\cdot) < 0$ and $V'''(\cdot) > 0$.[13] We normalize $V(\cdot)$ so that $V(0) = 0$.

The timing of the model can be specified as follows. First the court publicly announces the criteria that she will follow to settle a possible dispute. Nature then draws x, θ, and κ. Negotiation then takes place between the contracting parties. Recall that the buyer has all the bargaining power at this stage, hence negotiation is

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[13]We use $V'(\cdot)$, $V''(\cdot)$ and $V'''(\cdot)$ to denote respectively the first, second and third derivative of the seller's utility function $V(\cdot)$. Recall that $V'''(\cdot) > 0$ is a necessary condition for the degree of risk aversion to be decreasing in income.

a simple take-it-or-leave-it offer of a contract from the buyer to the seller. A contract may specify an ex-ante transfer; if it does, the transfer is made immediately after a contract is agreed upon. After the negotiation of an ex-ante contract the buyer chooses the level of specific investment e that increases the value of the widget to him by eR.

The state of nature ω is then realized and is observed by both parties to the contract. Moreover, if $\omega \in \Theta$ we also assume that the parties to a contract observe the exact value of the cost c_i, $i \in \{L, H\}$. The court on her part does not observe the realization of c_i, but only observes θ and whether $\omega \in \Theta$.

Each party to the contract now has the option to bring the other side to court. If this happens, the court is required to rule on the status of the contract. This ruling is based on all information available to the court: the contract itself, the value of θ and whether $\omega \in \Theta$. The ruling consists of the court's decision to *void* or to *enforce* the contract. In particular we do not allow the court to force the parties to trade at a given price determined by the court herself.[14]

In modeling the court's decision we do not allow the court to condition her ruling on the identity of the plaintiff — the party that initiates the court case — or utilize a message game in which the parties are required to report their information to the court, as in Maskin and Tirole (1999).[15] Notice, however, that provided that the court's ruling is limited to the choice of whether to enforce or void the contract and such ruling is taken only after both parties have reported all relevant information, we are not imposing any binding restriction. The court cannot infer the realized state from the identity of the plaintiff since the buyer and the seller ex-post are engaged in a zero-sum negotiation whatever the state of nature realized. Moreover, by ruling only after both parties have made their report on the realized state, subgame-perfect-implementation mechanisms cannot be used by the court to induce the parties to

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[14]The analysis of court rules that give the court more discretion than simply upholding or voiding contracts (for example by altering the price for the widget), while desirable, is beyond the scope of this paper.

[15]Again, we refer the reader to our discussion of this point in Section 5 below.

reveal the truth. On the other hand, the parties' negotiation does not satisfies Maskin-monotonicity, and therefore truth-telling cannot be implemented (Maskin 1999).

In the case in which the court decides to void the existing contract, renegotiation takes place between the buyer and the seller. Renegotiation is modeled as a take-it-or-leave-it offer from the seller to the buyer of a price at which to trade. When renegotiation occurs, following the court's decision to void the contract, the parties' outside options are represented by the payoffs associated with no trade. These payoffs are normalized to zero.

Finally, trade occurs according to the terms of the original contract, if the court decides to enforce it, or according to the terms of the renegotiated agreement, if the court decides to void the original ex-ante contract.

3.   The Optimal Ex-ante Contract

Given our assumptions above the parties to a contract can only specify in an ex-ante contract a constant price at which to trade p, and an ex-ante transfer from the buyer to the seller t. If the parties decide to draw up such an ex-ante contract, it is then left to the court to determine whether or not to protect them against the possibly very large risk associated with the unforeseen contingencies Θ.

We identify the optimal court's ruling solving the model backwards from the last stage. We begin with the renegotiation that follows the court's decision to void the contract. Denote $\hat{e}$ the given level of investment chosen by the buyer. Since the seller has all the bargaining power at the renegotiation stage he will receive all the gains from trade available to the parties. This implies that following renegotiation, the seller's utility is $V(\hat{e}\,R + \Delta)$.

Consider now the court's decision if one of the two parties brings the other to court. Recall that the court observes θ and whether $\omega \in \Theta$ and can condition her ruling only on this information. Therefore, without loss of generality, we can specify the court's decision rule to be such that if $\omega \in \Theta$ the court enforces all contracts when $\theta \in \mathcal{E}$ where $\mathcal{E} \subseteq [0, 1/2]$.[16] In other words when the size and probability of

the unforeseen contingency are within predetermined bounds, the court provides the parties with insurance by voiding the existing contract.

As we will see in Lemma 1 below, it will never be optimal for the court to void a contract if it observes $\omega \notin \Theta$. However, the general specification of the court's decision rule must allow for this possibility. If $\omega \notin \Theta$ the court enforces the contract only when $\omega \in \Gamma(\theta)$, where $\Gamma(\theta) \subseteq (\Omega \setminus \Theta)$.[17] In other words the court may void the parties' contract when $\omega \notin \Theta$. Moreover the set of ωs for which the court voids may, in principle, depend on the value of θ which the court observes. We will denote $\gamma(\theta)$ the measure of the set $\Gamma(\theta)$, so that obviously $\gamma(\theta) \leq 1 - \theta$.

The court determines and announces $\mathcal{E}$ and $\Gamma(\theta)$ prior to the parties' negotiation of the ex-ante contract. The parties take the court's decision rule as given when they decide which ex-ante contract to draw up.

Before we analyze the parties' negotiation of the ex-ante contract, we need to specify the seller's and buyer's outside options if the ex-ante negotiation breaks down. Notice that even in the absence of an ex-ante contract the parties can still trade the widget ex-post. As we mentioned above, in any ex-post negotiation the seller has all the bargaining power. Hence, in any ex-post agreement he appropriates all the gains from trade and receives utility $V(\bar{e}\,R + \Delta)$, where $\bar{e}$ is the level of specific investment chosen by the buyer in the absence of any ex-ante contract. The buyer receives a zero share of the gains from trade.

Notice that the advantage for the parties to trade ex-post is that they do not face any uncertainty, and therefore the seller is provided with full insurance. However, since the returns to the buyer from his ex-ante investment are zero, he will choose an investment level such that $\psi'(\bar{e}) = 0$. In other words, when trade takes place ex-post because there is no ex-ante contract the buyer has no incentive to invest: $\bar{e} = 0$. We can then conclude that, in the absence of an ex-ante contract the buyer's

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[16]Of course, $\mathcal{E}$ is assumed to be a Lebesgue-measurable set.

[17]Of course, $\Gamma(\theta)$ is also assumed to be a Lebesgue-measurable set, for every $\theta \in [0, 1/2]$. The reason for establishing different notation between the two sets $\mathcal{E}$ and $\Gamma(\theta)$ is simply that this helps to simplify some of our manipulations below.

payoff is 0 while the seller's level of utility is $V(\Delta)$. The seller is fully insured but no relationship-specific investment is undertaken by the buyer. The buyer's outside option when the ex-ante contract is negotiated is 0, while the seller's outside option is $V(\Delta)$.

Next, we turn to the parties' negotiation of the ex-ante contract. Recall that ex-ante the buyer makes a take-it-or-leave-it offer to the seller of a contract (p, t). Given the court's decision rule, and a level of investment $\hat{e}$, the seller's expected utility associated with (p, t) can now be written as follows.

$$
\begin{aligned}
V^*(p, t) = \ & \int_{\mathcal{E}} \theta \left[q_H V(p + t - c_H) + (1 - q_H) V(p + t - c_L) \right] \, 2 \, d\theta + \\
& + \int_{\left([0, \frac{1}{2}] \setminus \mathcal{E} \right)} \theta V(\hat{e}R + \Delta + t) \, 2 \, d\theta + \\
& + \int_0^{\frac{1}{2}} \left[\gamma(\theta) \, V(p + t - c_N) + (1 - \theta - \gamma(\theta)) \, V(\hat{e}R + \Delta + t) \right] \, 2 \, d\theta.
\end{aligned}
\tag{4}
$$

Notice that the first integral in (4) refers to the case in which $\omega \in \Theta$ and the contract is upheld by the court. The second integral in (4) captures those cases in which $\omega \in \Theta$ and the court voids the ex-ante contract, while the last integral in (4) covers the cases in which $\omega \notin \Theta$, both when the contract is voided and when it is upheld.

Using (2) above and taking again as given the court's decision rule and a level of investment $\hat{e}$, the buyer's expected profit associated with (p, t) can be computed as follows.

$$
\begin{aligned}
B^*(p, t) = \ & \int_0^{\frac{1}{2}} \gamma(\theta)[\hat{e}R + \Delta + c_N - p] \, 2 \, d\theta + \\
& + \int_{\mathcal{E}} \theta[\hat{e}R + \Delta + c_N - p] \, 2 \, d\theta - t - \psi(\hat{e})
\end{aligned}
\tag{5}
$$

Notice that if we set $\bar{\gamma} = \int_0^{\frac{1}{2}} \gamma(\theta) \, 2 \, d\theta$ and $\bar{\theta}_{\mathcal{E}} = \int_{\mathcal{E}} \theta \, 2 \, d\theta$, definition (5) can be rewritten more simply as

$$
B^*(p, t) = \left(\bar{\gamma} + \bar{\theta}_{\mathcal{E}} \right) [\hat{e}R + \Delta + c_N - p] - t - \psi(\hat{e})
\tag{6}
$$

and from (6) it is immediate that given (p, t) and the court's decision rule the buyer will select a level of relationship-specific investment $\hat{e}$ such that

$$\psi'(\hat{e}) = \left(\bar{\gamma} + \bar{\theta}_{\mathcal{E}}\right) R \tag{7}$$

We can now state the buyer's optimization problem for choosing an ex-ante contract. Given the court's decision rule, the buyer's take-it-or-leave-it offer to the seller is the solution, if it exists, to the following problem.

$$
\begin{aligned}
\max_{p,t} \quad & \left(\bar{\gamma} + \bar{\theta}_{\mathcal{E}}\right) \left[\hat{e}R + \Delta + c_N - p\right] - t - \psi(\hat{e}) \\
s.t. \quad & V^*(p, t) \geq V(\Delta) \\
& B^*(p, t) \geq 0 \\
& \psi'(\hat{e}) = \left(\bar{\gamma} + \bar{\theta}_{\mathcal{E}}\right) R
\end{aligned}
\tag{8}
$$

where the first two constraints guarantee that it is optimal for both the seller and the buyer to sign an ex-ante contract rather than to trade ex-post. If the feasible set of Problem (8) is in fact empty, then no ex-ante contract will be signed and trade will take place ex-post. However, when the court's decision rule is chosen so as to maximize the parties' welfare an ex-ante contract will be signed.

Remark 1. *For some specifications of the court's decision rule the feasible set of Problem (8) is clearly not empty.*

For example, suppose the court never voids the contract if $\omega \notin \Theta$, that is, $\Gamma(\theta) = (\Omega \setminus \Theta)$ for every $\theta \in [0, 1/2]$. Suppose further that the court always voids the contract if $\omega \in \Theta$ so that $\mathcal{E} = \emptyset$. In this case, the agents do not face any uninsurable risk from unforeseen contingencies, and can take advantage of a fixed price for the case $\omega \notin \Theta$ so that the buyer will undertake a positive amount of relationship-specific investment. It is clear that in this case there is an ex-ante contract that is preferred to no contract by both the buyer and seller.

Finally, notice that when an ex-ante contract is preferred to trading ex-post it is immediate by standard arguments that the solution to Problem (8) is in fact unique.

Assume that the court's decision rule is such that it is optimal for the parties to draw up an ex-ante contract (in other words assume that the court's decision rule is such that the opportunity set of Problem (8) is not empty). Let $\bar{B}$ be the expected profit of the buyer that the solution to Problem (8) yields. Then it is obviously the case that the optimal ex-ante contract is also the solution to the following problem.

$$
\begin{aligned}
\max_{p,t} \quad & V^*(p,t) \\
s.t. \quad & V^*(p,t) \geq V(\Delta) \\
& B^*(p,t) \geq \bar{B} \\
& \psi'(\hat{e}) = (\bar{\gamma} + \bar{\theta}_{\mathcal{E}})\, R
\end{aligned}
\tag{9}
$$

Notice that if the court's decision rule is such that $(\bar{\gamma} + \bar{\theta}_{\mathcal{E}}) = 0$ we obtain a trivial special case. When $(\bar{\gamma} + \bar{\theta}_{\mathcal{E}}) = 0$ the court voids the contract for every ω. Therefore, in this case the expected profit of the buyer is 0 and the expected utility of the seller is $V(\Delta)$, whatever the contract (p,t). In this case, since both parties are indifferent, we assume that they prefer to implement the same outcome by having no contract at all.

Our characterization of the optimal contract (p^*, t^*) given the court's decision rule can now be summarized as follows.

Proposition 1. *Let a decision rule for the court be given, and assume that it is such that it is optimal for the parties to draw up an ex-ante contract. Let the optimal ex-ante contract be denoted by (p^*, t^*). Then the price p^* satisfies*

$$
p^* - c_N \geq \hat{e}\, R + \Delta
\tag{10}
$$

while the transfer t^* is such that

$$
V^*(p^*, t^*) = V(\Delta)
\tag{11}
$$

The fact that (11) of Proposition 1 must hold is a simple consequence of the fact

that the seller's expected utility is increasing in t, while the buyer's expected profit is an decreasing function of t.

The intuition behind (10) of Proposition 1 is not hard to explain. Recall that the uninsurable risk embodied in the variation between c_L and c_H has mean zero. Moreover, in those states in which the contract is renegotiated, the seller necessarily gets a pay-off (on top of the transfer t) of $\hat{e} R + \Delta$. The price p^* is chosen so as to provide the seller with the optimal *partial* insurance against the fluctuations of cost between c_L and c_H. Of course, this means equating the seller's expected marginal utility in these two eventualities with the seller's marginal utility that he achieves when the contract is voided by the court. Since the seller's marginal utility is a convex function this implies that the price p^* minus the average of c_L and c_H must be above $\hat{e} R + \Delta$.

4.   The Court's Optimal Decision Rule

We are now equipped with the characterization (Proposition 1 above) of the optimal contract (p^*, t^*) given an arbitrary decision rule for the court. This is enough to proceed to characterize the court's optimal decision rule.

Recall that our court is a Stackelberg leader. She announces her decision rule, taking into account the parties' behavior given this rule, so as to maximize their welfare. From Proposition 1 we know that, as a result of the fact that the buyer makes a take-it-or-leave-it offer of an ex-ante contract to the seller, the seller's expected utility will be $V(\Delta)$, regardless of the court's decision rule. Therefore, the court's decision rule can be characterized as the solution to the problem of maximizing the buyer's expected profit subject to appropriate constraints.

The court's maximization problem can be written as follows. Choose a Lebesgue-measurable $\mathcal{E}$, and a (measurable) collection of Lebesgue-measurable sets $\Gamma(\theta)$ as θ

varies in $[0, 1/2]$ so as to solve

$$
\begin{aligned}
\max_{\Gamma(\theta), \mathcal{E}} \quad & \left(\bar{\gamma} + \bar{\theta}_{\mathcal{E}}\right) \left[\hat{e}R + \Delta + c_N - p\right] - t - \psi(\hat{e}) \\
s.t. \quad & V^*(p^*, t^*) \geq V(\Delta) \\
& B^*(p^*, t^*) \geq 0 \\
& \psi'(\hat{e}) = \left(\bar{\gamma} + \bar{\theta}_{\mathcal{E}}\right) R
\end{aligned}
\tag{12}
$$

where (p^*, t^*) is the optimal ex-ante contract characterized in Proposition 1 above.

We begin with two partial characterizations of the court's optimal decision rule. Our first claim asserts that, provided a solution to Problem (12) exists, it will be such that the court never voids the parties' ex-ante contract when $\omega \notin \Theta$. In other words, it is never optimal for the court to void the contract if the contingency that occurs in not an unforeseen one.

Lemma 1. *It is optimal for the court to enforce the contract whenever $\omega \notin \Theta$. More formally, assume that a solution to Problem (12) exists. Then any solution to this problem will satisfy*

$$
\gamma(\theta) = 1 - \theta.
$$

for every $\theta \in [0, 1/2]$, up to a set of θs of Lebesgue-measure zero.

The intuition behind Lemma 1 is very simple to outline. The court's decision to void the contract provides the parties with insurance against unforeseen contingencies. If a contingency $\omega \notin \Theta$ that is not unforeseen arises, it is optimal for the court to enhance the buyer's incentives to undertake the relationship-specific investment by enforcing the ex-ante contract.

We now turn to a further partial characterization of the court's optimal decision rule. We are concerned with the "shape" of the court's optimal decision rule for those ω that are in Θ. We first show that this part of the court's optimal decision rule consists of a *threshold* level θ^*. The court will void the ex-ante contract when $\theta < \theta^*$ is observed, and will uphold the ex-ante contract otherwise.

Lemma 2. *Assume that a solution to Problem (12) exists. Then any solution to this problem has the feature that there exists a $\theta^* \in [0, 1/2]$ such that the court will enforce the ex-ante contract if $\omega \in \Theta$ and $\theta \geq \theta^*$ and will void it if $\theta < \theta^*$. That is, $\mathcal{E} = [\theta^*, 1/2]$, up to a set of θs of Lebesgue-measure zero.*

The intuition behind this second partial characterization of the optimal court decision rule can be described as follows. The court is trading off the insurance she provides to the parties when she voids the contract with the decrease in incentives to invest that results from voiding. Incentives are adversely affected because when the court voids, at the margin, the buyer will not receive a full return from his investment. Hence, the higher the probability that the court voids, the lower is his incentive to invest. This negative effect on investment depends only on the probability that the court will void the contract. On the other hand, the value of the insurance to the parties from voiding is greater when θ is smaller, since, by assumption, the spread between c_L and c_H becomes higher as θ becomes smaller. Hence, whatever decrease in incentives is accepted, the optimal thing for the court to do is to void for the smallest values θs. In other words, whatever the overall probability that the court voids the ex-ante contract, the set of values of θ for which the contract is in fact voided must take the "threshold" form described in Lemma 2.

We have now all the elements to complete the characterization of the court's optimal decision rule.

According to Lemma 1 the optimal court's policy is to enforce every contract if $\omega \notin \Theta$, while, according to Lemma 2, if an unforeseen contingency occurs so that $\omega \in \Theta$ it is optimal for the court to use a threshold θ^* and void the contract if $\theta < \theta^*$ and uphold it if $\theta \geq \theta^*$. We summarize our characterization of the court's optimal decision rule in Proposition 2 below. Aside from incorporating the content of Lemmas 1 and 2, Proposition 2 asserts that an optimal decision rule for the court does in fact exist, that it is unique up to a set of θs of Lebesgue measure zero, and that the threshold θ^* used by the court is interior in the sense that $0 < \theta^* < 1/2$.

Proposition 2. *An optimal decision rule for the court exists and it is unique up to a set of θs of Lebesgue-measure zero.*

The court's unique optimal decision rule can be described as follows. The court upholds the ex-ante contract if a contingency $\omega \notin \Theta$ occurs. On the other hand, if an unforeseen contingency arises, $\omega \in \Theta$, the court voids the contract if $\theta < \theta^*$ and upholds the contract if $\theta \geq \theta^*$, where θ^* is a threshold value in the open interval $(0, 1/2)$.

We have already outlined the intuition behind part of the characterization of the court's optimal decision rule presented in Proposition 2. To understand why the threshold θ^* used by the court cannot be either 0 or $1/2$ it is enough to refer back to the specification of the risk that the unforeseen contingencies entails that we described in Section 2 above. Recall that, essentially, as θ approaches $1/2$ the unforeseen contingencies entail a negligible amount of risk for the contracting parties. As θ approaches $1/2$, both c_L and c_H approach c_N. Therefore, as θ becomes larger the benefits of voiding the ex-ante contract shrink to zero as the value of the insurance that voiding provides shrinks to zero. On the other hand, the costs of voiding the ex-ante contract do not vanish as θ^* approaches $1/2$. Indeed, the marginal cost (in terms of diminished incentives for the buyer to undertake relationship-specific investment) of increasing θ^* (the threshold for voiding the ex-ante contract) increases with the size of θ^*. Therefore it will be optimal to enforce the ex-ante contract when a $\omega \in \Theta$ and a θ sufficiently close to $1/2$ is observed.

Consider now the nature of the risk associated with the unforeseen contingencies for small θ, approaching 0. As we specified in Section 2 above, the difference between c_L and c_H becomes unboundedly large as θ approaches zero. Therefore, in this case the tradeoff between incentives and insurance is exactly reversed from the case of θ approaching $1/2$ that we have just described. As θ approaches 0, the gain in incentives from upholding the ex-ante contract is bounded above (it can never exceed R) while upholding the ex-ante contract becomes more and more costly as the parties are faced with an ever increasing amount of uninsurable risk.

We conclude this section with an observation that is a direct consequence of Proposition 2. Recall that it is always possible for the court to announce a decision rule that will make both contracting parties indifferent between drawing up an ex-ante contract and having no contract at all. This is the trivial case in which the court' decision rule is to void the ex-ante contract for every state $\omega \in \Omega$. Since the court's optimal decision rule characterized in Proposition 2 is *not* to void the ex-ante contract for every possible $\omega \in \Omega$ it immediately follows that an ex-ante contract is strictly preferable to having no contract at all. Indeed, the buyer's expected profit, given the court's optimal decision rule, is strictly above 0 (the expected profit of the buyer when the parties do not draw up an ex-ante contract at all).

5. Concluding Remarks

Our results depend on what courts of law are allowed to do. Our qualitative conclusions, for example, that the particular rules the court adopts affect the nature and efficiency of the contracts that are written, would hold for a wide variety of court rules. However, as we pointed out in the introduction, our results do depend on ruling out the kinds of implementation devices in Maskin and Tirole (1999), and hence, it is useful to explain our exclusion of them.

It is perhaps easiest first to describe the game forms in Maskin and Tirole (1999) by demonstrating one for a simple buyer-seller problem. We suppress any investment decisions to illustrate how a Maskin-Tirole game elicits truthful revelation of the buyer and seller's valuations at the time trade is to take place. The buyer and seller are both risk averse. The buyer's valuation is 0 independent of circumstances. The value of the object to the buyer would normally be 6, but both parties understand that under some contingencies, which the parties can foresee but not describe, the value would only be 4. Suppose that the parties want to split the gains from trade, that is, sell the object at price 3 in normal circumstances when the buyer's value is 6, but reduce the price to 2 in the event that the value to the buyer is 4.

Under a Maskin-Tirole scheme, when contracting takes place, the parties contract over the price as a function of the value to buyer, on the assumption that the value

can be truthfully elicited at the time production is to take place. At the time production is to take place, the parties will have observed any events that were ex-ante indescribable, and both the buyer and the seller will know the buyer's value. The buyer is then asked to report his value, and the seller can "challenge" the buyer on his announced value should he announce 4. In the event of a challenge, the buyer is immediately assessed a penalty of 10.

Following this, the buyer is offered the object at a price of 5.[18] If the buyer accepts the object, this is taken to be evidence that the seller was correct in his challenge, and the seller receives a large reward. In the event that the buyer rejects the object, this is taken to be evidence that the buyer's challenge was invalid, and the seller is assessed a large penalty.[19]

The unique subgame perfect equilibrium of this game is truthful announcement. Following an untruthful announcement of 4 by the buyer, the seller has an incentive to challenge, since subgame perfection assures that correct challenges will be confirmed by the buyer's ultimate acceptance of the object when it is offered at a price of 5. Since an untruthful announcement of 4 will be challenged, a buyer with valuation of 6 will reveal that value. Consequently, since the parties can rely on a Maskin-Tirole game to assure truthful revelation of values at the time trade is to take place, at the ex-ante stage before any contingencies have been realized, they can contract contingent on the to-be-revealed-later values.

This result is very interesting, in that it forces a more serious look at the foundations of incomplete contracting. However, we clearly do not see parties contracting in this way, and we will discuss some possible reasons.

One reason has to do with the possibility of renegotiation. Maskin and Tirole (1999) contains an extended discussion of the difficulties that the possibility of renegotiation presents.[20] Consider, for example, the consequences in the above example

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[18]Any number between 4 and 6 would do equally well.

[19]If the seller's cost of production could take on more than a single value, an analogous procedure would be undertaken to elicit his cost.

[20]Both Segal (1999) and Segal and Whinston (2001) are concerned with the impact of renegotiation on the outcome of possible message games in a contracting problem.

if a buyer who has value 6 announces 4, and the announcement is challenged by the seller. The seller does this in the expectation that he will be rewarded when the buyer subsequently accepts the object when it is offered to him at price 5. But what if the buyer, at the time he announces his value is 4, also announces that he will reject the object when it is subsequently offered at price of 5? This is a non-credible threat within the formal Maskin-Tirole game, and hence has no bearing on a subgame perfect equilibrium. However, it should be noted that if the buyer *were* to reject the object, the game ends with the seller paying a large penalty for challenging, *and* the unrealized gains from trade between the buyer and seller. If the buyer were to offer to purchase the object at a price of 5 from the seller at this "post-game" stage, it is in the interest of the seller to accede to the request. But if the post-game offer is accepted, it is perfectly rational for the buyer to refuse the object when it is offered in the formal Maskin-Tirole game as a means of verifying the validity of the seller's challenge.

In summary, truthful revelation is the unique subgame perfect equilibrium of the Maskin-Tirole game. There will, however, be unrealized gains from trade that remain at the (disequilibrium) node following a valid challenge and the subsequent rejection of the object. The plausibility of a Maskin-Tirole game rests on the assumption that it is possible to prevent those gains from being realized in the future. Maskin and Tirole (1999) have a long discussion of how parties might prevent the gains from being realized. One suggestion is that if either party can present evidence to the court of such a subsequent transaction (or an equivalent set of transactions with third parties), he is entitled to collect a large penalty from the other party. In order to accomplish this, there must be clear and indisputable evidence that the transaction has taken place, and that the transaction was intended to thwart the parties' initial contractual intentions. There would seem to be huge difficulties in determining whether any single future sale of an object by the seller to any third party might be motivated by an interest in thwarting the initial contract. For example, the buyer might conspire with a third party to buy an object from the seller, purchase the object from the third party, and take evidence of the transactions to court, claiming the penalty from the

seller.

Maskin and Tirole (1999) suggest that, alternatively, the contract could ban *all* subsequent transactions with third parties in the event that there are any challenges. This, of course, leaves the parties open to a new type of hold-up problem in which one or the other of the parties can threaten the other party to choose a disequilibrium action.

The important point of this discussion for our paper is only that there are a number of unresolved issues about the degree to which Maskin-Tirole games ameliorate contractual problems arising from indescribable contingencies. As is clear from the discussion in Maskin and Tirole (1999), the answer depends on the "technology" of contracting. What precisely can be ascertained by a court as to the transactions parties engage in, and the motivations underlying those transactions? What strategies can the parties invent to take advantage of, or circumvent, the monitoring available to the court or other third parties? Whether or not a more detailed Maskin-Tirole game could be constructed taking account of "fine details" of the contracting environment requires a model with a careful specification of the technology of contracting. It is premature to presume that the indescribability of contingencies is irrelevant in the absence of such a model. We have taken the conservative approach of constructing a "positive" model that specifies precisely the court's capabilities in a way that approximates those seen in practice.

The advantages of fully specifying the court as a player in the model are apparent. The choices available to the model-builders become clear-cut. A set of available strategies and a set of preferences must be specified for each actor in the model. This includes both the contracting parties and the court. Once this is specified, we have a fully specified game, which can be analyzed in familiar ways. In this sense, once the court is specified as a player, the debate about the appropriateness and plausibility of Maskin-Tirole games becomes a debate about plausible modeling choices for the strategy space and the preferences of the court.[21]

---

[21] It should be noted that Maskin and Tirole (1999) do *not* interpret their results as yielding a blueprint for the design of a court. They simply show the existence of message games that implement

In this paper we have taken a particularly simple specification of the court's strategy set and of its preferences. Let us briefly discuss both these issues and how they relate to our analysis above.

There is a sense in which *any* restrictions (except for strictly physical ones) on the court's strategy set take us back into a "partial equilibrium" approach. If there are restrictions on the court's strategy set, who put them there if the model is truly a closed one? This paper is but one step in the direction of a model that is truly closed in this sense.[22]

Once we take the view that *some* external considerations must be taken as given, it is easy to see why our modeling choice of a "simple" strategy set for the court is plausible. Courts typically face a large pool of possible disputes, and have very little prior specific knowledge about each case. It is clearly efficient to develop court procedures that are "detail free" wherever possible in the sense of being robust to even large variations in the parameters characterizing the situations to which they apply. Our courts that can only void or uphold contracts rather than dictate new terms of trade are a simple way to capture some of these considerations.

The restricted strategy space for the court that we have worked with in this paper can also be interpreted as a crude way to model the effects a richer domain for the preferences of the court. In particular it is clear that in a dynamic world courts must care about the *reputation* they accumulate about their rulings. In our model the court *announces* the rule that it will follow in case of a dispute. In a richer dynamic model this would be substituted by the reputation that the court has. At this point the rationale for simple behavior becomes, again, apparent. In practice, simple rules will have greater "penetration" as the reputation of the court among the pool of (possibly

---

the first-best outcome. These are interpreted as game forms that the parties themselves design and play. The role of the court in their interpretation is the standard one of a "minimal" court that only enforces the "rules of the game".

[22]In a different context — the design of a "legal system" for society as a whole — Mailath, Morris, and Postlewaite (2000) explore a model in which all "laws" are cheap-talk. They find that the role of the legal system in this case is limited to selecting among the multiple equilibria of the game determined by the physical description of the environment. See also the discussion in Schwartz and Watson (2000).

simple-minded) contracting parties who might take their disputes before the court.

Appendix

Proof of Lemma 1: Consider the first order conditions associated with Problem (9). After elementary manipulations we obtain that the following must hold.

$$
\bar{\gamma}\, V'(p^* + t^* - c_N) + \int_{\mathcal{E}} \theta \left[q_H V'(p^* + t^* - c_H) + (1 - q_H)V'(p^* + t^* - c_L) \right]\, 2\, d\theta =
$$
$$
= \; (\bar{\gamma} + \bar{\theta}_{\mathcal{E}})\, V'(\hat{e}R + \Delta + t^*) \tag{A.1}
$$

The convexity of the seller's marginal utility, $V'''(\cdot) > 0$, and (2) above imply that

$$
q_H V'(p^* + t^* - c_H) + (1 - q_H)V'(p^* + t^* - c_L) > V'(p^* + t^* - c_N). \tag{A.2}
$$

Substituting (A.2) into (A.1) yields

$$
V'(\hat{e}R + \Delta + t^*) > V'(p^* + t^* - c_N)
$$

which together with the fact that $V''(\cdot) < 0$ implies (10).

Finally, the fact that (11) must hold follows from the fact that the seller's expected utility $V^*(p, t)$ is monotonic increasing in t while the buyer's expected surplus $B^*(p, t)$ is monotonic decreasing in t. ∎

Proof of Lemma 1: We proceed by contradiction. Suppose that the court's optimal decision rule is such that a for a set of θs of positive measure, we have that $\gamma(\theta) < 1 - \theta$. As before, let $\bar{\gamma} = \int_0^{\frac{1}{2}} \gamma(\theta) 2 d\theta$. We can now show that there exists another court decision rule, which prescribes the same $\mathcal{E}$, and a collection of sets $\Gamma'(\theta)$ with an associated $\gamma'(\theta)$ such that $\bar{\gamma}' = \int_0^{\frac{1}{2}} \gamma'(\theta) 2 d\theta$ and $\bar{\gamma}' > \bar{\gamma}$. Further we can show that if the court switches to this new policy, assuming the ex-ante contract does not change, the payoffs of both parties increase. Of course this will imply that after the contract is adjusted in response to the new court decision rule the expected profit of the buyer must increase even further. This represents a violation of our contradiction hypothesis and clearly proves our claim.

Let the investment level $\hat{e}$ be given and consider the seller's expected utility, as in (4) above, when the contract is (p^*, t^*) as in Lemma 1. The terms associated with the states $\omega \notin \Theta$ are

$$
\bar{\gamma}\, V(p^* + t^* - c_N) + (1 - \bar{\theta} - \bar{\gamma})\, V(\hat{e}R + \Delta + t^*) \tag{A.3}
$$

Recall now that from Lemma 1 we know that $p^* + t^* - c_N > \hat{e}R + \Delta + t^*$. Therefore the quantity in (A.3) is clearly an increasing function of $\bar{\gamma}$. Therefore, for a given $\hat{e}$ the expected utility of the seller increases as the court's decision rule switches from $\Gamma(\theta)$ to $\Gamma'(\theta)$.

Consider next the buyer expected profit, as in (5), when the contract is (p^*, t^*) as in Lemma 1 and the investment level $\hat{e}$ is given. Again, clearly this is increasing in $\bar{\gamma}$. Therefore, for a given $\hat{e}$ the expected profit of the buyer increases as the court' decision rule switches from $\Gamma(\theta)$ to $\Gamma'(\theta)$. Since the level of investment is chosen optimally by the buyer (see (7) above), this is already sufficient to conclude that the buyer's expected profit must increase after the switch in the court's decision rule.

Recall now, from (7) above, that the buyer's investment level is also increasing in $\bar{\gamma}$.

Since the seller's expected utility is increasing in the investment choice $\hat{e}$ we can now conclude that switching the court's decision rule from $\Gamma(\theta)$ to $\Gamma'(\theta)$ increases the seller's expected utility after the corresponding change in the investment level has taken place. ∎

Proof of Lemma 2: We proceed by contradiction. Assume there is a solution to the court's Problem (12) with the following features. The set $\mathcal{E}$ has positive Lebesgue measure and there does not exists a $\theta^* \in [0, 1/2]$ such that $\mathcal{E} = [\theta^*, 1/2]$ up to a set of θs of Lebesgue-measure zero. We will show that in this case there exists a new decision rule for the court that uses a set $\mathcal{E}'$ such that a θ^* as above exists and such that, for a given contract (p^*, t^*) as in Lemma 1, the buyer's expected profit is unchanged while the expected utility of the seller has increased. Therefore if we switch the court's decision rule from $\mathcal{E}$ to $\mathcal{E}'$ the expected profit of the buyer after the change in the optimal contract has been taken into account must increase. This will clearly suffice to prove our claim.

Let the contract (p^*, t^*) be given and let θ' be such that

$$\bar{\theta}_{\mathcal{E}'} = \int_{\theta'}^{\frac{1}{2}} \theta \, 2 \, d\theta = \int_{\mathcal{E}} \theta \, 2 \, d\theta = \bar{\theta}_{\mathcal{E}} \tag{A.4}$$

The new decision rule for the court is simply defined as $\mathcal{E}' = [\theta', 1/2]$.

Notice next that using (A.4) and (7) it is immediate that the buyer's choice of investment is unchanged after the change in the court's decision rule. Therefore, since we are keeping the contract constant at (p^*, t^*), the buyer's expected profit does not change after the change in the court's decision rule.

Consider now the seller expected utility associated with the contract (p^*, t^*) and the court's policy $\mathcal{E}$. Using (4) and Lemma 1, without loss of generality we can write this as

$$\int_{\mathcal{E}} \theta \left[q_H V(p^* + t^* - c_N - g(\theta)) + (1 - q_H)V(p^* + t^* - c_N + f(\theta)) \right] 2 \, d\theta \; +$$
$$+ \; (\bar{\theta} - \bar{\theta}_{\mathcal{E}}) \; V(\hat{e}R + \Delta + t^*) + (1 - \bar{\theta}) \, V(p^* + t^* - c_N). \tag{A.5}$$

Notice next that, using (A.4) above we have that the second and third term of the seller's expected utility do not change if we change the court's decision rule from $\mathcal{E}$ to $\mathcal{E}'$. Therefore, we focus on the first term of the seller's expected utility and we let

$$\mathcal{G}(\theta) = \left[q_H V(p^* + t^* - c_N - g(\theta)) + (1 - q_H)V(p^* + t^* - c_N + f(\theta))\right]. \tag{A.6}$$

Now define the following three sets of values of θ

$$\begin{aligned}
\mathcal{S}_1 &= \{\mathcal{E} \cap \mathcal{E}'\} \\
\mathcal{S}_2 &= \{\mathcal{E} \cap ([0, 1/2] \setminus \mathcal{E}')\} \\
\mathcal{S}_3 &= \{([0, 1/2] \setminus \mathcal{E}) \cap \mathcal{E}'\}
\end{aligned} \tag{A.7}$$

We can then write the first term of (A.5) as:

$$\int_{\mathcal{E}} \theta\, \mathcal{G}(\theta) d\theta = \int_{\mathcal{S}_1} \theta\, \mathcal{G}(\theta) d\theta + \int_{\mathcal{S}_2} \theta\, \mathcal{G}(\theta) d\theta. \tag{A.8}$$

If we switch the court's decision rule from $\mathcal{E}$ to $\mathcal{E}'$ the corresponding term becomes

$$\int_{\mathcal{E}'} \theta\, \mathcal{G}(\theta) d\theta = \int_{\mathcal{S}_1} \theta\, \mathcal{G}(\theta) d\theta + \int_{\mathcal{S}_3} \theta\, \mathcal{G}(\theta) d\theta. \tag{A.9}$$

Obviously, the quantity in (A.9) is strictly larger than the quantity in (A.8) if and only if

$$\int_{\mathcal{S}_3} \theta\, \mathcal{G}(\theta) d\theta > \int_{\mathcal{S}_2} \theta\, \mathcal{G}(\theta) d\theta. \tag{A.10}$$

Notice now that the function $\mathcal{G}(\theta)$ is monotonic increasing in θ. Indeed, using (2), $g'(\theta) < 0$ and $V''(\cdot) < 0$ we obtain:

$$\frac{d\mathcal{G}(\theta)}{d\theta} = -q_H\, g'(\theta)\, [V'(p^* + t^* - c_N - g(\theta)) - V'(p^* + t^* - c_N + f(\theta))] > 0. \tag{A.11}$$

We can now change the variable in the integrals in (A.10) using $y = \theta^2$ and denote

$$\begin{aligned}
Y_2 &= \{y \mid y = \theta^2, \theta \in \mathcal{S}_1\} \\
Y_3 &= \{y \mid y = \theta^2, \theta \in \mathcal{S}_2\}
\end{aligned} \tag{A.12}$$

Therefore the inequality in (A.10) is satisfied if and only if

$$\int_{Y_3} \mathcal{G}(\sqrt{y})\, dy > \int_{Y_2} \mathcal{G}(\sqrt{y})\, dy. \tag{A.13}$$

Notice next that using (A.4) we get that

$$\int_{Y_3} dy = \int_{Y_2} dy \tag{A.14}$$

Finally, recall that using (A.7) and (A.12) we know that for every $y_2 \in Y_2$ and every $y_3 \in Y_3$ we must have that $y_3 \geq y_2$. Therefore, using the fact that $\mathcal{G}(\sqrt{y})$ is an increasing function of y and (A.14), we can now deduce that (A.13) must hold, and therefore that (A.10) must hold as well.

Therefore, for a given contract (p^*, t^*), the seller's expected utility increases if the court changes her decision rule from $\mathcal{E}$ to $\mathcal{E}'$. Since we have already shown that this change leaves the expected profit of the seller unchanged, this is enough to prove our claim. ∎

Proof of Proposition 2: Using Lemmas 2 and 1, we know that an optimal decision rule for the court exists, then, up to a set of θs of Lebesgue-measure zero it is a solution to the following problem.

$$
\begin{aligned}
\max_{\theta^*} \quad & \left[1 - (\theta^*)^2\right] \left[\hat{e}R + \Delta + c_N - p^*\right] - t^* - \psi(\hat{e}) \\
s.t. \quad & \int_{\theta^*}^{\frac{1}{2}} \theta \left[q_H V(p^* + t^* - c_H) + (1 - q_H)V(p^* + t^* - c_L)\right] 2 \, d\theta \; + \\
& \quad + \; (\theta^*)^2 \, V(\hat{e}R + \Delta + t^*) + (1 - \bar{\theta}) \, V(p^* + t^* - c_N) \geq V(\Delta) \\
& \psi'(\hat{e}) = \left[1 - (\theta^*)^2\right] R
\end{aligned}
\tag{A.15}
$$

Since a solution to Problem (A.15) exists unique by standard arguments, this observation is sufficient to prove our existence and uniqueness claim.

Therefore, there only remains to show that the solution to Problem (A.15) has $\theta^* \in (0, 1/2)$. Our strategy is to show that the solution to the following problem has $\theta^* \in (0, 1/2)$, which will obviously suffice to prove our claim.

$$
\begin{aligned}
\max_{\theta^*} \quad & \int_{\theta^*}^{\frac{1}{2}} \theta \left[q_H V(p^* + t^* - c_H)) + (1 - q_H)V(p^* + t^* - c_L)\right] 2 \, d\theta \; + \\
& \quad + \; (\theta^*)^2 \, V(\hat{e}R + \Delta + t^*) + (1 - \bar{\theta}) \, V(p^* + t^* - c_N) \\
s.t. \quad & \left[1 - (\theta^*)^2\right] \left[\hat{e}R + \Delta + c_N - p^*\right] - t^* - \psi(\hat{e}) \geq \bar{B} \\
& \psi'(\hat{e}) = \left[1 - (\theta^*)^2\right] R
\end{aligned}
\tag{A.16}
$$

where $\bar{B}$ is the buyer's expected profit associated with the solution of Problem (A.15).

Consider the Lagrangean of Problem (A.16) above

$$\mathcal{L}(\theta^*, \lambda) = \int_{\theta^*}^{\frac{1}{2}} \theta \left[q_H V(p^* + t^* - c_N - g(\theta))) + (1 - q_H)V(p^* + t^* - c_N + f(\theta)) \right] 2d\theta +$$
$$+ (\theta^*)^2 V(\hat{e}R + \Delta + t^*) + (1 - \bar{\theta}) V(p^* + t^* - c_N) +$$
$$+ \lambda \left[\left(1 - (\theta^*)^2\right) (\hat{e}R + \Delta + c_N - p^*) - t^* - \psi(\hat{e}) - \bar{B} \right]$$

Therefore, the first order condition with respect to θ^* of Problem (A.16) can be written as (taking into account the dependence of $\hat{e}$ on θ^* implied by the last constraint in Problem (A.16)):

$$\frac{\partial \mathcal{L}(\theta^*, \lambda)}{\partial \theta^*} = 2 \theta^* V(\hat{e}R + \Delta + t^*) - 2 \frac{(\theta^*)^3 R^2}{\psi''(\hat{e})} V'(\hat{e}R + \Delta + t^*) -$$
$$- 2 \theta^* \left[q_H V(p^* + t^* - c_N - g(\theta))) + (1 - q_H)V(p^* + t^* - c_N + f(\theta)) \right] - \qquad \text{(A.17)}$$
$$- \lambda 2 \theta^* (\hat{e}R + \Delta + c_N - p^*) = 0$$

Consider now the value of the left-hand side of (A.17) when $\theta^* = 1/2$. Recall that when $\theta^* = 1/2$ no risk is associated with the unforeseen contingencies contract under the contract (p^*, t^*). This is because $g(1/2) = f(1/2) = 0$. Using the same method as in the proof of Lemma 1 we then immediately have that for $\theta^* = 1/2$ it must be the case that $p^* - c_N = \hat{e}R + \Delta$. Therefore we obtain that

$$\frac{\partial \mathcal{L}(1/2, \lambda)}{\partial \theta^*} = - \frac{R^2}{4 \psi''(\hat{e})} V'(\hat{e}R + \Delta + t^*) < 0$$

which directly implies that in the solution to Problem (A.16) it is impossible that $\theta^* = 1/2$.

Consider now the limit of the left-hand side of (A.17) as θ^* approaches zero. Using the function $\mathcal{G}(\cdot)$ defined in (A.6) of the proof of Lemma 2, we can write this limit as

$$\lim_{\theta^* \to 0} \frac{\partial \mathcal{L}(\theta^*, \lambda)}{\partial \theta^*} = \lim_{\theta^* \to 0} -2 \theta^* \mathcal{G}(\theta^*)$$

Notice that, using (A.6) again, as θ^* approaches zero, the value of $\mathcal{G}(\theta^*)$ diverges to $-\infty$ Using l'Hôpital's rule we can then rewrite the limit above as follows

$$\lim_{\theta^* \to 0} \frac{\partial \mathcal{L}(\theta^*, \lambda)}{\partial \theta^*} = \lim_{\theta^* \to 0} (\theta^*)^2 \mathcal{G}'(\theta^*) \qquad \text{(A.18)}$$

Substituting the expression for $\mathcal{G}'(\cdot)$ that we computed in (A.11) into (A.18) yields

$$\lim_{\theta^* \to 0} -q_H (\theta^*)^2 g'(\theta^*) \left[V'(p^* + t^* - c_N - g(\theta^*)) - V'(p^* + t^* - c_N + f(\theta^*)) \right] \qquad \text{(A.19)}$$

Notice next that the term in square brackets in (A.19) remains positive as θ^* approaches zero. This is simply because both $f(\cdot)$ and $g(\cdot)$ diverge to $+\infty$ as θ^* approaches zero, and because $V''(\cdot)$

< 0. Therefore the sign of the limit in (A.19) is the same as the sign of $\lim_{\theta^* \to 0} - (\theta^*)^2 \, g'(\theta^*)$. Finally, recall that by assumption (3) above $\lim_{\theta^* \to 0} (\theta^*)^2 \, g'(\theta^*) < 0$. Therefore, we can now conclude that

$$\lim_{\theta^* \to 0} \frac{\partial \mathcal{L}(\theta^*, \lambda)}{\partial \theta^*} > 0 \qquad (A.20)$$

which directly implies that it is impossible that the solution to problem (A.16) has $\theta^* = 0$. ∎

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