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Price Maintenance**

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**USC Center for Law, Economics & Organization
Research Paper No. C03-5**



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Los Angeles, CA 90089-0071

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A Signal Jamming Theory of Resale Price Maintenance^{*}

Thomas W. Gilligan

Department of Finance and Business Economics
Marshall School of Business
University of Southern California
Los Angeles, CA 90089-1421

May 1, 2003

Abstract

This paper contains a theoretical analysis demonstrating that a retail price floor can increase the expected profits of an upstream firm when it is asymmetrically informed about the state of product demand. The retail price floor serves to eliminate the incentives of the upstream firm to misrepresent its private information and, thus, reduces the transaction costs associated with the strategic use of information. The wholesale and retail prices (and profits) that emerge in the equilibrium with the asymmetrically informed upstream firm given a retail price floor are identical to those that obtain when prices reflect only common prior knowledge about the state of demand. In this way, the retail price floor serves to jam or block the transmission of the upstream firm's private knowledge and increase, for some parameter values, its profits. When used for this purpose, the retail price floor reduces social welfare and lowers the expected retail margin, an observed empirical regularity. (JEL L1, L2, L4, L15, L22, L42)

Keywords: Industrial organization, vertical restraints, resale price maintenance.

^{*} I would like to thank my colleagues Tony Marino, John Matsusaka and Jan Zabojnik for their helpful comments and suggestions.

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Thomas W. Gilligan

The retail price floor, sometimes referred to as a minimum price restraint or resale price maintenance, is often an integral part of a marketing plan in which a producer or upstream firm distributes its product through independent retailers or downstream firms. The motives for this particular vertical restraint are widely debated and a discriminating empirical test of the competing theories has not yet been conducted. The legal treatment of the retail price floor has varied over time and across jurisdictions and remains controversial to this day.¹

While the motives and consequences of the retail price floor continue to be debated, there are at least two apparent regularities that characterize its use. First, the retail price floor is often observed in the distribution of products facing considerable demand uncertainty.² Such

¹ See Mathewson and Winter (1998) for an introduction to and review of the economic theories, empirical evidence and legal treatment of the retail price floor.

² For Deneckere, Marvel and Peck (1996,1997), demand uncertainty is a principle assumption and focus of their analysis. They provide some useful and entertaining illustrations of the magnitude of uncertainty that characterizes the demand for some products. They also note that for products with a substantial fashion, fad or stylistic component, forecasts of demand with any degree of precision are difficult to make (1996, p. 888). Likewise, Butz (1997) employs the assumption of uncertainty to explore the retail price floor as a device that controls rivalry among downstream firms once the low state of demand is realized. Marvel and McCafferty (1984) note that the use of a retail price floor is often associated with the distribution of high-quality goods and provide some pertinent examples, suggesting that the retail price floor is used in industries in which product quality is somewhat variable.

uncertainty may be endemic to certain markets or arise as a result of the novelty of particular goods.³ Second, the retail price floor is often used when the promotional services of downstream distributors are unimportant, or at least not evident, for product demand.⁴ That is, the retail price floor appears to be employed for reasons that are unrelated to the resolution of moral hazard problems in the distribution channel.

In this paper we develop a new motivation for the adoption of the retail price floor consistent with these regularities. Our argument relies on two key assumptions, namely, that the demand for a product is stochastic and the upstream firm is asymmetrically informed about the state of demand. Within this framework we show that the transaction costs associated with the

³ The retail price floor is used more often early in a product's life than in its mature or declining stage. Many authors have identified this regularity. For example, Marvel and McCafferty (1984) note that the retail price floor is "often adopted by new entrants to apparently competitive industries" (p. 347). Mathewson and Winter (1998) make a similar observation and offer an explanation that the retail price floor "is often used in the early part of a product's life cycle to aid in the establishment of the distribution system" (p. 60). There is even some empirical evidence that other vertical restraints (e.g., exclusive territorial restraints) are more valuable to manufacturers early in the product life cycle (Bergen, Heide and Dutta, 1998).

⁴ Telser's (1960) "dealer services" hypothesis is unquestionably the most commonly cited efficiency rationale for the retail price floor. A necessary condition for the relevance of this theory is that dealers engage in activities susceptible to moral hazard. The retail price floor is often used with products where no such activities are evident. This is a common observation and, indeed, a principle motivation for contemporary interest in the economics of the retail price floor. Nearly all of the theoretical papers herein cited make reference to this regularity.

strategic use of information can cause the upstream firm to prefer that it did not know the true state of product demand and that, instead, the strategic choices of both the upstream and downstream firms be based solely on common prior knowledge about demand. We then show that the adoption of a retail price floor can support a unique equilibrium in which the private information of the upstream firm is not imbued in the strategic decisions of the firms and that, for certain parameterizations, the profits to the upstream firm are greater than those that obtain under asymmetric information. Thus, the retail price floor can increase the profits to the upstream firm by eliminating the costs associated with the strategic use of information.

Our analysis is related to the recent theoretical research on the retail price floor. Much of this literature assumes stochastic demand. Deneckere, Marvel and Peck (1996, 1997) and Butz (1997, 1998) illustrate the role a retail price floor can play in managing retailer inventories in the face of demand uncertainty and increasing the profits of the upstream firm. In addition, some of this literature assumes that information about stochastic demand is unevenly distributed within the value chain. Marvel and McCafferty (1984) expose the logic of the retail price floor when downstream firms have better information about product quality than do consumers while Mathewson and Winter (1983) illustrate the value of the retail price floor when downstream firms have superior information about local markets. Our analysis differs only in that the potential for superior information is assumed to reside with the upstream firm.

Nearly all of the theoretical literature regarding the motives and consequences of the retail price floor fall within one of two broad categories. On the one hand is the class of theories that purport to show how the retail price floor retards competition by promoting cooperative behavior among horizontal competitors at either the wholesale or retail level. On the other hand is the class of theories that claim to illustrate that the retail price floor can result in efficiencies

that are beneficial to both producers and consumers.⁵ Our analysis is different in that we provide neither a pro-cartel nor an efficiency-based rationale for the retail price floor. We show simply that an upstream firm with the potential to obtain private information about consumer demand can reduce the transaction costs of asymmetric information and increase its expected profits by adopting a retail price floor. When the upstream firm uses the retail price floor for this purpose, both retailers and consumers are harmed and aggregate economic welfare is diminished. This reduction in welfare results because the retail price floor reduces both the expected quantity sold and the expected retail margin.⁶

Our model predicts that the retail price floor can serve to reduce the expected retail margin. This prediction is in obvious contrast to theories that argue that the retail price floor is used to enhance retail margins and promote dealer services. Butz shows that a retail price floor is often coincident with reduced retail margins and argues that this results since the retail price floor can be shown to reduce the inventory costs of downstream firms (1998). We provide another and, indeed, more parsimonious model explaining the correlation between reduced retail margins and the use of a retail price floor.

We begin with a brief description of the linear distribution model and identify its equilibria and performance given the symmetric distribution of information. We then show how asymmetric information affects the model and explore the consequences of a retail price floor. We end by identifying the conditions that can lead to higher expected upstream and downstream

⁵ Again, Mathewson and Winter (1998) is a good source for review of this literature.

⁶ Our welfare conclusions, thus, are similar to Deneckere, Marvel and Peck (1996) who show that the gains to the upstream firm from the use of the retail price floor come principally from consumer surplus. In our model, the upstream firm's gains come from consumers and retailers.

profits as a result of the minimum retail price constraint and explore some normative and positive implications of our analysis.

I. Linear Distribution Model and the Symmetric Distribution of Information

Consider a model in which the demand for a product produced by a risk-neutral upstream firm called the wholesaler is given by $q = \mathbf{q}_i - p$ where q is the quantity demanded, $\mathbf{q}_i \in \{\mathbf{q}_L, \mathbf{q}_H\}$, $0 < \mathbf{q}_L < \mathbf{q}_H$, is a random variable, and p is the product's retail price. For simplicity, we assume that \mathbf{q}_H and \mathbf{q}_L occur with equal probability and, further, normalize \mathbf{q}_L to unity so that \mathbf{q}_H represents the ratio of the high to low demand state variables. Alternatively, $\mathbf{q}_H - 1$ represents, in percentage terms, the relative size of the high and low demand state variables. Given these assumptions, $E(\mathbf{q}_i) = (1 + \mathbf{q}_H) / 2$.

Regardless its type (i.e., the value of \mathbf{q}_i , $i = L, H$), the wholesaler's average cost of production is constant and equal to zero and its profits are $\mathbf{p}_w = wq$ where w is the wholesale price. An independent, risk-neutral downstream firm or retailer distributes the product at zero variable costs earning profits of $\mathbf{p}_r = (p - w)q$. Since the economic relationships between consumers and the retailer and the retailer and the wholesaler are moderated by simple linear tariffs, we refer to this one parameter (\mathbf{q}_H) framework as the linear distribution model.

The behavior of the wholesaler and retailer depend on the informational structure of the model. To begin, we consider the benchmarks of symmetric information. There are obviously two possibilities to consider.

A. Complete Information

Complete information (CI) is said to exist if and only if the wholesaler and retailer know the value q_i , $i = L, H$ prior to the formulation of both the wholesale and retail prices. Under complete information, nature selects a value for the high demand state variable (q_H) and reveals the state of demand (i.e., high or low) to both the wholesaler and retailer. The wholesaler begins a sequence of actions when it chooses a wholesale price based on information about the demand state and the anticipated action of the retailer. The retailer then chooses a retail price based on the wholesale price and demand state information. Subsequent to the retailer's price choice sales are made and profits earned according to the adopted wholesale and retail prices.

Given complete information, the subgame perfect Nash equilibrium of the linear distribution model is given by $p_{CI}(q_i) = 3q_i/4$ and $w_{CI}(q_i) = q_i/2$, $i = L, H$. Prices are monotonically increasing in the value of the demand state variable. Retailer and wholesaler profits are strictly convex functions of the value of the demand state variable and are given by $p_r^{CI} = q_i^2/16$ and $p_w^{CI} = q_i^2/8$. Since either demand state is equally likely, prior to the revelation of the demand state variable the expected profits of the retailer and wholesaler are given by $E(p_r^{CI}) = (1 + q_H^2)/32$ and $E(p_w^{CI}) = (1 + q_H^2)/16$, respectively.⁷

⁷ For any q_i , $i = L, H$, and wholesale price w , the retailer chooses $p = (q_i + w)/2$. Substituting this expression into the wholesaler's profit function and optimizing illustrates that $w_{CI}(q_i) \equiv \arg \max [w(q_i - p_{CI}(q_i))]$. Substituting $w_{CI}(q_i)$ into the retailer's profit function and optimizing yields $p_{CI}(q_i) \equiv \arg \max [(p - w_{CI}(q_i))(q_i - p)]$. $\sum_{i=L,H} w_{CI}(q_i)[q_i - p_{CI}(q_i)]/2$ equals

It is important to note a well-known property of the equilibrium of the linear distribution model under complete information, namely that the joint profits of the wholesaler and retailer are not maximized. The sequential and uncoordinated nature of the pricing decisions in this model generate a retail price that is fifty percent greater than the retail price that would maximize joint profits.⁸ This is often referred to as the *double marginalization* problem and is sometimes cited as a motivation for antitrust enforcement or for integrating parts of the value chain (Spengler, 1950). As we show below, independent price setting behavior can create some additional difficulties when information about product demand is unevenly distributed throughout the distribution chain. Paradoxically, one solution to these difficulties can be the adoption of a retail price floor.

B. Symmetric Uncertainty

Information is also distributed evenly when both the retailer and wholesaler have the same incomplete information about the value of the demand state variable when formulating their pricing decisions. Symmetric uncertainty (SU) is said to exist when both the retail and wholesale price choices are formulated using only common prior information about the value of the demand state variable. Under symmetric uncertainty, nature selects a value for the high demand state variable (\mathbf{q}_H), which becomes common knowledge, but does not reveal the state of demand (i.e., high or low) to either the wholesaler or retailer. The wholesaler then begins a

the expected profits of the wholesaler while the retailer's expected profits equal

$$\sum_{i=L,H} [p_{ci}(\mathbf{q}_i) - w_{ci}(\mathbf{q}_i)][\mathbf{q}_i - p_{ci}(\mathbf{q}_i)]/2.$$

⁸ By summing the profits of the wholesaler and the retailer, joint profits are given by

$$p(\mathbf{q}_i)[\mathbf{q}_i - p_i(\mathbf{q}_i)], \quad i = L, H, \quad \text{which are maximized for } p(\mathbf{q}_i) = \mathbf{q}_i/2.$$

sequence of actions when it chooses a wholesale price based on its prior information about the demand state and the anticipated action of the retailer. The retailer then chooses a retail price based on the wholesale price and its prior information about the demand state variable. Subsequent to the retailer's price choice, sales are realized and profits earned according to the functions defined above.

Given symmetric uncertainty, the wholesale and retail price choices cannot be conditioned on the state of product demand, but rather must be derived given the anticipated or expected value of the demand state variable. Under this restriction the Nash equilibrium of the linear distribution model depends on the value of the high demand state variable. When the retail price is sufficiently low to result in sales in either demand state, the Nash equilibrium wholesale and retail prices are given by $p_{SU} = 3(1+q_H)/8$ and $w_{SU} = (1+q_H)/4$, respectively.⁹ It is easy to verify that sales occur in the low demand state if and only if $q_H < 5/3$.¹⁰ When the retail price precludes sales in the low demand state, the Nash equilibrium is given by $p_{SU} = 3(q_H)/4$ and $w_{SU} = q_H/2$, the same retail and wholesale price, respectively, that obtain in the complete information equilibrium for $q_i = q_H$. Expected profits of the retailer and wholesaler under symmetric uncertainty are given by $E(p_r^{SU}) = (1+q_H)^2/64$ and

⁹ Given the wholesale price w and only prior common knowledge about the state of demand, the retailer will choose a retail price that satisfies $p = (E(q_i) + w)/2$. Substituting this expression into the wholesaler's profits function yields $p_w = w(1+q_H - 2w)/4$, which is maximized at $w_{SU} = (1+q_H)/4$. Substitution yields $p_{SU} = 3(1+q_H)/8$.

¹⁰ For $p_{SU} = 3(1+q_H)/8$, $q(q_L) = 1 - p_{SU} = [1 - 3(1+q_H)/8]$, which must be positive for sales to occur in the low demand state. This is true if and only if $q_H < 5/3$.

$E(\mathbf{p}_w^{SU}) = (1 + \mathbf{q}_H)^2 / 32$ when $\mathbf{q}_H < 5/3$ and $E(\mathbf{p}_r^{SU}) = \mathbf{q}_H^2 / 64$ and $E(\mathbf{p}_w^{SU}) = \mathbf{q}_H^2 / 32$ otherwise.¹¹

C. Comparing the Symmetric Information Benchmarks

While one can shown that the expected retail and wholesale prices are identical in the symmetric information benchmarks, the expected profits of both the wholesaler and retailer are lower under symmetric uncertainty than they are given complete information. Substitution reveals that $E(\mathbf{p}_r^{CI}) > E(\mathbf{p}_r^{SU})$ and $E(\mathbf{p}_w^{CI}) > E(\mathbf{p}_w^{SU})$ for all values of \mathbf{q}_H . Both parties are worse off when the wholesaler and retailer base their pricing decisions solely on common prior information about the value of the demand state variable. This result is evident given the strict convexity of the profit functions in the value of the demand state variable.

II. Linear Distribution Model Under Asymmetric Information

Suppose that only the wholesaler knows \mathbf{q}_i , $i = L, H$, prior to the formation of prices in the linear distribution model. This may be true, for example, when the wholesaler is better equipped to judge the appeal of products to consumers. This may also be true when the marketing research function in the distribution chain is concentrated at the wholesale level. In such a case the linear distribution model is characterized by asymmetric information (AI).

¹¹ For $\mathbf{q}_H < 5/3$, $\sum_{i=L,H} w_{SU} (\mathbf{q}_i - p_{SU}) / 2$ are the expected profits of the wholesaler given

$p_{SU} = 3(1 + \mathbf{q}_H) / 8$ and $w_{SU} = (1 + \mathbf{q}_H) / 4$. The retailer's expected profits are given by the

$\sum_{i=L,H} [p_{SU} - w_{SU}] [\mathbf{q}_i - p_{SU}] / 2$. Otherwise the wholesaler's and retailer's expected profits are

given by $\mathbf{q}_H^2 / 16$ and $\mathbf{q}_H^2 / 32$, respectively.

Given asymmetric information, a high-type wholesaler may wish to mimic the behavior of the low-type wholesaler in order to induce a lower retail price and secure a larger quantity of demand. To demonstrate this, assume that the complete information equilibrium of the linear distribution model also obtains in the case of asymmetric information. Then a high-type wholesaler can earn $p_w^{CI}(\mathbf{q}_H | w = \mathbf{q}_H / 2) = \mathbf{q}_H^2 / 8$ by revealing its true type (i.e., selecting the wholesale price $w_{CI}(\mathbf{q}_H) = \mathbf{q}_H / 2$) or $p_w^{CI}(\mathbf{q}_H | w = 1/2) = (4\mathbf{q}_H - 3)/8$ by mimicking the behavior of a low-type wholesaler (i.e., selecting the wholesale price $w_{CI}(1) = 1/2$). Calculations show that $p_w^{CI}(\mathbf{q}_H | w = \mathbf{q}_H / 2) \geq p_w^{CI}(\mathbf{q}_H | w = 1/2)$ if and only if $\mathbf{q}_H \geq 3$. When $\mathbf{q}_H < 3$, the high-type wholesaler has an incentive to mimic the behavior of the low-type wholesaler. Thus, for $\mathbf{q}_H < 3$ the complete information equilibrium is not a suitable candidate for the equilibrium to the linear distribution model under asymmetric information in the sense that the retailer cannot learn the value of the demand state variable by observing the price choice of the wholesaler.

A. Equilibrium Under Asymmetric Information

Any equilibrium of the linear distribution model under asymmetric information must consider the wholesaler's incentives to behave strategically given its private information and the retailer's attempt to infer the state of demand based on the wholesaler's price selection. Such equilibria should also require that the retailer's inference satisfy some acceptable estimation procedure. The concept of a perfect Bayesian equilibrium satisfies these three criteria. Like the ordinary Nash equilibrium, a perfect Bayesian equilibrium insures that the wholesaler's strategy maximize its profits given the retailer's strategy. Further, a perfect Bayesian equilibrium requires that the retailer's strategy maximize its profits given the retailer's posterior beliefs about

the value of the demand state variable. And lastly, a perfect Bayesian equilibrium computes the retailer's posterior beliefs using the wholesaler's strategy, common prior beliefs about the demand state variable, and Bayes Rule.

One potential limitation of the application of a perfect Bayesian equilibrium in the present context is that it places no restrictions on retailer beliefs in response to out-of-equilibrium wholesale price choices. What should the retailer believe if it observes a wholesale price that is not part of any equilibrium to the linear distribution model?¹² We refine the perfect Bayesian equilibrium of the linear distribution model under asymmetric information by insisting that the retailer, in response to observing an out-of-equilibrium wholesale price, assign positive probability only to that wholesaler type that is least harmed by the retailer's response. This refinement recognizes that out-of-equilibrium wholesale price choices are deviations (i.e., not best responses given the equilibrium beliefs and strategy of the retailer) that are, by assumption, more likely to be made by the wholesaler type harmed least by the resulting response. This is a common theme in many prominent refinements and can be shown, for example, to be consistent with those proposed by Cho and Kreps (1987) and Banks and Sobel (1987).

For reasons that will soon become apparent, we restrict $q_H \leq 2$ and maintain this assumption for the remainder of the analysis. Moreover, let $m(q_i | w)$ represent the retailer's posterior probability that the value of the demand state variable is q_i , $i = L, H$, given the

¹² For example, one might conjecture that $w = 0$ is not an equilibrium strategy for the wholesaler. Nonetheless, a retailer must form a guess about the value of the demand state variable in order to best respond to this opportunity. More importantly, what the retailer believes about the state of demand when $w = 0$ could affect the equilibrium behavior of both the retailer and wholesaler.

wholesaler price w . The following proposition identifies an equilibrium to the linear distribution model under asymmetric information.

Proposition 1: For $q_H \leq 2$, the unique separating perfect Bayesian equilibrium to the linear distribution model under asymmetric information is given by

$$w_{AI}(q_i) = \begin{cases} q_H / 2 & \text{if } q_i = q_H \\ w_L^*(q_H) & \text{if } q_i = 1 \end{cases}$$

$$p_{AI}(w) = \begin{cases} (q_H + w) / 2 & \text{if } w > w_L^* \\ (1 + w) / 2 & \text{if } w \leq w_L^* \end{cases}$$

$$m(q_i | w) = \begin{cases} 1 & \text{if } q_i = q_H \text{ and } w > w_L^* \\ 1 & \text{if } q_i = 1 \text{ and } w \leq w_L^* \\ 0 & \text{otherwise} \end{cases}$$

where $w_L^*(q_H) = \{(2q_H - 1) - [(3q_H - 1)(q_H - 1)]^{1/2}\} / 2 < q_L / 2$.

Proof: See appendix.

This equilibrium has the same features as the one contained in Albaek and Overgaard (1993) and is constructed by recognizing that the high-type wholesaler can always convey its private information and earns its complete information profits by choosing $w = q_H / 2$. We then find a lower wholesale price $w_L^*(q_H)$ that, conditional on the retailer believing that $q_i = 1$, eliminates any (strict) incentive for the high-type wholesaler to mimic the behavior of a low-type wholesaler. That is, it must be the case that $p_w^{AI}(q_H | w = q_H / 2) \geq p_w^{AI}(q_H | w = w_L^*(q_H))$, or $q_H^2 / 8 \geq w_L^*(q_H)[q_H - p_{AI}(w_L^*(q_H) | q_i = 1)]$. Since the high-type wholesaler now has no (strict) incentive to mimic the behavior of the low-demand wholesaler, the retailer believes that only a

low-type wholesaler adopts a price at or below $w_L^*(q_H)$; $m(q_L | w \leq w_L^*(q_H)) = 1$. A wholesale price above the critical value $w_L^*(q_H)$ signals a high-type wholesaler; $m(q_H | w > w_L^*(q_H)) = 1$. Given this belief structure, only the low-type wholesaler chooses $w_L^*(q_H)$ while the high-type wholesaler selects $q_H/2 > w_L^*(q_H)$. Thus, the retailer's beliefs are consistent with Bayes rule and the equilibrium strategies and, as the appendix shows, with the proposed refinement as well.

It is important to note two properties of the equilibrium described in Proposition 1. First, the wholesale and retail prices are the same under complete and asymmetric information when $q_i = q_H$. For high values of the demand state variable, the equilibria of the linear distribution model are identical under complete and asymmetric information.

Second, the wholesale and retail prices are different under complete and asymmetric information when $q_i = 1$. One can show that $w_L^*(q_H) < 1/2$ for $1 < q_H \leq 2$ and that this function has a unique minimum of $w_L^*(q_H) = 1/3$ when $q_H = 4/3$.¹³ The wholesale and retail prices under asymmetric information are lower than those that obtain under complete information. Figure 1 is a plot of the equilibrium wholesale and retail prices in the low demand state as a function of the value of the high demand state variable given complete and asymmetric information. And as inspection of this figure suggests, one can also show that the retail margin is greater under asymmetric information. For example, substitution reveals that $p_{CI}(1) - w_{CI}(1) = 3/4 - 1/2 = 1/4$ while $p_{AI}[w_{AI}(1)] - w_{AI}(1) = 2/3 - 1/3 = 1/3$.

¹³ Derivation yields $\partial w_L^*(q_H) / \partial q_H = 1 - (3q_H - 2) / 2[(3q_H - 1)(q_H - 1)]^{1/2}$ which equals zero when $q_H = 4/3$.

B. Expected Profits Under Complete and Asymmetric Information

The pricing distortions introduced by the strategic use of information in the linear distribution model degrade the expected profits of the wholesaler but enhance the expected profits of the retailer relative to the case of complete information.

Corollary 1: $E(\mathbf{p}_w^{CI}) > E(\mathbf{p}_w^{AI})$ and $E(\mathbf{p}_r^{CI}) < E(\mathbf{p}_r^{AI})$.

Proof: When $\mathbf{q}_i = \mathbf{q}_H$, the equilibrium behaviors of the wholesaler and retailer are the same under complete and incomplete information. For $\mathbf{q}_i = 1$, $\mathbf{p}_w^{CI}(1) = 1/8$ while $\mathbf{p}_w^{AI}(1) = w_L^*[1 - p_{AI}(w_L^*)] = w_L^*(1 - w_L^*)/2$ and $\mathbf{p}_r^{CI}(1) = 1/16$ while $\mathbf{p}_r^{AI}(1) = (1 - w_L^*)^2/4$. Calculations show that $w_L^*(1 - w_L^*)/2 < 1/8$ and $(1 - w_L^*)^2/4 > 1/16$ for $1/3 < w_L^* < 1/2$.

Relative to the case of complete information both the retail margin, the retailer's average revenue, and the quantity sold are greater under asymmetric information when $\mathbf{q}_i = 1$ which, necessarily, raises the retailer's expected profits. However, the lower wholesale price, the wholesaler's average revenue, is not offset by increased sales yielding lower expected wholesaler profits. Somewhat surprisingly, relative to the case of complete information the retailer is strictly better off and the wholesaler is strictly worse off when the wholesaler is asymmetrically informed about the state of demand.

C. Expected Profits Under Symmetric Uncertainty and Asymmetric Information

Similar to the case of complete information, the retailer is always better off when the wholesaler has asymmetric information than it is when neither have information about the value of the state of demand. Even with the distortion of the wholesale and retail prices necessary to

elicit the wholesaler's private information, the retailer prefers the equilibrium with asymmetric information to that of symmetric uncertainty.

Corollary 2: For $1 < \mathbf{q}_H \leq 2$, $E(\mathbf{p}_r^{SU}) < E(\mathbf{p}_r^{AI})$.

Proof: For $\mathbf{q}_H \leq 2$, using Proposition 1 one can compute $E(\mathbf{p}_r^{AI}) = (1 - w_L^*)^2 / 8 + \mathbf{q}_H^2 / 32$.

From above, for $5/3 \leq \mathbf{q}_H$, $E(\mathbf{p}_r^{SU}) = \mathbf{q}_H^2 / 64$. Assuming $E(\mathbf{p}_r^{SU}) \geq E(\mathbf{p}_r^{AI})$

yields $(1 - w_L^*)^2 / 8 \leq -\mathbf{q}_H^2 / 64$, a contradiction given $1/2 > w_L^*(\mathbf{q}_H) > 1/3$ and $5/3 \leq \mathbf{q}_H$. For

$\mathbf{q}_H < 5/3$, $E(\mathbf{p}_r^{SU}) = (1 + \mathbf{q}_H)^2 / 64$. Assume for the moment that $E(\mathbf{p}_r^{SU}) \geq E(\mathbf{p}_r^{AI})$. Since

$w_L^*(\mathbf{q}_H) < 1/2$, this assumption implies that $1 < \mathbf{q}_H(2 - \mathbf{q}_H)$, which is also a contradiction.

Relative to the case of symmetric uncertainty, the retailer's expected profits are higher when the wholesaler has asymmetric information about product demand.

The wholesaler is often better off when it has private information about the value of the demand state variable. For values of the high demand state that satisfy $\hat{\mathbf{q}}_H < \mathbf{q}_H \leq 2$ where $\hat{\mathbf{q}}_H = 1 + \{2[1 - 4w_L^*(\hat{\mathbf{q}}_H)(1 - w_L^*(\hat{\mathbf{q}}_H))]\}^{1/2}$, the wholesaler prefers the equilibrium with asymmetric information to that of symmetric uncertainty even with the distortion of the wholesale and retail prices when $\mathbf{q}_i = \mathbf{q}_L$. However, for smaller values of the high demand state variable satisfying $1 < \mathbf{q}_H \leq \hat{\mathbf{q}}_H$, the distortion in prices under asymmetric information when $\mathbf{q}_i = \mathbf{q}_L$ lowers the wholesaler's expected profits below those expected under symmetric uncertainty.

Corollary 3: For $q_H \leq \hat{q}_H$, $E(\mathbf{p}_w^{SU}) \geq E(\mathbf{p}_w^{AI})$ where $\hat{q}_H = 1 + \{2[1 - 4w_L^*(\hat{q}_H)(1 - w_L^*(\hat{q}_H))]\}^{1/2}$.

Otherwise, $E(\mathbf{p}_w^{SU}) < E(\mathbf{p}_w^{AI})$.

Proof: For $q_H \leq 2$, using Proposition 1 one can compute $E(\mathbf{p}_w^{AI}) = w_L^*(1 - w_L^*)/4 + q_H^2/16$.

From above we know that $E(\mathbf{p}_w^{SU}) = q_H^2/32$ for $5/3 \leq q_H$. Assume that $E(\mathbf{p}_w^{SU}) \geq E(\mathbf{p}_w^{AI})$ for

$5/3 \leq q_H$. But this implies that $w_L^*(1 - w_L^*)/4 \leq -q_H^2/32$, a contradiction given $1 < q_H$ and

$1/2 > w_L^*(\hat{q}_H) \geq 1/3$. Therefore, $E(\mathbf{p}_w^{SU}) < E(\mathbf{p}_w^{AI})$ for $5/3 \leq q_H \leq 2$. We first show that

$4/3 < \hat{q}_H < 5/3$. $E(\mathbf{p}_w^{SU}) = (1 + q_H)^2/32$ for $q_H \leq 5/3$. Let \hat{q}_H solve

$E(\mathbf{p}_w^{SU}) - E(\mathbf{p}_w^{AI}) = (1 + \hat{q}_H)^2/32 - w_L^*(\hat{q}_H)(1 - w_L^*(\hat{q}_H))/4 - \hat{q}_H^2/16 = 0$. Then, computations

yield $\hat{q}_H = 1 + \{2[1 - 4w_L^*(\hat{q}_H)(1 - w_L^*(\hat{q}_H))]\}^{1/2}$. Assume that $\hat{q}_H \leq 4/3$. This implies that

$1/3 \geq \{2[1 - 4w_L^*(\hat{q}_H)(1 - w_L^*(\hat{q}_H))]\}^{1/2} > 1$ given $w_L^*(q_H) < 1/2$, a contradiction. Assume also

that $\hat{q}_H \geq 5/3$. This implies that $2/3 \leq \{2[1 - 4w_L^*(\hat{q}_H)(1 - w_L^*(\hat{q}_H))]\}^{1/2} < (1/3)^{1/2}$ given

$w_L^*(q_H) > 1/3$, again a contradiction. Thus, $4/3 < \hat{q}_H < 5/3$. Lastly, it is possible to show that

$E(\mathbf{p}_w^{SU}) - E(\mathbf{p}_w^{AI})$ is strictly positive for $1 < q_H \leq \hat{q}_H$ and negative thereafter.

That is, for relative small values of the high demand state variable the wholesaler is made worse off by having private information about the realization of the demand state variable.

For $1 < q_H \leq \hat{q}_H$, the strategic costs of information transmission, the distortion of prices necessary to preclude wholesaler masquerading, exceed the benefits of informed decision-making for the wholesaler. It is worth noting that $4/3 < \hat{q}_H < 5/3$ in the linear distribution model. That is, when the value of the high demand state variable is less than approximately fifty

percent greater than the value of the low demand state variable, the wholesaler with private information is worse off than it is given only prior knowledge about product demand.

D. Transaction Costs of Asymmetric Information

The reduction in expected wholesaler profits in the linear distribution model under asymmetric information relative to the profits that obtain under symmetric uncertainty show that the cost of preventing the wholesaler from misrepresenting its type can exceed the benefits of private information about the value of the demand state variable. Recall that the motive for the wholesaler to misrepresent its private information (i.e., masquerade its type) is to cause the retailer to adopt a lower retail price and, thus, increase the quantity of demand, which for certain parameterizations can be profitable to the high-type wholesaler. A sensible conjecture is that the incentive of the high-type wholesaler to masquerade its type is attenuated if the retailer's ability to reduce its price in response to the belief that $q_i = q_L$ is somehow restricted. Might a retail price floor be a way to manage the strategic use of information in the linear distribution model?

III. Linear Distribution Model Under Asymmetric Information and a Retail Price Floor

Suppose that the retailer can commit to a retail price floor prior to the wholesaler learning the value of the demand state variable. This might be true if the retailer has advertised and, thus, made an implied promise to consumers to sell the good at a particular price. This might also be true if the retailer has agreed to the retail price floor as a condition of its distribution relationship with the wholesaler. This latter explanation exemplifies the classical application of resale price maintenance. Figure 2 is a timeline characterizing the sequences of actions and information disclosure in the linear distribution model under asymmetric information with a retail price floor (AIF).

A. *Equilibrium Under Asymmetric Information with a Retail Price Floor*

Suppose momentarily that the price floor to which the retailer commits is given by $p_f = 3E(\mathbf{q}_i)/4 = 3(1+\mathbf{q}_H)/8$. Notice that this is precisely the price the retailer would select given its priors on the state of demand and a wholesale price of $w = E(\mathbf{q}_i)/2$, the price the wholesaler would select based, too, only on its priors. The following proposition identifies an equilibrium to the linear distribution model under asymmetric information with a retail price floor of $p_f = 3E(\mathbf{q}_i)/4 = 3(1+\mathbf{q}_H)/8$.

Proposition 2: Given $\mathbf{q}_H \leq 5/3$, asymmetric information and a retail price floor of $p_{AIF}(w) \geq 3(1+\mathbf{q}_H)/8$, the unique perfect Bayesian equilibrium is given by $w_{AIF}(\mathbf{q}_i) = (1+\mathbf{q}_H)/4$ for \mathbf{q}_i , $i = L, H$,

$$p_{AIF}(w) = \begin{cases} (\mathbf{q}_H + w)/2 & \text{if } w > (1+\mathbf{q}_H)/4 \\ 3(1+\mathbf{q}_H)/8 & \text{if } w \leq (1+\mathbf{q}_H)/4 \end{cases}$$

$$\mathbf{m}(\mathbf{q}_i | w) = \begin{cases} 1 & \text{if } \mathbf{q}_i = \mathbf{q}_H \text{ and } w > (1+\mathbf{q}_H)/4 \\ 1 & \text{if } \mathbf{q}_i = (1+\mathbf{q}_H)/2 \text{ and } w = (1+\mathbf{q}_H)/4 \\ 1 & \text{if } \mathbf{q}_i = 1 \text{ and } w < (1+\mathbf{q}_H)/4 \\ 0 & \text{otherwise} \end{cases}$$

Proof: See the appendix.

Given that both the high and low-type wholesaler choose $w = E(\mathbf{q}_i)/2 = (1+\mathbf{q}_H)/4$, the wholesale price conveys no information and, thus, the retailer's posterior belief about the value of the demand state variable is $\mathbf{m}(E(\mathbf{q}_i) | E(\mathbf{q}_i)/2) = 1$. The retailer chooses $p = 3E(\mathbf{q}_i)/4 = 3(1+\mathbf{q}_H)/8$ since this maximizes its profits given these posterior beliefs. The

retailer's out-of-equilibrium beliefs are intuitive in the sense that deviations above and below the equilibrium wholesale price cause the retailer to believe the wholesaler type is high and low, respectively. Given these beliefs, neither a high or low-type wholesaler wishes to charge a price different from $w = E(\mathbf{q}_i)/2 = (1 + \mathbf{q}_H)/4$ since, given the retailer's beliefs, deviating from the equilibrium price choice can only lower the profits of the wholesaler.

What is the function of the retail price floor? How does it support an equilibrium in which the private information of the wholesaler is not reflected in either the wholesale or retail price? Simply put, by eliminating the incentive of the high-type wholesaler to masquerade as a low-type wholesaler. Recall that the benefits of the masquerade to the high-type wholesaler result from the reduced price that a retailer charges when it believes that $\mathbf{q}_i = 1$. The price floor prevents the retailer from dropping its price in response to signals that convey the low realization of the demand state variable. Since for any fixed retail price the wholesaler's profits are strictly increasing in the wholesale price, the wholesaler has no incentive to reduce its wholesale price. Even if the retailer believes that $\mathbf{q}_i = 1$ and, thus, it prefers a retail price $p < p_{AIF}(w)$ given $w < w_{AIF}$, the constraint restricts the retailer from taking such an action. By limiting the strategic options of the retailer, the retail price floor eliminates the incentive of the wholesaler to masquerade as a low-type.

B. Optimal Retail Price Floor

The following corollary establishes that, indeed, the price floor of $p_f = 3E(\mathbf{q}_i)/4$ maximizes the expected profits of the wholesaler.

Corollary 4: The price floor $p_f^* = 3E(\mathbf{q}_i)/4$ maximizes $E(\mathbf{p}_w^{AIF})$.

Proof: Given Proposition 2, $E(\mathbf{p}_w^{AIF}) = [\bar{w}(\mathbf{q}_L - p_f) + \bar{w}(\mathbf{q}_H - p_f)]/2$ since either demand state is equally likely. For any fixed retail price the wholesaler's profits are strictly increasing in the wholesale price. The highest wholesale price that would cause the retailer with beliefs $E(\mathbf{q}_i) = (1 + \mathbf{q}_H)/2$ to be indifferent between p_f and a slightly higher price satisfies $\bar{w} = (4p_f - 1 - \mathbf{q}_H)/2$; $p_f = [E(\mathbf{q}_i + \bar{w})]/2$. Making the substitution and maximizing $E(\mathbf{p}_w^{AIF})$ with respect to p_f yields $p_f^* = 3E(\mathbf{q}_i)/4$.

Relative to the cases of complete or asymmetric information, the retail price floor trades off the benefits of a lower retail price and higher profits when $\mathbf{q}_i = \mathbf{q}_H$ against the costs of a higher retail price and lower profits when $\mathbf{q}_i = \mathbf{q}_L$. This tradeoff is balanced when the wholesaler and retailer choose prices based on their unconditional and common expectation of the value of the demand state variable. The assumption that $\mathbf{q}_H \leq 5/3$ insures that the retail price is sufficiently low so that sales occur in the low demand state.

C. Expected Profits Under Asymmetric Information without and with a Retail Price Floor

Since the equilibrium behaviors in the linear distribution model under symmetric uncertainty and asymmetric information with a price floor are identical, it is obvious that $E(\mathbf{p}_w^{SU}) = E(\mathbf{p}_w^{AIF})$ and $E(\mathbf{p}_r^{SU}) = E(\mathbf{p}_r^{AIF})$. This fact, together with the results presented in Corollary 3, form the basis for our main result.

Corollary 5: $E(\mathbf{p}_w^{AIF}) > E(\mathbf{p}_w^{AI})$ for $\mathbf{q}_H < \hat{\mathbf{q}}_H$ where $\hat{\mathbf{q}}_H = 1 + \{2[1 - 4w_L^*(\hat{\mathbf{q}}_H)(1 - w_L^*(\hat{\mathbf{q}}_H))]\}^{1/2}$.

A wholesaler with better information than the retailer can increase its expected profits by, prior to its learning the value of the demand state variable, imposing a retail price floor equal to the retail price that would arise given symmetric uncertainty. Figure 3 plots the percentage increase in the wholesaler's expected profits under asymmetric information given the use of a retail price floor. It is important to note that the maximum gain the retail price floor generates in the linear distribution model is less than four percent.

D. Social Welfare and the Retail Price Floor

When the wholesaler employs the retail price floor to limit the use of its private information in the formation of wholesale and retail prices, the expected profits of the wholesaler are larger but the expected profits of the retailer are lower. It is also true that the optimal retail price floor is greater than the expected retail price under asymmetric information.

Corollary 6: $p_f^* > E(p_{AI})$.

Proof: From above $p_f^* = 3(1 + \mathbf{q}_H)/8$ and $E(p_{AI}) = [3\mathbf{q}_H + 2(1 + w_L^*(\mathbf{q}_H))]/8$. Assume that $p_f^* \leq E(p_{AI})$. This implies that $3/2 \leq [1 + w_L^*(\mathbf{q}_H)]$, which is false since $w_L^*(\mathbf{q}_H) < 1/2$ for all $1 < \mathbf{q}_H \leq \hat{\mathbf{q}}_H$.

Thus, expected consumer surplus is lower when the retail price floor is employed to distribute the product under asymmetric information.

It is also possible to show that the retail price floor reduces the sum of wholesaler and retailer expected profits.

Corollary 7: $E[p_w^{AIF}(q_H)] + E[p_r^{AIF}(q_H)] < E[p_w^{AI}(q_H)] + E[p_r^{AI}(q_H)]$.

Proof: The sum of wholesaler and retailer expected profits under asymmetric information and a retail price floor are given by $E[p_w^{AIF}(q_H)] + E[p_r^{AIF}(q_H)] = 3(1+q_H)^2/64$. The sum of wholesaler and retailer expected profits under asymmetric information absent the retail price floor are $E[p_w^{AI}(q_H)] + E[p_r^{AI}(q_H)] = \{4[1 - w_L^*(q_H)] + 3q_H^2\}/32$. Assume that expected producer profits are at least as large when the retail price floor is used. Then, $8\{1 - [w_L^*(q_H)]^2\} \leq 3[1 + q_H(2 - q_H)]$. Since $w_L^*(q_H) < 1/2$ for $1 < q_H \leq \hat{q}_H$, the assumption that producer surplus is higher with the retail price floor implies that $1 < q_H(2 - q_H)$, which is false for $1 < q_H \leq \hat{q}_H$.

Since the retail price floor reduces both expected producer and expected consumer surplus, its use lowers expected social welfare. The private benefits of the retail price floor to the wholesaler are strictly less than the costs the retail price floor imposes on the retailer and consumers.

E. Empirical Properties and the Retail Price Floor

When the wholesaler employs the retail price floor to limit the use of its private information in the formation of wholesale and retail prices, the expected retail margin is reduced.

Corollary 8: $E(p_{AI} - w_{AI}) > E(p_{AIF} - w_{AIF})$.

Proof: $E(p_{AI} - w_{AI}) = (q_H + 1 - w_L^*)/4$ while $E(p_{AIF} - w_{AIF}) = (1 + q_H)/8$. Assume to the contrary that $E(p_{AI} - w_{AI}) \leq E(p_{AIF} - w_{AIF})$. This implies that $w_L^* \geq (1 + q_H)/2$, which is a contradiction given $q_H > 1$ and $w_L^*(q_H) < 1/2$.

Butz (1998) has shown that the retail price floor is sometimes coincident with lower retail margins. He argues that lower retail margins result since resale price maintenance can be shown to reduce the inventory costs of downstream firms. We show that reduced expected retail margins can also result when the wholesaler, to cloak its private information about product demand, uses the retail price floor.

IV. Conclusions

The analysis contained in this paper demonstrates that an upstream firm or wholesaler can often benefit by using a retail price floor to distribute its product through independent retailers. By eliminating the incentives of a wholesaler to misrepresent its private knowledge about the state of demand for the product, the retail price floor can attenuate the transaction costs associated with the strategic use of information in the linear distribution model. Since these costs are borne by the informed party, the benefits of the retail price floor ennuui to the wholesaler and harm the retailer and consumers. The use of a retail price floor to jam the signal imbued in the wholesale price reduces social welfare and reduces the expected retail margin.

As a practical matter, when might a wholesaler, to cloak its information about product demand, use a retail price floor? Two conditions appear necessary. First, the demand for the product must be stochastic. This is likely to be true of products with novel or changing characteristics such as those that are entirely new or unique or those modified on a periodic basis. This may also be true for those products subject to random consumer tastes and preferences; products subject to fads or some other important credence feature. Second, the upstream firm should be in a better position to judge the appeal of the product to consumers.

The distribution of branded products through independent retailers offering a wide variety of goods and services appear to fit this requirement. This is particularly true when the marketing research function resides in the upstream firm. Many contemporary applications of the retail price floor have these two characteristics. For instance, many of the most visible antitrust actions regarding the retail price floor have occurred in the clothing and footwear, consumer electronics, recreational equipment and cosmetics industries. These industries sell an array of constantly changing products through a variety of types of independent retailers.

Appendix

Proof to Proposition 2: We begin by showing that i) the retailer's actions are optimal given its beliefs and that ii) the wholesaler's action are optimal given the retailer's equilibrium strategy. We then demonstrate that iii) the retailer's beliefs are based on priors updated by Bayes's Rule given the equilibrium strategies and that iv) the retailer's out-of-equilibrium beliefs assign positive probability only to the type least harmed by the retailer's response. Finally, we argue that v) this separating equilibrium is unique.

i) If $w > w_L^*(q_H)$, the belief is that $q_i = q_H$ and $(q_H + w)/2 \equiv \arg \max (p - w)(q_H - p)$.

For $w \leq w_L^*(q_H)$, $(q_L + w)/2 \equiv \arg \max (p - w)(q_L - p)$ since the retailer believes that $q_i = q_L$.

ii) If $q_i = q_H$ and $w > w_L^*(q_H)$, $q_H/2 \equiv \arg \max w(q_H - p_{AI}(w))$. It is easy to confirm that $q_H/2 > w_L^*(q_H)$ for $q_H < 3$ and, therefore, $w_{AI}(q_H) = w_L^*(q_H)$. If $q_i = q_L$ and the wholesaler chooses some $w \leq w_L^*(q_H)$, $1/2 \equiv \arg \max w(q_L - p_{AI}(w))$. However, since $w_L^*(q_H) < 1/2$ and wholesaler profits are strictly increasing for all $w \leq w_L^*$, $w_{AI}(1) = w_L^*(q_H)$.

iii) First note that equilibrium wholesaler profits for $q_i = q_H$ and $w = q_H/2$ are $q_H^2/8$. Next note that the variable $w_L^*(q_H)$ is defined such that $q_H^2/8 = w_L^*(q_H)[2q_H - 1 - w_L^*(q_H)]/2$. Call this equality E1. The RHS of E1 represents the profits a high-type wholesaler can earn by setting the wholesale price to $w_L^*(q_H)$ given retailer beliefs. The text demonstrates that $1/3 \leq w_L^*(q_H) < 1/2$ for $1 < q_H < 3$. If $w = q_H/2$, then $\pi(q_H | q_H/2) = 1$ since the LHS of E1 exceeds the RHS of E1; the high-type wholesaler earns strictly higher profits by choosing $w = q_H/2$ rather than $w = w_L^*(q_H)$. $1 < q_H < 3$. If $w = w_L^*(q_H)$, then $\pi(1 | w_L^*(q_H)) = 1$ since

the RHS of E1 exceeds the LHS of E1; the low-type wholesaler earns strictly higher profits by choosing $w = w_L^*(\mathbf{q}_H)$ rather than $w = \mathbf{q}_H / 2$.

iv) Suppose the wholesaler selects some non-equilibrium price $w > w_L^*$, (i.e., $w \neq \mathbf{q}_H / 2$). Since the retailer believes that $\mathbf{q}_i = \mathbf{q}_H$, the high-type wholesaler reduces its profits by $L(\text{high-type} | \mathbf{q}_H) = [\mathbf{q}_H^2 / 4 - w(\mathbf{q}_H - w)] / 2$ while the low-type wholesaler harms itself by $L(\text{low-type} | \mathbf{q}_H) = \{w_L^*(\mathbf{q}_H)[1 - w_L^*(\mathbf{q}_H)] - w(2 - \mathbf{q}_H - w)\} / 2$. One can (tediously) show that $L(\text{high-type} | \mathbf{q}_H) < L(\text{low-type} | \mathbf{q}_H)$ for $1 < \mathbf{q}_H \leq 2$ and, therefore, that the retailer's out-of-equilibrium belief that $\mathbf{q}_i = \mathbf{q}_H$ is consistent with the proposed refinement when it sees a $w > w_L^*$. For $w < w_L^*$, since the retailer believes that $\mathbf{q}_i = 1$, the high-type wholesaler reduces its profits by $L(\text{high-type} | 1) = [\mathbf{q}_H^2 / 4 - w(2\mathbf{q}_H - 1 - w)] / 2$ while the low-type wholesaler harms itself by $L(\text{low-type} | 1) = \{w_L^*(\mathbf{q}_H)[1 - w_L^*(\mathbf{q}_H)] - w(1 - w)\} / 2$. Since one can show that $L(\text{high-type} | 1) > L(\text{low-type} | 1)$ for $1 < \mathbf{q}_H \leq 2$, the retailer's out-of-equilibrium belief that $\mathbf{q}_i = \mathbf{q}_L$ is consistent with the refinement when it sees a $w < w_L^*$.

v) (Sketch of uniqueness) It is impossible to find another wholesale price pair $\{w'_{AI}(\mathbf{q}_L), w'_{AI}(\mathbf{q}_H)\}$ that both maximizes the profits of the high-type wholesaler and provides no strict incentive to mimic the low-type for $\mathbf{q}_i = \mathbf{q}_H$.

Proof to Proposition 3: As above, we begin by showing that i) the retailer's actions are optimal given its beliefs and that ii) the wholesaler's actions are optimal given the retailer's equilibrium strategy. Unlike above, we must insure that the retailer's equilibrium strategy satisfies the retail price floor. Finally, we demonstrate that iii) the retailer's beliefs are based on

priors updated by Bayes's Rule given equilibrium strategies, that iv) the retailer's out-of-equilibrium beliefs assign positive probability only to the type least harmed by the retailer's response, and that v) the equilibrium is unique.

i) If $w > (1 + \mathbf{q}_H)/4$, $(\mathbf{q}_H + w)/2 \equiv \arg \max(p - w)(\mathbf{q}_H - p)$ since the retailer's belief is that $\mathbf{q}_i = \mathbf{q}_H$ and. For $w < (1 + \mathbf{q}_H)/4$, $(1 + w)/2 \equiv \arg \max(p - w)(\mathbf{q}_L - p)$ since the belief is that $\mathbf{q}_i = 1$ and. However, since $(1 + w)/2 < 3(1 + \mathbf{q}_H)/8$ for $w < (1 + \mathbf{q}_H)/4$, the retail price floor is binding. For $w = (1 + \mathbf{q}_H)/4$, $(1 + \mathbf{q}_H + 2w)/4 \equiv \arg \max(p - w)(\mathbf{q}_L - p)$ since the retailer believes that $E(\mathbf{q}_i) = (1 + \mathbf{q}_H)/2$.

ii) If $\mathbf{q}_i = \mathbf{q}_H$, the wholesaler earns $(1 + \mathbf{q}_H)(5\mathbf{q}_H - 3)/32$ in equilibrium. For deviations $w > w_{AIF}(\mathbf{q}_H)$, its profits are maximized at $\mathbf{q}_H^2/8$ when $w = \mathbf{q}_H/2$. It is clear that for all $1 < \mathbf{q}_H$, $\mathbf{q}_H^2/8 < (1 + \mathbf{q}_H)(5\mathbf{q}_H - 3)/32$, so the high-type wholesaler has no incentive to choose some $w > w_{AIF}(\mathbf{q}_H)$. For deviations $w < w_{AIF}(\mathbf{q}_H)$, the wholesaler's profits are strictly increasing in the wholesale price given the retail price floor and maximized when $w = w_{AIF}(\mathbf{q}_H)$. If $\mathbf{q}_i = \mathbf{q}_L$, the wholesaler earns $(1 + \mathbf{q}_H)(8 - \mathbf{q}_H)/32$ in equilibrium. For deviations $w > w_{AIF}(\mathbf{q}_H)$, its profits are maximized at $(2 - \mathbf{q}_H)^2/8$ when $w = (2 - \mathbf{q}_H)/2$. It is clear that for all $1 < \mathbf{q}_H$, $(2 - \mathbf{q}_H)^2/8 < (1 + \mathbf{q}_H)(5\mathbf{q}_H - 3)/32$, so the low-type wholesaler has no incentive to choose some $w > w_{AIF}(\mathbf{q}_H)$. For deviations $w < w_{AIF}(\mathbf{q}_H)$, the wholesaler's profits are strictly increasing in the wholesale price given the retail price floor and maximized when $w = w_{AIF}(\mathbf{q}_H)$.

iii) Since $w_{AIF} = (1 + \mathbf{q}_H)/4$ for both \mathbf{q}_i , $i = L, H$, the retailer's equilibrium posterior beliefs are identical to its priors.

iv) Suppose the wholesaler selects some non-equilibrium price $w > w_{AIF}$. Since the retailer believes that $q_i = q_H$, the high-type wholesaler reduces its profits by $L(\text{high-type} | q_H) = [(1 + q_H)(5q_H - 3)]/32 - w(q_H - w)/2$ while the low-type wholesaler reduces its profits by $L(\text{low-type} | q_H) = [(1 + q_H)(8 - q_H)]/32 - w(2 - q_H - w)/2$. Since $L(\text{high-type} | q_H) < L(\text{low-type} | q_H)$, the retailer's out-of-equilibrium belief that $q_i = q_H$ is consistent with the proposed refinement when it sees a $w > w_{AIF}$. Suppose the wholesaler selects some non-equilibrium price $w < w_{AIF}$. Since the retailer believes that $q_i = 1$, the high-type wholesaler reduces its profits by $L(\text{high-type} | q_H) = [(1 + q_H)(5q_H - 3)]/32 - w(2q_H - 1 - w)/2$ while the low-type wholesaler reduces its profits by $L(\text{low-type} | q_H) = [(1 + q_H)(8 - q_H)]/32 - w(1 - w)/2$. Since $L(\text{high-type} | 1) > L(\text{low-type} | 1)$, the retailer's out-of-equilibrium belief that $q_i = 1$ is consistent with the proposed refinement when it sees a $w < w_{AIF}$.

v) (Sketch of uniqueness) Corollary 4 shows that all other proposed price floors fail to maximize the profits of the wholesaler.

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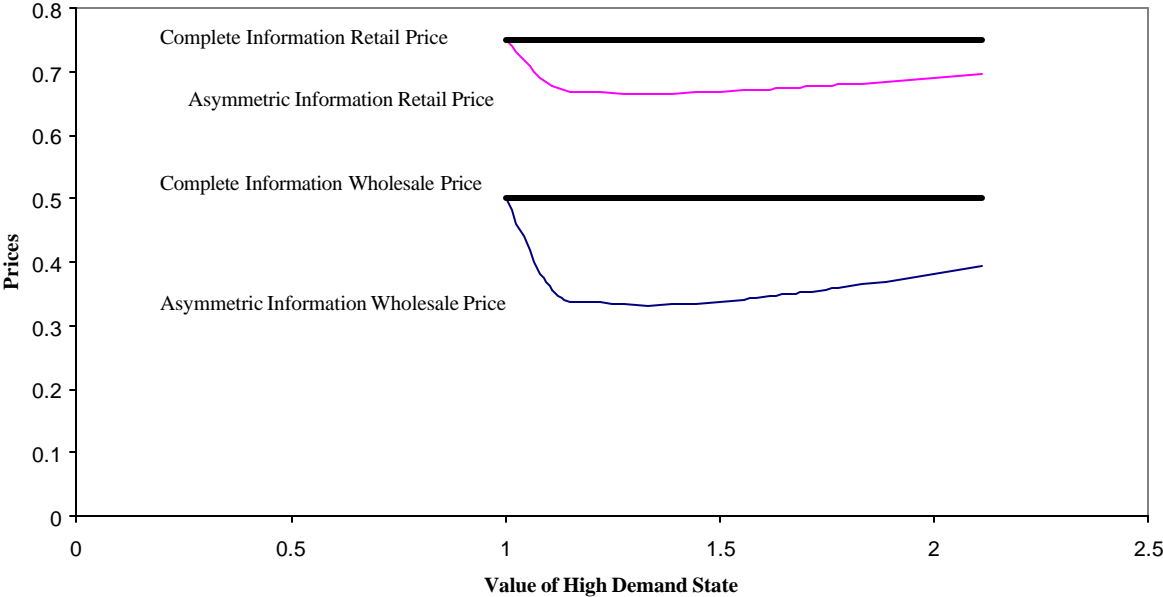
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**FIGURE 1:
Equilibrium Prices in the Low Demand State**



**FIGURE 3:
Increase in Wholesaler Expected Profits with a Retail Price Floor**

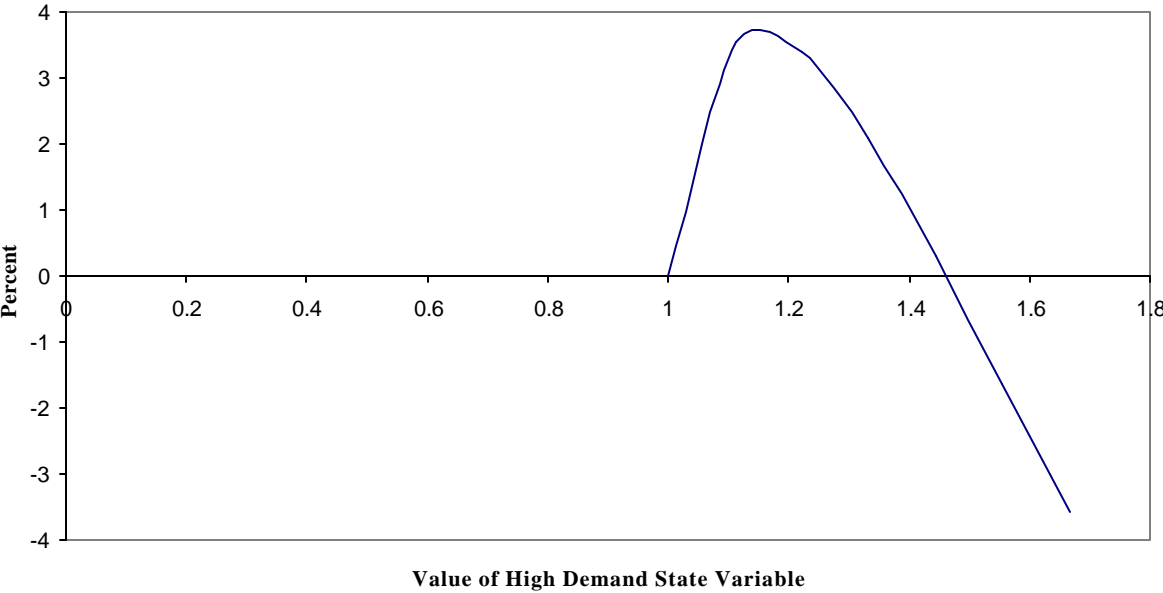


FIGURE 2:
Timeline of the Linear Distribution Model with a Retail Price Floor

