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Isabelle Brocas and Juan D. Carrillo

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University of Southern California Law School
Los Angeles, CA 90089-0071

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Isabelle Brocas
USC and CEPR

Juan D. Carrillo
USC and CEPR

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Abstract

This paper presents a model where individuals have imperfect information about their preferences (or the environment) and there is an opportunity cost of learning. It shows that the endogenous decision to collect information before taking an action creates a *systematic bias* in the aggregate behavior of a population of rational, profit-maximizing agents. More precisely, individuals will favor actions with large payoff-variance, i.e., those which may potentially generate the highest benefits even if they may also generate the biggest losses. The paper thus concludes that systematically biased choices do not necessarily imply that agents have irrational, systematically biased beliefs. It also provides testable implications about the propensity of individuals to incur different types of errors. Some applications such as biases in judicial decision-making and career choices are discussed.

*We thank participants in several seminars for helpful comments. Address for correspondence: Juan D. Carrillo, Department of Economics, University of Southern California, 3620 S. Vermont Ave., Los Angeles, CA - 90089, e-mail: <juandc@usc.edu>.

1 Motivation

It has been long argued in psychology that individuals have a systematically biased view of themselves and of the world in general. The recent economic literature has approached the issue of biased perceptions and biased behavior from different angles. One line of research takes human cognitive limitations as exogenously given (based mainly on introspection, casual observation and the previously mentioned research) and discusses their economic consequences. To give only one representative example, in behavioral finance there are numerous recent studies on the effects, persistence and evolution of entrepreneurial optimism and entrepreneurial overconfidence in business decision-making.¹ Another line of research questions whether biased perceptions and choices are *always* the result of a limited cognitive ability to process information. In other words, instead of exogenously assuming the existence of a bias in beliefs, this second strand of the literature provides microeconomic foundations for some observed systematic biases in behavior. The arguments are based on hyperbolic discounting (Carrillo and Mariotti (2000), Bénabou and Tirole (2001 and 2002)), regret and rejoicing (Loomes and Sugden (1997)), anticipatory feelings (Caplin and Leahy (2001 and 2002), Palacios-Huerta (2004)), self-signaling (Bodner and Prelec (1997)) and other utilities derived from beliefs (Yariv (2001 and 2002)) among others. These theories have received an important support due to their intuitive appeal and their capacity to render the *homo-economicus* more human. At the same time, all these models rely on some elements that depart from the standard neoclassical utility paradigm: hyperbolic discounting instead of exponential discounting or a utility enjoyed from beliefs rather than only from outcomes. Mainly for this reason, they have also provoked some fierce criticisms (see e.g. Read (2001) and Rubinstein (2003) for arguments against hyperbolic discounting and Eliaz and Spiegler (2003) for arguments against a direct inclusion of beliefs in the utility function).

In this paper, individuals with imperfect knowledge about themselves (or about some element of the environment) choose between alternatives with different risks. We argue that if learning is feasible but costly, then the endogenous decision to collect information generates a *systematic* and *testable* bias in the aggregate behavior of a population of indi-

¹See e.g. Daniel et al. (1998), Manove (1999), Manove and Padilla (1999), Bernardo and Welch (2001) and Gervais and Odean (2001) out of a long list.

viduals who do not derive utility from beliefs and are time-consistent, profit maximizers, rational processors of information. The paper thus falls in the second line of research discussed above, except that no element of our theory departs from the standard postulates of dynamic choice under uncertainty.²

To present and illustrate our theory, consider the following stylized example. A city has two judges identical in most respects: given the same belief about the culpability of a prisoner, not only they both prefer the same sentence (convict or release), they even incur the same utility loss if their preferred sentence is not executed. There is however one subtle difference between them: releasing the prisoner has the greatest variance in payoffs for the first judge (i.e., highest utility if innocent and lowest utility if guilty) whereas convicting the prisoner has the greatest variance in payoffs for the second judge (i.e., highest if guilty and lowest if innocent). Judges initially share the same imperfect information about the culpability of the accused and can acquire extra evidence at the expense of delaying the sentence. Should the prisoner be concerned about which judge is assigned to his case?

Given the identical behavior and utility loss of both judges for any given belief, one could think that they are equally likely to commit any given mistake. However, this intuition is incorrect: the first judge is more likely to release guilty suspects and less likely to convict innocent suspects than the second judge. Therefore, all prisoners will strictly prefer to be on a trial with the first judge, independently of their culpability.

The key for the result is the opportunity cost of learning. Suppose that the preliminary evidence states that the suspect is likely to be innocent. In this case, the first judge has a higher opportunity cost than the second one to keep accumulating evidence: he is tempted to stop the information acquisition process, and enjoy the high expected payoff of his (hopefully correct) decision to release the prisoner. Conversely, when the preliminary evidence states that the prisoner is guilty, his cost of continuing the acquisition of information is lower than for the second judge, given his relatively smaller variance in payoffs between convicting a guilty and an innocent suspect. He is therefore more likely to keep learning, with the corresponding likelihood of reversing his prior. Summing up,

²Needless to say, we do not claim that imperfect knowledge and endogenous information acquisition provide an explanation for *all* the biases documented in psychology and economics. In this respect, the paper just adds one new element to the discussion which has some interesting properties: simple and standard premises together with testable implications.

these two judges would behave identically if the amount of information collected were fixed or exogenous. However, the asymmetry in the total payoff of making the right decision combined to the costly endogenous choice of learning implies that, in expectation, they will commit systematically different types of errors.

The reader may find obvious that each judge favors the action that has the potential to yield highest payoff (in equilibrium, the first judge releases more innocent and the second one convicts more guilty suspects). However, one should realize that by adopting such attitude, judges are also committing more often the mistakes that are most costly (the first judge releases more guilty and the second one convicts more innocent suspects).

The result has also a different lecture. Suppose that agents with imperfect self-knowledge about their talent decide between high-risk and low-risk careers. With endogenous learning, the fraction of agents who eventually opt for the high-risk (respectively low-risk) career will be greater (respectively smaller) than the objective fraction of individuals with a talent for it. This alternative interpretation is also explored in the paper.

The models developed independently by Zabochnik (2004) and Santos-Pinto and Sobel (2004) are closely related to ours. Both works concentrate on a single activity that requires ability and show that agents may perceive themselves as better than their objective ranking. The argument in the first paper is based on an opportunity cost of learning (as in our paper) and an exogenous utility function convex in ability. Under appropriate initial conditions on the discount factor, the initial ability and the degree of convexity of the utility, only individuals with an expected ability below a certain threshold experiment, generating the bias. The second paper assumes that agents with heterogeneous preferences about which skills are valuable can invest in improving them. Each agent invests optimally given his preferences but evaluates the skills of others according to his own criteria rather than the criteria of others. As a result, and contrary to ours and Zabochnik's paper, the fraction of individuals who place themselves in the top x percentile of the population exceeds x for all x (rather than only for some x). Our setting is different from these two papers in that our agents choose between two alternatives with different risks. The systematic bias in favor of one activity automatically implies a systematic bias against the other. That way, we obtain testable predictions about which activity will be favored and

which one will be avoided, and therefore also about the propensity to commit different types of errors *exclusively as a function of the payoff-variance of the different alternatives*.

Last, since we build a model of costly learning with an optimal stopping rule, the paper is tangentially related to the literatures on optimal experimentation (see e.g. Aghion et al. (1991), Bolton and Harris (1999) or Keller and Rady (1999) for some representative examples of papers in this literature) and investment under uncertainty (see e.g. Dixit and Pindyck (1994) for a survey).

The plan of the paper is the following. We first present a model with two agents with imperfect information about the state of nature who choose between two actions. For any given belief, the difference in expected utility between the two actions is the same for both agents (section 2). We show that their different incentives to acquire information affects their behavior and expected errors (section 3). We then argue that the model and the results immediately extend to the case of one agent with imperfect self-knowledge who learns about his own preferences and manipulates his own choices (section 4). Last, we offer some concluding remarks (section 5).

2 A model of biased behavior

2.1 States, actions and utilities

We consider the following model. There are two types of agents in the economy ($i \in \{1, 2\}$). A type- i agent chooses an action $\gamma_i \in \{a, b\}$. His utility $u_i(\cdot)$ depends on his action γ_i and the state of the economy $s \in \{A, B\}$, which is common to all agents. Agents initially have imperfect knowledge about the state. They share a prior belief p that the true state is A , that is, $\Pr(A) = p$ and $\Pr(B) = 1 - p$.

Type-1 and type-2 agents have different preferences, which translate into different functional representations of their utility. However, we will assume that for any given belief $p \in [0, 1]$, they both have the same difference in expected utility between the two possible actions. This means not only that they have the same preferred action when confronted to the same evidence, but also that they have the same willingness to pay in order to have the freedom of choosing which action they take. We will say that these two types of agents “FOR IDENTICAL BELIEFS ARE IDENTICAL IN BEHAVIOR AND UTILITY

DIFFERENCE” (IBIBUD). The property is summarized as follows.

Definition *Type-1 and type-2 agents are IBIBUD if and only if:*

$$E[u_1(a) - u_1(b)] = E[u_2(a) - u_2(b)] \quad \forall p$$

which, in particular, implies that $\arg \max_{\gamma_1} E[u_1(\gamma_1)] \equiv \arg \max_{\gamma_2} E[u_2(\gamma_2)]$ for all p .

For expositional purposes, we will consider a simple representation of the utility functions of type-1 and type-2 agents:

$$u_1(a) = \begin{cases} h & \text{if } s = A \\ -h & \text{if } s = B \end{cases} \quad \text{and} \quad u_1(b) = \begin{cases} -l & \text{if } s = A \\ l & \text{if } s = B \end{cases}, \quad (1)$$

$$u_2(a) = \begin{cases} l & \text{if } s = A \\ -l & \text{if } s = B \end{cases} \quad \text{and} \quad u_2(b) = \begin{cases} -h & \text{if } s = A \\ h & \text{if } s = B \end{cases}, \quad (2)$$

with $h > l > 0$. Under this representation, action a has the greatest variance in payoffs for agent 1 and action b the greatest variance in payoffs for agent 2. Given (1) and (2), the IBIBUD property translates into:

$$E[u_i(a) - u_i(b)] = (h+l)(2p-1) \quad \forall i \Rightarrow \gamma_i = a \text{ if } p > 1/2 \text{ and } \gamma_i = b \text{ if } p < 1/2 \quad \forall i.$$

Figure 1 provides a graphical representation of these utilities.

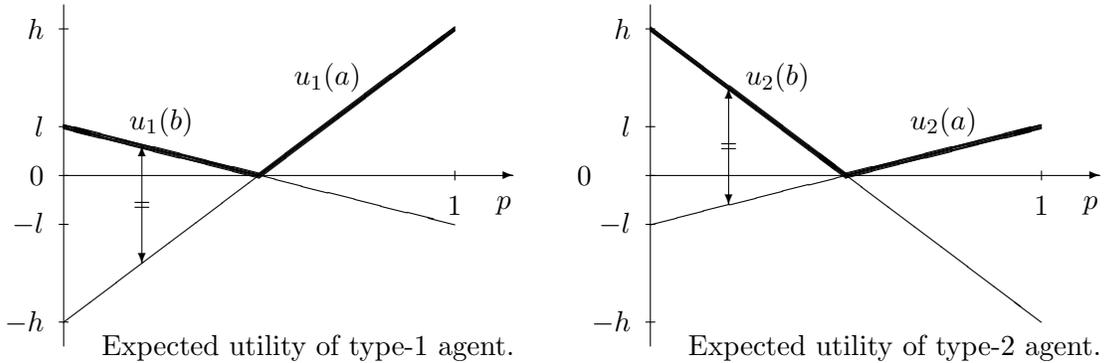


Figure 1. Utility representations for type-1 and type-2 agents.

The theory on reveal preferences states that utility functions are only modelling devices used to represent rank-orders of preferences; actions and (sometimes) payments incurred

in order to choose actions are usually the observable variables from which we deduce the preferences of individuals and construct the utility representations. Therefore, one could think that two agents with preferences that are obviously different given their different utility representations, but who satisfy the IBIBUD property (as the ones represented by types 1 and 2) should be indistinguishable, as long as the set of choices is restricted to a and b . This intuition is correct either when information is freely available or exogenously given, but what happens when we allow individuals to decide how much costly information they collect?

2.2 Information

In order to answer this question, we need to introduce the information acquisition technology. Learning is formalized in the simplest possible way. We denote by $\tau_{i,t}$ the decision of agent i at a given date $t \in \{0, 1, \dots, T - 1\}$, where T is finite but arbitrarily large. At each date, his options are either to take the optimal action conditional on his current information ($\tau_{i,t} = \gamma_i \in \{a, b\}$) or to wait until the following period ($\tau_{i,t} = w$). The action is irreversible, so if the agent undertakes it, then payoffs are realized and the game ends. Waiting has costs and benefits. On the one hand, the delay implied by the decision to wait one more period before acting is costly. We denote by $\delta (< 1)$ the discount factor. On the other hand, the agent obtains between dates t and $t + 1$ one signal $\sigma \in \{\alpha, \beta\}$ imperfectly correlated with the true state. Information increases the confidence of the agent's beliefs and therefore improves the quality of his future decision. As long as the agent waits, he keeps the option of undertaking action a or b in a future period, except at date T in which waiting is not possible anymore, so the agent's options are reduced to actions a and b .³ The relation between signal and state is the following:

$$\Pr[\alpha | A] = \Pr[\beta | B] = \theta \quad \text{and} \quad \Pr[\alpha | B] = \Pr[\beta | A] = 1 - \theta,$$

where $\theta \in (1/2, 1)$ captures the accuracy of information: as θ increases, the informational content of a signal σ increases (when $\theta \rightarrow 1/2$ signals are completely uninformative, and when $\theta \rightarrow 1$ one signal perfectly informs the agent about the true state).⁴

³A finite horizon game ensures the existence of a unique stopping rule at each period that can be computed by backward induction. By setting T arbitrarily large we can determine the limiting properties of this optimal stopping rule.

⁴It is formally equivalent to increase the correlation between signal and state or to increase the number of signals between two dates (both can be captured with the parameter θ).

Suppose that a number n_α of signals α and a number n_β of signals β are revealed during the $n_\alpha + n_\beta$ periods in which the agent waits. Using standard statistical techniques, it is possible to compute the agent's posterior belief about the state of the economy:

$$\begin{aligned} \Pr(A | n_\alpha, n_\beta) &= \frac{\Pr(n_\alpha, n_\beta | A) \Pr(A)}{\Pr(n_\alpha, n_\beta | A) \Pr(A) + \Pr(n_\alpha, n_\beta | B) \Pr(B)} \\ &= \frac{\theta^{n_\alpha - n_\beta} \cdot p}{\theta^{n_\alpha - n_\beta} \cdot p + (1 - \theta)^{n_\alpha - n_\beta} \cdot (1 - p)} \end{aligned}$$

It is interesting to notice that the posterior depends only on the difference between the number of signals α and the number of signals β . So, roughly speaking, two different signals “cancel out”. The relevant variable which will be used from now on is $n \equiv n_\alpha - n_\beta \in \mathbb{Z}$. Also, we define the posterior probability $\mu(n) \equiv \Pr(A | n_\alpha, n_\beta)$.⁵ Last, when solving the model, we will treat n as a real number (instead of an integer as we should in order to be rigorous). This mathematical abuse is made only for technical convenience and it does not affect the substance of our results.

Before solving the game, we want to provide a stylized example that illustrates the meaning of the IBIBUD property, the utility representations (1) and (2), and the costly decision to acquire information.

2.3 An example: court judgement under civil law

As informally suggested in the introduction, the main ingredients of our model may capture judicial decisions. Judge i ($\in \{1, 2\}$) must choose whether to release (action a) or convict (action b) an offender. The prisoner is either innocent (state A) or guilty (state B). The judge's prior belief of the offender being innocent is $p = \Pr(A)$. The judge can acquire information about the culpability of the accused (signals σ) at the cost of delaying his sentence. According to (1) and (2), for any belief p , the differential in utility between convicting and releasing the offender is the same for both types of judges (IBIBUD property). In particular, they both prefer to release the prisoner if his probability of being innocent is above a certain threshold (in our case, $1/2$) and convict him otherwise. The main difference is that letting the prisoner free is the riskiest choice for judge 1 (payoff $u_1(a) \in \{-h, h\}$) whereas convicting him is the riskiest choice for judge 2

⁵Some properties of $\mu(n)$ are: (i) $\lim_{n \rightarrow -\infty} \mu(n) = 0$, (ii) $\lim_{n \rightarrow +\infty} \mu(n) = 1$, and (iii) $\mu(n+1) > \mu(n) \forall n$.

(payoff $u_2(b) \in \{-h, h\}$).⁶

3 Information acquisition and optimal decision-making

3.1 Option value of waiting and optimal stopping rule

In this section, we show that under costly acquisition of information two IBIBUD agents may behave in a systematically different way. Given the information revelation structure presented in section 2.2, agents face a trade-off between delay and information. This trade-off has been extensively analyzed in the literature on investment under uncertainty. In order to find the optimal stopping rule, we first determine the value function V_i^t that a type- i agent maximizes at date t . It can be written as:

$$V_1^t(n) = \begin{cases} \max\left\{h(2\mu(n)-1), \delta\left[\nu(n)V_1^{t+1}(n+1) + (1-\nu(n))V_1^{t+1}(n-1)\right]\right\} & \text{if } \mu(n) \geq \frac{1}{2} \\ \max\left\{l(1-2\mu(n)), \delta\left[\nu(n)V_1^{t+1}(n+1) + (1-\nu(n))V_1^{t+1}(n-1)\right]\right\} & \text{if } \mu(n) < \frac{1}{2} \end{cases} \quad (3)$$

$$V_2^t(n) = \begin{cases} \max\left\{l(2\mu(n)-1), \delta\left[\nu(n)V_2^{t+1}(n+1) + (1-\nu(n))V_2^{t+1}(n-1)\right]\right\} & \text{if } \mu(n) \geq \frac{1}{2} \\ \max\left\{h(1-2\mu(n)), \delta\left[\nu(n)V_2^{t+1}(n+1) + (1-\nu(n))V_2^{t+1}(n-1)\right]\right\} & \text{if } \mu(n) < \frac{1}{2} \end{cases} \quad (4)$$

where $\nu(n) = \mu(n)\theta + (1-\mu(n))(1-\theta)$. In words, at date t and given a difference of signals n that implies a posterior $\mu(n) > 1/2$, type-1 agent chooses between taking action a with expected payoff $h\mu - h(1-\mu)$ or waiting. In the latter case, signal α (respectively β) is received with probability ν (respectively $1-\nu$) and the value function in the following period $t+1$ becomes $V_1^{t+1}(n+1)$ (respectively $V_1^{t+1}(n-1)$), discounted at the rate δ . For $\mu(n) < 1/2$, the argument is the same, except that the optimal action if the agent does not wait is b with payoff $-l\mu + l(1-\mu)$. The reasoning for a type-2 agent is the same. Given (3) and (4), we can determine the optimal strategy for each type. This technical result is key for the subsequent analysis.

Lemma 1 *For all $\delta < 1$ and $h > l > 0$, there exist $(n_{1,t}^*, n_{1,t}^{**}, n_{2,t}^*, n_{2,t}^{**})$ at each date t s.t.:*

(i) $\tau_{1,t} = b$ if $n \leq n_{1,t}^*$, $\tau_{1,t} = a$ if $n \geq n_{1,t}^{**}$ and $\tau_{1,t} = w$ if $n \in (n_{1,t}^*, n_{1,t}^{**})$.

(ii) $\tau_{2,t} = b$ if $n \leq n_{2,t}^*$, $\tau_{2,t} = a$ if $n \geq n_{2,t}^{**}$ and $\tau_{2,t} = w$ if $n \in (n_{2,t}^*, n_{2,t}^{**})$.

*By the symmetry of types 1 and 2: $\mu(n_{1,t}^{**}) = 1 - \mu(n_{2,t}^*)$ and $\mu(n_{1,t}^*) = 1 - \mu(n_{2,t}^{**})$.*

⁶In other words, judge 1 is most willing to release an innocent and most averse to convict an innocent whereas judge 2 is most willing to convict a guilty and most averse to release a guilty prisoner.

Last, but most importantly: $\mu(n_{1,t}^*) < \mu(n_{2,t}^*) < 1/2 < \mu(n_{1,t}^{**}) < \mu(n_{2,t}^{**})$.

Proof. See Appendix. □

The idea is simple. Agents trade-off the costs of delaying their choice between actions a and b with the benefits of acquiring a more accurate information. When $\mu(n) > 1/2$, waiting becomes more costly as n increases, because delaying the action one extra period reduces the expected payoff by an amount proportional to $2\mu(n) - 1$. Similarly, when $\mu(n) < 1/2$, waiting becomes more costly as n decreases, because delaying the action reduces the expected payoff by an amount proportional to $1 - 2\mu(n)$. In other words, at each date t , there are two cutoffs $\mu(n_{i,t}^{**}) > 1/2$ and $\mu(n_{i,t}^*) < 1/2$ for a type- i agent. When $\mu \geq \mu(n_{i,t}^{**})$, the individual is “reasonably confident” that the true state is A , and when $\mu \leq \mu(n_{i,t}^*)$, he is “reasonably confident” that the true state is B . In either case, the marginal gain of improving the information about the true state is offset by the marginal cost of a reduction in the expected payoff due to the delay it implies. As a result, he strictly prefers to stop learning and take his optimal action. For intermediate beliefs, that is when $\mu(n) \in (\mu(n_{i,t}^*), \mu(n_{i,t}^{**}))$, a type- i agent prefers to keep accumulating evidence.

The most interesting property of these cutoffs is that:

$$\mu(n_{1,t}^{**}) - 1/2 < 1/2 - \mu(n_{1,t}^*) \quad \text{and} \quad \mu(n_{2,t}^{**}) - 1/2 > 1/2 - \mu(n_{2,t}^*)$$

It states that the confidence of a type-1 agent on the true state being A when he takes action a is smaller than his confidence on the true state being B when he takes action b . The opposite is true for a type-2 agent. Comparing the two agents, it means that a type-1 agent will need fewer evidence in favor of A in order to decide to stop collecting news and take action a and more evidence in favor of B in order to stop collecting news and take action b than a type-2 agent.

The intuition for this result is simply that, given the delay associated to the accumulation of evidence, the marginal cost of learning is a function of the agent’s belief and expected payoff of taking an action. For a type-1 individual, it is proportional to $h(1 - \delta)$ when $\mu > 1/2$ and to $l(1 - \delta)$ when $\mu < 1/2$. As a result, and other things being equal, it is relatively less interesting to keep experimenting when the action currently optimal is a rather than b . The argument for a type-2 agent is symmetric. The shape of these cutoffs is graphically represented in Figure 2.

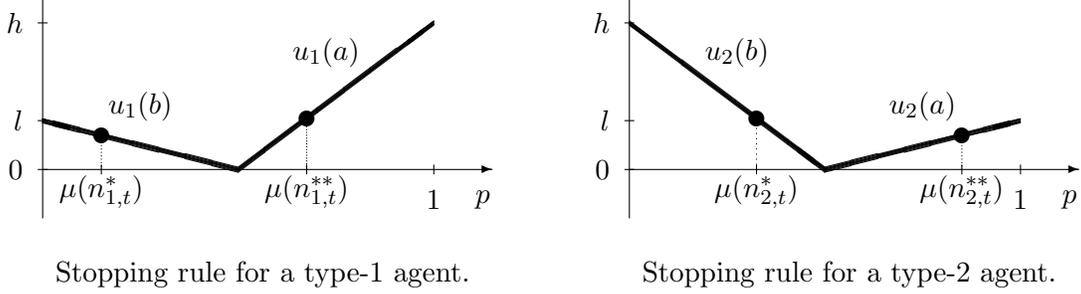


Figure 2. Stopping rules for type-1 and type-2 agents.

Suppose now that $T \rightarrow \infty$. This means that $n_{i,t}^* \rightarrow n_i^*$ and $n_{i,t}^{**} \rightarrow n_i^{**}$ for all t . Denote by $\Pr(\tau_i = \gamma_i \mid s)$ the probability that a type- i individual eventually undertakes action $\gamma_i \in \{a, b\}$ when the true state is $s \in \{A, B\}$. Also, let $\mu^{**} \equiv \mu(n_1^{**})$ and $\mu^* \equiv \mu(n_1^*)$ (which means that $\mu(n_2^*) = 1 - \mu^*$ and $\mu(n_2^{**}) = 1 - \mu^{**}$). Last, suppose that type-1 and type-2 agents start with the same prior belief $p \in (1 - \mu^{**}, \mu^{**})$. Each agent chooses the amount of information he collects before undertaking an action and the signals obtained by the agents are independent. Their optimal stopping rule is given by Lemma 1. In the main proposition of the paper, we compare the relative probabilities that each agent undertakes action a and action b .

Proposition 1 For all $p \in (1 - \mu^{**}, \mu^{**})$, $\delta < 1$, $h > l > 0$ and when $T \rightarrow \infty$ we have:

- (i) $\Pr(\tau_1 = a \mid B) > \Pr(\tau_2 = a \mid B)$ and $\Pr(\tau_1 = b \mid A) < \Pr(\tau_2 = b \mid A)$.

Judge 1 releases more guilty suspects and convicts fewer innocent suspects than judge 2.

- (ii) $\frac{\partial \Pr(\tau_1 = a \mid s)}{\partial h} > 0 > \frac{\partial \Pr(\tau_2 = a \mid s)}{\partial h}$ and $\frac{\partial \Pr(\tau_1 = a \mid s)}{\partial l} < 0 < \frac{\partial \Pr(\tau_2 = a \mid s)}{\partial l}$ for all s .

Keeping IBIBUD, as the difference in payoffs between actions ($h - l$) increases, the difference in behavior between judges 1 and 2 also increases.

Proof. Part (i) is a direct consequence of $\mu(n_2^{**}) > \mu(n_1^{**})$ and $\mu(n_2^*) > \mu(n_1^*)$. Part (ii) results from the fact that, also by Lemma 1, $\frac{\partial n_1^*}{\partial h} < 0$, $\frac{\partial n_1^{**}}{\partial h} < 0$, $\frac{\partial n_1^*}{\partial l} > 0$, $\frac{\partial n_1^{**}}{\partial l} > 0$ and by symmetry $\frac{\partial n_2^*}{\partial h} > 0$, $\frac{\partial n_2^{**}}{\partial h} > 0$, $\frac{\partial n_2^*}{\partial l} < 0$, $\frac{\partial n_2^{**}}{\partial l} < 0$.

These comparative statics fulfill the purpose of our analysis. However, for the reader interested, the analytical expressions of the probabilities $\Pr(\tau_i \mid s)$ are derived in Brocas

and Carrillo (2004, Lemma 1) for an initial prior p and exogenous stopping posteriors μ^* and μ^{**} .⁷ These are given by: $\Pr(\tau_1 = a \mid A) = \frac{p - \mu^*}{\mu^{**} - \mu^*} \frac{\mu^{**}}{p}$, $\Pr(\tau_1 = a \mid B) = \frac{p - \mu^*}{\mu^{**} - \mu^*} \frac{1 - \mu^{**}}{1 - p}$, $\Pr(\tau_2 = a \mid A) = \frac{p - (1 - \mu^{**})}{\mu^{**} - \mu^*} \frac{1 - \mu^*}{p}$, $\Pr(\tau_2 = a \mid B) = \frac{p - (1 - \mu^{**})}{\mu^{**} - \mu^*} \frac{\mu^*}{1 - p}$. \square

Proposition 1 shows that, even if type-1 and type-2 agents are IBIBUD, they will make systematically different choices, at least in a stochastic sense. As shown in Lemma 1, a type-1 agent is relatively more likely to stop collecting news when the preliminary evidence points towards the optimality of action a than when it points towards the optimality of action b (i.e., when the first few signals are mainly α and not β). Stated differently, the evidence in favor of A needed to induce a type-1 agent to take action a is smaller than the evidence in favor of B needed to induce him to take action b . The opposite is true for a type-2 agent. As a result, in equilibrium, a type-1 agent is more likely to take action a by mistake (i.e., when the true state is B) and less likely to take action b by mistake (i.e., when the true state is A) than a type-2 agent (part (i) of the proposition). Note that the endogenous choice to acquire information is crucial for this result: by definition of IBIBUD, the two types of agents would take action a with the same probability if the number of signals they receive were externally or exogenously imposed. Last, as the difference in the variance of payoffs ($h - l$) increases, the likelihood that the two agents behave differently also increases: type-1 takes more often action a by mistake and less often action b by mistake whereas the opposite is true for type-2 (part (ii) of the proposition). In terms of our judicial example, Proposition 1 suggests that despite the similarities between these two judges, their actual behavior can be substantially different: judge 1 releases guilty suspects more often and convicts innocent suspects less often than judge 2.

The systematic differences in the decisions (and therefore errors) made by the two types of agents have some immediate yet interesting implications.

Proposition 2 (i) *Different types of agents have different effects on the welfare of third parties: independently of their culpability, all suspects prefer judge 1 rather than judge 2.*

(ii) *Agents' types can be inferred from decisions but also from the delay in making them: judge 1 takes less time to release and more time to convict suspects than judge 2.*

⁷The paper uses similar techniques to study a different issue. It analyzes a principal/agent model with incomplete contracting and determines the rents obtained by the former due to his ability to control the flow of public information.

Note that, in our model, agents select a stopping rule that increases the probability of taking the action with highest payoff (the utility for judge 1 of releasing an innocent and for judge 2 of convicting a guilty is h in both cases). This might seem a trivial conclusion. However, the other side of the coin is that, with this strategy, agents are also increasing the probability of making the mistakes that are most costly (the utility for judge 1 of releasing a guilty and for judge 2 of convicting an innocent is $-h$ in both cases). The paper thus provides one simple reason, namely the payoff-variance of the different alternatives in the different states of nature, that explains systematic biases for and against some actions or, more exactly, the propensity of individuals to incur different types of errors.

We conclude this section with some remarks on the model and interpretations.

Remark 1. As it should be clear, when we refer to a “bias” we do not mean that agents are fooled, deceived or misled. Systematic “mistakes” are, by definition, impossible in our setting, given the agents’ rationality in their acquisition and interpretation of information and their optimal choice of action conditional on the information they possess. However, their decisions can be systematically tilted towards some actions and away from others. Technically, the point is simply that the endogenous decision to acquire information cannot affect the first-order moment of beliefs (i.e., the average belief in the population always coincides with the true average). However, it may influence the higher-order moments and, in particular, the *skewness* in the distribution of beliefs. Given a limited set of actions, two populations whose distribution of beliefs have the same average but different skewness will exhibit different aggregate behaviors.

Remark 2. The model relies on irreversibility of actions or no learning after the decision is made. Irreversibility is quite natural in the judicial example, but either assumption can be too extreme in other contexts. Nevertheless, one should realize that partial irreversibility is enough to generate a short run bias towards the riskier of the two alternatives. Moreover, if the environment changes stochastically, information becomes obsolete over time, preventing the agent from learning the state with certainty. In that case, the decision bias may persist in the long run even under partial reversibility.

Remark 3. The model argues that, *ceteris paribus*, agents favor actions with higher rather than lower variance in payoffs and, at some point, we have used the rather convenient labels of “riskier” vs. “safer”. Needless to say, we are not building a general theory of

decision-making under risk: if, in a particular setting, risk-aversion factors crowd-out any other motivation or if the risky action has an infinitely negative payoff in one state, then agents will choose the safe alternative independently of their type. Our model is not design to address this issue.

3.2 Comparative statics

The importance of the effects highlighted in Proposition 1 is an empirical question, interesting but largely beyond the scope of this paper. In this section, we simply want to provide simple numerical examples that give an idea of the propensity of agents to make different types of mistakes. Consider the extreme situation in which $h > 0$ and $l \rightarrow 0$.⁸ From the proof of Proposition 1, the probability that a type- i agent makes the wrong decision is:

$$\begin{aligned} \Pr(\tau_1 = a \mid B) &= \frac{p}{1-p} \times \frac{1-\mu^{**}}{\mu^{**}} & \text{and} & \quad \Pr(\tau_1 = b \mid A) \rightarrow 0 \\ \Pr(\tau_2 = a \mid B) &\rightarrow 0 & \text{and} & \quad \Pr(\tau_2 = b \mid A) = \frac{1-p}{p} \times \frac{1-\mu^{**}}{\mu^{**}} \end{aligned}$$

A type-1 agent will never take action b mistakenly, and a type-2 agent will never take action a mistakenly. Simple comparative statics about the likelihood of taking the wrong action given a prior probability p and a stopping posterior μ^{**} are illustrated in Figure 3.

⁸This means that $n_1^* \rightarrow -\infty$, $n_2^{**} \rightarrow +\infty$ and therefore $\mu^* \rightarrow 0$. This assumption is by no means necessary. However, it allows us to make neat comparative statics by reducing the number of parameters to two (p and μ^{**}).

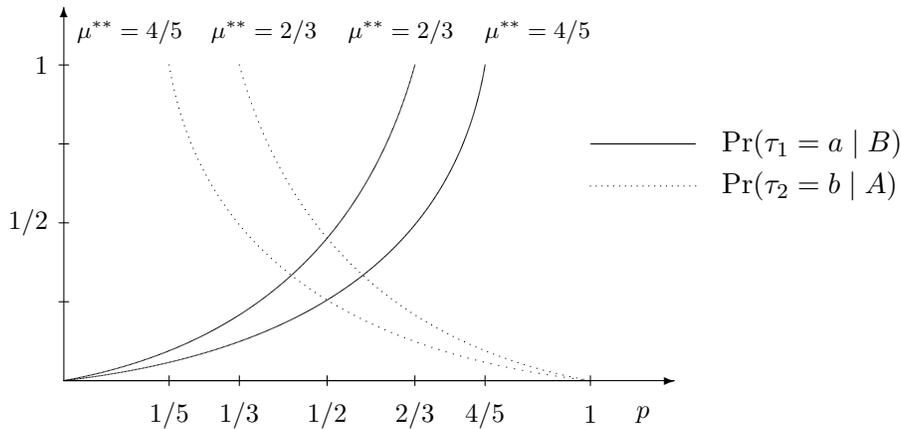


Figure 3. Frequency of mistakes by type-1 and type-2 agents.

As an illustrative case, suppose that half the convicts are guilty ($p = 1/2$) and the stopping rule is $\mu^{**} = 2/3$. Judge 1 releases all the innocent convicts and one-half of the guilty ones ($3/4$ of the population) and convicts one-half of the guilty suspects (the remaining $1/4$ of the population). By contrast, judge 2 releases only one-half of the innocent suspects and convicts all the guilty suspects and one-half of the innocent ones. Last, note that μ^{**} is increasing in δ , and $\lim_{\delta \rightarrow 1} \mu^{**} = 1$. As individuals become more patient, they acquire more information and incur fewer mistakes. If they are infinitely patient, the cost of waiting vanishes. It then becomes optimal for both types to be (almost) perfectly informed before choosing any action, and (almost) no mistake occurs in equilibrium.

3.3 Welfare analysis

Suppose that a welfare maximizing principal can ask several agents their opinion about which action a or b should be taken. For simplicity, we assume that each agent is interested in maximizing the probability of providing the correct appraisal (a if A and b if B), independently of whether the suggestion is followed by the principal or not. Such behavior is rational if the appraisal is publicly observed, the state is ex-post revealed and agents have career-concerns: their payoff is then a function of the quality of their suggestion, and not a function of the final action taken. In this setting, each agent's optimal rule for the acquisition of information coincides with that of Lemma 1, so increasing the number of

agents can only decrease the probability of an incorrect decision.⁹ We assume that the number of agents is fixed but the principal can choose the proportion of type-1 and type-2 individuals. Given that the two types of agents have different biases in the errors they commit, is it optimal to select all agents of the same type or to have appraisals from agents of both types?

To answer this question, we consider the simplest version of our model (which can be generalized in a number of dimensions). We denote by γ_i^j the recommendation made by the j th agent of type- i . We suppose that $l \rightarrow 0$, so that $\Pr(\gamma_1^j = b \mid A) = 0$ and $\Pr(\gamma_2^j = a \mid B) = 0$ for all j . The total number of agents is fixed and equal to n . The principal chooses x , the number of type-1 agents, $n - x$ being the number of type-2 agents. Last, the principal's sole concern is to minimize the probability of a mistake, i.e., it is equally costly to take action a when $s = B$ than action b when $s = A$. If we denote by $\gamma_P \in \{a, b\}$ the action taken eventually by the principal, we have the following result.

Proposition 3 *If $p < 1/2$, then $x = n$. The principal chooses $\gamma_P = a$ if $\gamma_1^j = a \ \forall j$ and $\gamma_P = b$ otherwise. Also, $\Pr(\gamma_P = b \mid A) = 0$ and $\Pr(\gamma_P = a \mid B) = \left(\frac{p}{1-p} \times \frac{1-\mu^{**}}{\mu^{**}}\right)^n$.*

*If $p > 1/2$, then $x = 0$. The principal chooses $\gamma_P = b$ if $\gamma_2^j = b \ \forall j$ and $\gamma_P = a$ otherwise. Also, $\Pr(\gamma_P = b \mid A) = \left(\frac{1-p}{p} \times \frac{1-\mu^{**}}{\mu^{**}}\right)^n$ and $\Pr(\gamma_P = a \mid B) = 0$.*

Proof. Fix x . Given $l \rightarrow 0$, we have $\Pr(\gamma_1 = b \mid A) = 0$ and $\Pr(\gamma_2 = a \mid B) = 0$, so the only possible error arises when all type-1 agents announce $\gamma_1^j = a$ ($j \in \{1, \dots, x\}$) and all type-2 agents announce $\gamma_2^k = b$ ($k \in \{1, \dots, n - x\}$). The remaining question is whether, if this happens, the principal will take action a or action b .

- Suppose that the principal minimizes costs with $\gamma_P = a$. The expected loss is then:

$$L_A(x) = \Pr(B) \cdot \prod_{j=1}^x \Pr(\gamma_1^j = a \mid B) \cdot \prod_{k=1}^{n-x} \Pr(\gamma_2^k = b \mid B) = (1-p) \left(\frac{p}{1-p} \times \frac{1-\mu^{**}}{\mu^{**}}\right)^x$$

So, conditional on taking $\gamma_P = a$, the principal optimally sets $x = n$, and the loss is:

$$L_A(n) = (1-p) \left(\frac{p}{1-p} \times \frac{1-\mu^{**}}{\mu^{**}}\right)^n \tag{5}$$

⁹By contrast, if individuals were rewarded as a function of the quality of the final decision, then they would integrate the behavior of other agents in their choice to acquire information (and, possibly, free-ride accordingly). The optimal stopping rule would then be modified and it would not be always true that increasing the number of agents improves the quality of the final decision. For an analysis of this free-riding problem in juries under different voting rules, see Feddersen and Pesendorfer (1998).

- Suppose that the principal minimizes costs with $\gamma_P = b$. The expected loss is then:

$$L_B(x) = \Pr(A) \cdot \prod_{j=1}^x \Pr(\gamma_1^j = a \mid A) \cdot \prod_{k=1}^{n-x} \Pr(\gamma_2^k = b \mid A) = p \left(\frac{1-p}{p} \times \frac{1-\mu^{**}}{\mu^{**}} \right)^{n-x}$$

So, conditional on taking $\gamma_P = b$, the principal optimally sets $x = 0$, and the loss is:

$$L_B(0) = p \left(\frac{1-p}{p} \times \frac{1-\mu^{**}}{\mu^{**}} \right)^n \quad (6)$$

Last, from (5) and (6): $L_A(n) \leq L_B(0) \Leftrightarrow (1-p) \left(\frac{p}{1-p} \right)^n \leq p \left(\frac{1-p}{p} \right)^n \Leftrightarrow p \leq 1/2$. \square

Proposition 3 states that even if the principal can choose the source of information, a systematic bias in his choice will still be present (similar in nature as before but quantitatively smaller due to the greater amount of information obtained). The idea is simple. Since the principal dislikes equally both types of errors, he will select agents so as to minimize their likelihood of committing a mistake, independently of the nature. We know from Proposition 1 that the likelihood of providing an incorrect appraisal is inversely proportional to the distance between the prior belief and the posterior at which the agent decides to stop collecting evidence and recommends an action (formally, $\mu^{**} - p$ for a type-1 agent and $p - (1 - \mu^{**})$ for a type-2 agent). Hence, if $p < 1/2$, type-1 agents are relatively less likely to mislead the principal than type-2 agents ($|\mu^{**} - p| > |p - (1 - \mu^{**})|$), and therefore it is optimal to pick only type-1 agents. The opposite is true when $p > 1/2$. Overall, fewer mistakes occur as we increase the number of agents who provide an appraisal. However, the systematic bias in the final decision persists.

4 Self-perception, self-influence and self-biased behavior

Consider an adolescent who decides whether to devote his time and energy to pursue a career in sports or to continue his intellectual education. Although at early stages in life it is possible to combine training in both areas, each year of non-exclusive attention decreases the long-run expected return in either domain. Success in sports depends largely on “talent”, an intangible combination of physical strength and coordination, performance under pressure and other abilities intrinsic but unknown to individuals. Young players do not know their own talent, although training and repeated exposure to the activity provides information. The career choice of this individual can be formally represented using our previous model of a type-1 agent, with a being the decision to concentrate on sports, b the

decision to concentrate on intellectual education and w the decision to pursue both activities and wait before making a choice. States A and B denote respectively a person with high talent and low talent for sport (say, relative to his talent for intellectual activities), with p being the individual's own initial assessment of his talent. Last, assuming that the individual is only concerned with earnings, it seems reasonable to argue that payoffs have a higher variance in sports (h or $-h$) than in intellectual endeavors (l or $-l$).

According to our theory, an adolescent will first combine both activities and start learning about his talent. Limited information pointing towards a brilliant sports career (i.e., few α -signals) will persuade him to concentrate on this activity. By contrast, a large amount of information in favor of intellectual ability (i.e., many β -signals) will be needed to induce him to focus on an intellectual career. Overall, we will observe more (less) professional athletes (intellectuals) than the objective fraction of agents with a talent for sports (with a talent for intellectual activities). Also, more agents will mistakenly choose a sport career than mistakenly choose an intellectual career. This theory is obviously simplistic and, as such, should not be taken literally¹⁰ (also, the caveat mentioned in footnote 2 applies here). However, it demonstrates a general point: under imperfect self-knowledge and an opportunity cost of delaying choices, individuals will systematically bias their career decisions towards the alternative with the potential to generate the highest payoff, even at the risk of obtaining the lowest payoff.

Can we label this behavior as “self-influence” or “self-manipulation”? In a sense, the choice of this specific stopping rule in the acquisition of information can be seen as a (conscious or unconscious) attempt towards a self-bias: the individual actively convinces himself that he has good reasons to try and become a soccer superstar. However, we have also proved that such stopping rule is optimal for a rational person given his structure of payoffs and the opportunity cost of learning. Whether we think of it as self-influence or not, the main message is that from an observed or reported systematic bias in the behavior of a population of agents we cannot conclude the existence of an irrational bias in their belief.

¹⁰Note for example that learning about talent in the chosen activity never stops and switching the career focus is always feasible. However, recall that what matters for our theory is partial irreversibility of actions (see Remark 2). Therefore, the same results apply as long as the individual has a handicap in going back to one activity after a number of years entirely dedicated to the other, an assumption that seems reasonable.

5 Concluding remarks

Incentives for decision-making in judicial contexts have received the attention of economists in the past recent years (see Shin (1998), Feddersen and Pesendorfer (1998) and Dewatripont Tirole (1999) among others). Our paper can be seen as a further step in this important research area, as it shows how judges with very similar preferences may end up behaving quite differently and committing opposite types of mistakes. However, we think that our model and results are of interest for a larger class of problems. The paper has explored the distinction between (irrational) systematically biased beliefs and (rational) systematically biased behaviors that result from the endogenous and costly decision to acquire information. We have pointed out as our major conclusion that individuals will tend to bias their choices in favor of actions with highest variance in payoffs across states and away from actions with lowest variance in payoffs across states. In some applications (e.g. career choices), payoffs in the different activities are likely to be endogenously determined and possibly inversely related to the fraction of individuals who choose that career. Adding this general equilibrium element and studying whether this possibility increases or decreases the magnitude of the bias is an interesting extension left for future work.¹¹

Naturally, it would be absurd to pretend that our explanation can account for all the biases documented first in the psychology and now in the behavioral economics literature. First, because the ingredients of our model are not relevant in many settings.¹² Second, because some aggregate beliefs are impossible to reconcile with statistical inference.¹³ And third, because the behavioralist explanations reviewed in the introduction seem to do a good job in some situations. Yet, we feel that adding this extra element to the discussion can be very useful if we want to improve our understanding of the reasons, means and situations where individuals distort their choices.

¹¹We thank J. Zabochnik for suggesting this extension.

¹²Among other things, stakes have to be sufficiently small, otherwise the incentives of individuals to become perfectly informed before choosing their optimal action will crowd-out all other motivations (think for example of a patient deciding whether to learn from the doctor his health state concerning a *curable* disease). Also, incomplete information and costly learning have to be crucial elements at play.

¹³Bayesian inference can be compatible with more than half of the population believing to be above average (we just need to play with the skewness of the distribution). However, the average belief can never exceed the true average.

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Appendix: Proof of Lemma 1

Type-1 agent.

Date T . Denote $V_1^T(n) = \max\{h(2\mu(n) - 1); l(1 - 2\mu(n))\}$ and let:

$$Y_1^t(n) = V_1^t(n) - h(2\mu(n) - 1) \quad \text{and} \quad W_1^t(n) = V_1^t(n) - l(1 - 2\mu(n)).$$

For $t = T$, we have $Y_1^T(n) = \max\{0; (h + l)(1 - 2\mu(n))\}$ and $W_1^T(n) = \max\{0; (h + l)(2\mu(n) - 1)\}$. Since $\mu(n)$ is increasing in n , $W_1^T(n)$ is non-decreasing and $Y_1^T(n)$ is non-increasing in n . Besides, $\lim_{n \rightarrow +\infty} \mu(n) = 1$ and $\lim_{n \rightarrow -\infty} \mu(n) = 0$, so there exists \bar{n} defined by $\mu(\bar{n}) = 1/2$ such that for all $n > \bar{n}$ then $\tau_{1,T} = a$, and for all $n < \bar{n}$ then $\tau_{1,T} = b$.

Date $T - 1$.

Case-1: $n \geq \bar{n}$. $V_1^{T-1}(n) = \max\{h(2\mu(n) - 1); \delta\nu(n)V_1^T(n+1) + \delta(1 - \nu(n))V_1^T(n-1)\}$ and

$$Y_1^{T-1}(n) = \max\{0, -(1 - \delta)h(2\mu(n) - 1) + \delta\nu(n)Y_1^T(n+1) + \delta(1 - \nu(n))Y_1^T(n-1)\}$$

where $Y_1^{T-1}(n)$ is defined on $(\bar{n}, +\infty)$. Since $\nu(n)$ is increasing in n and $Y_1^T(n)$ is non-increasing in n , we can check that the right-hand side (r.h.s.) of $Y_1^{T-1}(n)$ is decreasing in n , and therefore there exists a cutoff $n_{1,T-1}^{**}$ such that for all $n > n_{1,T-1}^{**}$ then $\tau_{1,T-1} = a$, and for all $n \in [\bar{n}, n_{1,T-1}^{**})$ then $\tau_{1,T-1} = w$. To solve the previous equation, the cutoff has to be such that $n_{1,T-1}^{**} + 1 \geq \bar{n}$ and $n_{1,T-1}^{**} - 1 < \bar{n}$, and therefore it is the solution of:

$$0 = h \cdot f(n_{1,T-1}^{**}, \delta) - l \cdot g(n_{1,T-1}^{**}, \delta)$$

where $f(n_{1,T-1}^{**}, \delta) \equiv 2\mu(n_{1,T-1}^{**}) - 1 - \delta\nu(n_{1,T-1}^{**})(2\mu(n_{1,T-1}^{**} + 1) - 1)$ and $g(n_{1,T-1}^{**}, \delta) = \delta(1 - \nu(n_{1,T-1}^{**}))(1 - 2\mu(n_{1,T-1}^{**} - 1))$. Differentiating with respect to h , l and δ we have:¹⁴

$$\begin{aligned} \frac{\partial n_{1,T-1}^{**}}{\partial h} \left[l \cdot g_n(n_{1,T-1}^{**}, \delta) - h \cdot f_n(n_{1,T-1}^{**}, \delta) \right] &= f(n_{1,T-1}^{**}, \delta) \\ \frac{\partial n_{1,T-1}^{**}}{\partial l} \left[h \cdot f_n(n_{1,T-1}^{**}, \delta) - l \cdot g_n(n_{1,T-1}^{**}, \delta) \right] &= g(n_{1,T-1}^{**}, \delta) \\ \frac{\partial n_{1,T-1}^{**}}{\partial \delta} \left[l \cdot g_n(n_{1,T-1}^{**}, \delta) - h \cdot f_n(n_{1,T-1}^{**}, \delta) \right] &= h \cdot f_\delta(n_{1,T-1}^{**}, \delta) - l \cdot g_\delta(n_{1,T-1}^{**}, \delta) \end{aligned}$$

Given $f(n_{1,T-1}^{**}, \delta) > 0$, $g(n_{1,T-1}^{**}, \delta) > 0$, $l \cdot g_n(n_{1,T-1}^{**}, \delta) - h \cdot f_n(n_{1,T-1}^{**}, \delta) < 0$, $h \cdot f_\delta(n_{1,T-1}^{**}, \delta) - l \cdot g_\delta(n_{1,T-1}^{**}, \delta) < 0$, we finally have:

$$\frac{\partial n_{1,T-1}^{**}}{\partial h} < 0, \quad \frac{\partial n_{1,T-1}^{**}}{\partial l} > 0, \quad \frac{\partial n_{1,T-1}^{**}}{\partial \delta} > 0.$$

¹⁴The subscripts n and δ denote a partial derivative with respect to that argument.

Case-2: $n \leq \bar{n}$. $V_1^{T-1}(n) = \max\{l(1 - 2\mu(n)); \delta\nu(n)V_1^T(n+1) + \delta(1 - \nu(n))V_1^T(n-1)\}$ and

$$W_1^{T-1}(n) = \max\{0, -(1 - \delta)l(1 - 2\mu(n)) + \delta\nu(n)W_1^T(n+1) + \delta(1 - \nu(n))W_1^T(n-1)\}$$

where $W_1^{T-1}(n)$ is defined on $(-\infty, \bar{n})$. Since $\nu(n)$ is increasing in n and $W_1^T(n)$ is non-decreasing in n , we can check that the r.h.s. of $W_1^{T-1}(n)$ is increasing in n , and therefore there exists a cutoff $n_{1,T-1}^*$ such that for all $n \in (n_{1,T-1}^*, \bar{n}]$ then $\tau_{1,T-1} = w$, and for all $n < n_{1,T-1}^*$ then $\tau_{1,T-1} = b$. This cutoff has to be such that $n_{1,T-1}^* + 1 > \bar{n}$ and $n_{1,T-1}^* - 1 \leq \bar{n}$, so it is solution of:

$$0 = l \cdot x(n_{1,T-1}^*, \delta) - h \cdot y(n_{1,T-1}^*, \delta)$$

where $x(n_{1,T-1}^*, \delta) = 1 - 2\mu(n_{1,T-1}^*) - \delta(1 - \nu(n_{1,T-1}^*))(1 - 2\mu(n_{1,T-1}^* - 1))$ and $y(n_{1,T-1}^*, \delta) = \delta\nu(n_{1,T-1}^*)(2\mu(n_{1,T-1}^* + 1) - 1)$. Again, differentiating with respect to h , l and δ we have:

$$\begin{aligned} \frac{\partial n_{1,T-1}^*}{\partial h} \left[l \cdot x(n_{1,T-1}^*, \delta) - h \cdot y(n_{1,T-1}^*, \delta) \right] &= y(n_{1,T-1}^*, \delta) \\ \frac{\partial n_{1,T-1}^*}{\partial l} \left[h \cdot y(n_{1,T-1}^*, \delta) - l \cdot x(n_{1,T-1}^*, \delta) \right] &= x(n_{1,T-1}^*, \delta) \\ \frac{\partial n_{1,T-1}^*}{\partial \delta} \left[l \cdot x(n_{1,T-1}^*, \delta) - h \cdot y(n_{1,T-1}^*, \delta) \right] &= h \cdot y_\delta(n_{1,T-1}^*, \delta) - l \cdot x_\delta(n_{1,T-1}^*, \delta) \end{aligned}$$

Given $y(n_{1,T-1}^*, \delta) > 0$, $x(n_{1,T-1}^*, \delta) > 0$, $l \cdot x(n_{1,T-1}^*, \delta) - h \cdot y(n_{1,T-1}^*, \delta) < 0$, $h \cdot y_\delta(n_{1,T-1}^*, \delta) - l \cdot x_\delta(n_{1,T-1}^*, \delta) > 0$, we finally have:

$$\frac{\partial n_{1,T-1}^*}{\partial h} < 0, \quad \frac{\partial n_{1,T-1}^*}{\partial l} > 0, \quad \frac{\partial n_{1,T-1}^*}{\partial \delta} < 0.$$

The proof is completed using a simple recursive method.¹⁵

Case-1: $n \geq \bar{n}$. $V_1^{t-1}(n) = \max\{h(2\mu(n) - 1); \delta\nu(n)V_1^t(n+1) + \delta(1 - \nu(n))V_1^t(n-1)\}$ and

$$\begin{aligned} Y_1^t(n) &= \max\{0, -(1 - \delta)h(2\mu(n) - 1) + \delta\nu(n)Y_1^{t+1}(n+1) + \delta(1 - \nu(n))Y_1^{t+1}(n-1)\} \\ Y_1^{t-1}(n) &= \max\{0, -(1 - \delta)h(2\mu(n) - 1) + \delta\nu(n)Y_1^t(n+1) + \delta(1 - \nu(n))Y_1^t(n-1)\} \end{aligned}$$

Suppose that the following assumptions **(A1)**-**(A5)** hold.

(A1): $Y_1^t(n)$ is non-increasing in n and there exists $n_{1,t}^{**}$ such that $\tau_{1,t} = a$ if $n > n_{1,t}^{**}$ and $\tau_{1,t} = w$ if $n \in [\bar{n}, n_{1,t}^{**})$.

¹⁵For the reader unfamiliar with this method, the technique is very simple. Basically, we have already proved that some properties (that will be labelled below as **(A1)**-**(A5)** or **(A1')**-**(A5')**) depending on whether $n \leq \bar{n}$ or $n \geq \bar{n}$) hold at dates T and $T - 1$. The second step consists in assuming that these properties hold at a given date $t \in \{1, \dots, T-1\}$, that we leave unspecified. If, starting from this assumption, we are able to prove that the properties also hold at $t - 1$, then we have proved that the properties hold for all $t \in \{0, \dots, T\}$.

- (A2): $Y_1^t(n) \geq Y_1^{t+1}(n)$ and therefore $n_{1,t}^{**} > n_{1,t+1}^{**}$.
(A3): $Y_1^t(n, h) \leq Y_1^t(n, h')$ if $h > h'$ (and therefore $\partial n_{1,t}^{**}/\partial h < 0$).
(A4): $Y_1^t(n, l) \geq Y_1^t(n, l')$ if $l > l'$ (and therefore $\partial n_{1,t}^{**}/\partial l > 0$).
(A5): $Y_1^t(n, \delta) \geq Y_1^t(n, \delta')$ if $\delta > \delta'$ (and therefore $\partial n_{1,t}^{**}/\partial \delta > 0$).

Given (A1), the r.h.s. of $Y_1^{t-1}(n)$ is decreasing in n , so $Y_1^{t-1}(n)$ is non-increasing in n . Therefore, there exists a unique cutoff $n_{1,t-1}^{**}$ such that for all $n > n_{1,t-1}^{**}$ then $\tau_{1,t-1} = a$, and for all $n \in [\bar{n}, n_{1,t-1}^{**}]$ then $\tau_{1,t-1} = w$. Also, given (A2), the r.h.s. of $Y_1^{t-1}(n)$ is greater or equal than the r.h.s. of $Y_1^t(n)$ and therefore $Y_1^{t-1}(n) \geq Y_1^t(n)$. Overall, both (A1) and (A2) hold at date $t-1$. Furthermore, $n_{1,t-1}^{**} > n_{1,t}^{**}$. Now, denote:

$$Y_1^{t-1}(n, h) = \max\{0, -(1-\delta)h(2\mu(n)-1) + \delta\nu(n)Y_1^t(n+1, h) + \delta(1-\nu(n))Y_1^t(n-1, h)\}$$

$$Y_1^{t-1}(n, h') = \max\{0, -(1-\delta)h'(2\mu(n)-1) + \delta\nu(n)Y_1^t(n+1, h') + \delta(1-\nu(n))Y_1^t(n-1, h')\}$$

By (A3), if $h > h'$ then $Y_1^t(n+1, h) \leq Y_1^t(n+1, h')$ and $Y_1^t(n-1, h) \leq Y_1^t(n-1, h')$. Therefore, $Y_1^{t-1}(n, h) \leq Y_1^{t-1}(n, h')$. This means that (A3) holds at date $t-1$ and, as a consequence, that $\partial n_{1,t-1}^{**}/\partial h < 0$. Using a similar reasoning, it is immediate that (A4) and (A5) also hold at $t-1$ and therefore that $\partial n_{1,t-1}^{**}/\partial l > 0$ and $\partial n_{1,t-1}^{**}/\partial \delta > 0$.

Case-2: $n \leq \bar{n}$. $V_1^{t-1}(n) = \max\{l(1-2\mu(n)); \delta\nu(n)V_1^t(n+1) + \delta(1-\nu(n))V_1^t(n-1)\}$ and

$$W_1^t(n) = \max\{0, -(1-\delta)l(1-2\mu(n)) + \delta\nu(n)W_1^{t+1}(n+1) + \delta(1-\nu(n))W_1^{t+1}(n-1)\}$$

$$W_1^{t-1}(n) = \max\{0, -(1-\delta)l(1-2\mu(n)) + \delta\nu(n)W_1^t(n+1) + \delta(1-\nu(n))W_1^t(n-1)\}$$

Suppose that the following assumptions (A1')-(A5') hold.

- (A1'): $W_1^t(n)$ is non-decreasing in n and there exists $n_{1,t}^*$ such that $\tau_{1,t} = b$ if $n < n_{1,t}^*$ and $\tau_{1,t} = w$ if $n \in (n_{1,t}^*, \bar{n}]$.
(A2'): $W_1^t(n) \geq W_1^{t+1}(n)$ and therefore $n_{1,t}^* < n_{1,t+1}^*$.
(A3'): $W_1^t(n, h) \geq W_1^t(n, h')$ if $h > h'$ (and therefore $\partial n_{1,t}^*/\partial h < 0$).
(A4'): $W_1^t(n, l) \leq W_1^t(n, l')$ if $l > l'$ (and therefore $\partial n_{1,t}^*/\partial l > 0$).
(A5'): $W_1^t(n, \delta) \geq W_1^t(n, \delta')$ if $\delta > \delta'$ (and therefore $\partial n_{1,t}^*/\partial \delta < 0$).

Given (A1'), the r.h.s. of $W_1^{t-1}(n)$ is increasing in n , so $W_1^{t-1}(n)$ is non-decreasing in n . Therefore, there exists a unique cutoff $n_{1,t-1}^*$ such that for all $n < n_{1,t-1}^*$ then $\tau_{1,t-1} = b$, and for all $n \in (n_{1,t-1}^*, \bar{n}]$ then $\tau_{1,t-1} = w$. Also, given (A2'), the r.h.s. of $W_1^{t-1}(n)$ is greater or equal than the r.h.s. of $W_1^t(n)$ and therefore $W_1^{t-1}(n) \geq W_1^t(n)$. Overall, both (A1') and (A2') hold at date $t-1$. Furthermore, $n_{1,t-1}^* < n_{1,t}^*$. Now, denote:

$$W_1^{t-1}(n, h) = \max\{0, -(1-\delta)l(1-2\mu(n)) + \delta\nu(n)W_1^t(n+1, h) + \delta(1-\nu(n))W_1^t(n-1, h)\}$$

$$W_1^{t-1}(n, h') = \max\{0, -(1-\delta)l(1-2\mu(n)) + \delta\nu(n)W_1^t(n+1, h') + \delta(1-\nu(n))W_1^t(n-1, h')\}$$

By **(A3')**, if $h > h'$ then $W_1^t(n+1, h) \geq W_1^t(n+1, h')$ and $W_1^t(n-1, h) \geq W_1^t(n-1, h')$. Therefore, $W_1^{t-1}(n, h) \geq W_1^{t-1}(n, h')$. This means that **(A3')** holds at date $t-1$ and, as a consequence, that $\partial n_{1,t-1}^*/\partial h < 0$. Using a similar reasoning, it is immediate that **(A4')** and **(A5')** also hold at $t-1$ and therefore that $\partial n_{1,t-1}^*/\partial l > 0$ and $\partial n_{1,t-1}^*/\partial \delta < 0$.

Type-2 agent.

From equations (1) and (2), it is immediate to notice that type-1 and type-2 agents are fully symmetric. Therefore, if at date t there exists $n_{1,t}^{**}$ s.t. $\tau_{1,t} = a$ if $n > n_{1,t}^{**}$ and $\tau_{1,t} = w$ if $n \in [\bar{n}, n_{1,t}^{**})$, then there also exists $n_{2,t}^*$ s.t. $\tau_{2,t} = b$ if $n < n_{2,t}^*$ and $\tau_{2,t} = w$ if $n \in (n_{2,t}^*, \bar{n}]$. Furthermore, $n_{2,t}^*$ is such that $\bar{n} - n_{2,t}^* = n_{1,t}^{**} - \bar{n}$, that is $\mu(n_{1,t}^{**}) = 1 - \mu(n_{2,t}^*)$. Similarly, if at date t there exists $n_{1,t}^*$ s.t. $\tau_{1,t} = b$ if $n < n_{1,t}^*$ and $\tau_{1,t} = w$ if $n \in (n_{1,t}^*, \bar{n}]$, then there also exists $n_{2,t}^{**}$ s.t. $\tau_{2,t} = a$ if $n > n_{2,t}^{**}$ and $\tau_{2,t} = w$ if $n \in [\bar{n}, n_{2,t}^{**})$. Furthermore, $n_{2,t}^{**}$ is such that $n_{2,t}^{**} - \bar{n} = \bar{n} - n_{1,t}^*$, that is $\mu(n_{1,t}^*) = 1 - \mu(n_{2,t}^{**})$.

Note that if $h = l$, then for all t we have $\mu(n_{1,t}^*) = 1 - \mu(n_{1,t}^{**})$ and $\mu(n_{2,t}^*) = 1 - \mu(n_{2,t}^{**})$. As a result, $n_{2,t}^* = n_{1,t}^* < \bar{n}$ and $n_{2,t}^{**} = n_{1,t}^{**} > \bar{n}$. Also, we know that $\frac{\partial n_{1,t}^{**}}{\partial h} < 0$ and $\frac{\partial n_{1,t}^*}{\partial h} < 0$ (which, again by symmetry, implies that $\frac{\partial n_{2,t}^*}{\partial h} > 0$ and $\frac{\partial n_{2,t}^{**}}{\partial h} > 0$). Therefore, for all $h > l$ we have $n_{1,t}^* < n_{2,t}^* < \bar{n} < n_{1,t}^{**} < n_{2,t}^{**}$.

Summing up, when $\delta < 1$, $h > l > 0$ and $T \rightarrow +\infty$, we have $n_1^* < n_2^* < \bar{n} < n_1^{**} < n_2^{**}$ where $\mu(n_1^{**}) = 1 - \mu(n_2^*)$ and $\mu(n_1^*) = 1 - \mu(n_2^{**})$. Moreover, $\frac{\partial n_1^{**}}{\partial h} < 0$, $\frac{\partial n_1^*}{\partial l} > 0$, $\frac{\partial n_1^{**}}{\partial \delta} > 0$, $\frac{\partial n_1^*}{\partial h} < 0$, $\frac{\partial n_1^*}{\partial l} > 0$, $\frac{\partial n_1^*}{\partial \delta} < 0$ and $\frac{\partial n_2^*}{\partial h} > 0$, $\frac{\partial n_2^*}{\partial l} < 0$, $\frac{\partial n_2^*}{\partial \delta} < 0$, $\frac{\partial n_2^{**}}{\partial h} > 0$, $\frac{\partial n_2^{**}}{\partial l} < 0$, $\frac{\partial n_2^{**}}{\partial \delta} > 0$.