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Enforcement Games

Shmuel Leshem\*

Avraham D. Tabbach<sup>†</sup>

\*USC Law School, [sleshem@law.usc.edu](mailto:sleshem@law.usc.edu)

<sup>†</sup>Tel Aviv University, [adtabbac@post.tau.ac.il](mailto:adtabbac@post.tau.ac.il)

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# Commitment versus Flexibility in Enforcement Games\*

Shmuel Leshem and Avraham D. Tabbach

## Abstract

This paper studies the role of commitment in the design of enforcement mechanisms when enforcement can remedy harm from non-compliance. We consider a game between an enforcement authority ("enforcer") and an offender in which either the enforcer or the offender may act as a Stackelberg leader. The enforcer must choose whether to move first by committing to an enforcement strategy—thereby directly affecting the level of non-compliance; or rather let the offender make the first move—thereby calibrating the level of enforcement to the actual level of non-compliance. We show that the value of commitment to the enforcer depends on each player's responsiveness to a change in the other player's strategy choice. Commitment to an enforcement strategy is thus not always in the enforcer's interest.

**KEYWORDS:** Enforcement, inspection game, Stackelberg, strategic complements, strategic substitutes

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\*Leshem: University of Southern California; sleshem@law.usc.edu. Tabbach: Tel Aviv; e-mail: adtabbac@post.tau.ac.il.

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# 1 Introduction

Should enforcement authorities commit to an investigation policy or, rather, choose the level of investigation in response to the severity of non-compliance? This paper considers cases in which—as with most property crimes—detection of the offender remediates harm from non-compliance. The enforcement authority thus faces the dilemma of either committing to a (non-discriminatory) investigation strategy or choosing the level of investigation in response to the actual level of non-compliance. This paper explores the different trade-offs between deterrence and enforcement costs associated with each of these investigation strategies.

To illustrate this dilemma, suppose the Criminal Investigation Division of the EPA considers an enforcement policy. One alternative is to announce a plan to investigate a class of pollution incidents involving a single, identifiable, polluter—such as oil spills and toxic wastes—irrespective of their severity. A commitment to a strict, uniform enforcement policy deters polluters, but also involves an excessive level of enforcement. Another alternative is to decide on the intensity of the investigation in response to the severity of the pollution; for example, the size of the spill or the magnitude of the waste. In the absence of commitment to an enforcement strategy, a potential polluter must decide whether, and how much, to pollute in anticipation of the Division's best response. The Division, in turn, can adjust the intensity of its investigation to the severity of the pollution.

As another example, suppose the Criminal Investigation Division of a state police department has to choose an enforcement strategy. In an attempt to deter property crimes, the Division may announce, and commit to, a uniform investigation policy of any theft. Alternatively, the Division may choose the level of investigation after having observed the amount stolen. By refraining from committing to an investigation strategy, the Division induces a potential thief to take into account the effect of his decision on the Division's choice of level of investigation.

To understand the choice between these different investigation strategies, consider their effects on potential offenders as well as on the costs of enforcement. By committing to an investigation strategy, the enforcement authority directly affects the level of non-compliance, but overspends enforcement resources relative to the actual level of non-compliance. By choosing the level of investigation retrospectively, in contrast, the enforcement authority calibrates the level of investigation to the actual level of non-compliance, but only indirectly affects the level of non-compliance. This paper's main argument is that no one investigation strategy is superior to the other. In particular,

the enforcement authority might do better by letting the offender move first rather than taking the lead by committing to an investigation strategy.

A key assumption in our analysis is that the enforcement authority commits to a uniform enforcement scheme: A scheme which specifies the same level of investigation for different levels of non-compliance. Although enforcement authorities may be able to commit to a more nuanced enforcement scheme than that which we consider here, constraints likely exist on the design of optimal commitment. For example, a commitment to an enforcement strategy can be obtained by entering into contracts with employees. However, an enforcement authority might not be able to condition its contracts on the actual level of non-compliance. The trade-off we identify between commitment and flexibility thus characterizes the choice faced by an enforcement authority which cannot commit to a *fully* contingent enforcement scheme. It thereby more generally highlights the benefits and costs of commitment in various enforcement contexts.

To study the role of commitment in enforcement, we consider a game between an enforcement authority (enforcer) and an offender, in which either the enforcer or the offender may act as a Stackelberg leader. An offender-leadership game captures a strategic interaction in which the enforcer chooses an enforcement strategy after having observed the offender's choice of non-compliance. The possibility that the offender acts as a Stackelberg leader has been previously dismissed on the grounds that an offender cannot credibly commit to performing an illegal act and therefore his strategy is neither observable nor irreversible (Avenhaus et al., 2002, p. 29).<sup>1</sup> But as the examples above illustrate, the observability and irreversibility of the offender's strategy are often a result of the fact that non-compliance itself is observable and irrevocable. Accordingly, the offender need not commit to a non-compliance strategy to render his choice irreversible; rather, he should simply engage in non-compliance.

The essential features of the game between the enforcer and the offender are as follow. Both the offender's gains and the enforcer's harm are increasing with the offender's level of non-compliance. Detection of the offender remedies or prevents a certain fraction of the harm from non-compliance as well as eliminates the offender's gain; for example, detected offenders can be forced to pay for clean-up costs or to return their loot. Detection of the offender also subjects the offender to a sanction whose magnitude is proportional to

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<sup>1</sup>"The notion of leadership ... is particularly suitable for inspection games since an inspector can credibly announce his strategy and stick to it, *whereas the inspectee cannot do so if he intends to act illegally.*" (emphasis added)

the level of non-compliance. We assume that the offender's gains are strictly increasing and concave in the level of non-compliance, and that the enforcer's costs are strictly increasing and convex in the probability of detection. This implies that the offender's optimal level of non-compliance is *decreasing* with the enforcer's level of enforcement and that the enforcer's optimal level of enforcement is *increasing* with the offender's level of non-compliance. Thus, from the offender's perspective, this is a game with *strategic substitutes*, whereas from the enforcer's perspective, this is a game with *strategic complements*.

The players' objectives depend on the sequence of moves in the game, which reflects the information each player possesses on the other player's strategy, as well as the players' ability to commit to a strategy. To facilitate the comparison between the enforcer- and the offender-leadership games, we compare the equilibrium in each of these games to the equilibrium in a simultaneous-move game. In a simultaneous-move game, the offender and the enforcer choose their strategies without observing the other player's strategy choice. Accordingly, enforcement in a simultaneous-move game takes the form of monitoring non-compliance: employing enforcement resources to detect non-compliance. Although the enforcement technology may be different in a simultaneous-move game, the equilibrium of this game serves as a benchmark for comparing the Stackelberg equilibria of the sequential games.

In an enforcer-leadership game, the enforcer commits to an observable investigation strategy. The enforcer's commitment is unconditional on the actual level of non-compliance; for example, the enforcer may spend enforcement resources before the offender chooses a level of non-compliance (we comment later on the possibility that the enforcer commits to a conditional enforcement scheme). The offender, having observed the enforcer's committed level of enforcement, then chooses a level of non-compliance. The enforcer's objective is to minimize the sum of expected harm from non-compliance and enforcement costs by affecting the level of non-compliance (strategic effect) and by adjusting the level of enforcement to the offender's equilibrium level of non-compliance (direct effect). Because the enforcer can secure her Nash equilibrium payoff by choosing her Nash equilibrium strategy, she enjoys a *first-mover advantage* relative to a simultaneous-move game. To realize this advantage, the enforcer chooses a higher level of enforcement than the Nash equilibrium level. The offender's best response is to choose a lower level of non-compliance than the Nash equilibrium level. The offender therefore suffers a *second-mover disadvantage* as compared to a simultaneous-move game.

In an offender-leadership game, the offender engages in non-compliance in an irrevocable and observable way. The enforcer, having observed the offender's level of non-compliance, then chooses a level of investigation. The

offender's objective is to maximize his net gains from non-compliance by affecting the level of enforcement (strategic effect) and by adjusting the level of non-compliance to the enforcer's equilibrium level of enforcement (direct effect). Because the offender can secure his Nash equilibrium payoff by choosing his Nash equilibrium strategy, the offender enjoys a *first-mover advantage* relative to a simultaneous game. To realize this advantage, the offender chooses a lower level of non-compliance than the Nash equilibrium level. The enforcer's best response is to choose a lower level of enforcement than the Nash equilibrium level. The enforcer therefore enjoys a *second-mover advantage* as compared to a simultaneous game.<sup>2</sup>

The offender's choice as a leader to curb the level of non-compliance, and the enforcer's in-kind response as a follower, may help to explain anecdotal observations on the relationship between enforcement authorities and crime organizations. According to a common perception, enforcement authorities and crime organizations often adhere to a tacit agreement whereby the enforcement authority refrains from, or scales back, enforcement so long as the crime organization restrains its criminal activity. These implicit agreements can be interpreted as strategic interactions in which the crime organization first decides on the level of non-compliance and the enforcement authority chooses, in response, an optimal level of enforcement.

The arguments above establish that the enforcer's equilibrium payoff is higher either as a leader or as a follower than in a simultaneous-move game. Whether the enforcer's equilibrium payoff is higher as a leader or as a follower depends on the relation between her first- versus second-mover advantage. The enforcer's advantage as a leader relative to a simultaneous-move game is that she can directly induce the offender to choose a lower level of non-compliance. The equilibrium level of enforcement, however, is excessive relative to the equilibrium level of non-compliance. The enforcer's advantage as a follower relative to a simultaneous-move game is that the offender, anticipating the enforcer's response, chooses a lower level of non-compliance. The enforcer as a follower, moreover, chooses an optimal level of enforcement in response to the offender's choice of level of non-compliance. This implies that, although the level of non-compliance is lower in both leadership games relative to a simultaneous-move game, the level of enforcement is calibrated to the level of non-compliance only in an offender-leadership game. It follows that, if the level of non-compliance in an offender-leadership game is not excessively higher than in an enforcer-leadership game, the enforcer's equilibrium payoff is higher

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<sup>2</sup>This follows because the enforcer's payoff is decreasing along her reaction curve. We prove this formally in the paper.

as a follower than as a leader.

The level of non-compliance in each of the leadership games depends on the responsiveness of the follower to the leader's strategy. The *more* responsive the offender-follower is to a change in the level of enforcement, the lower is the equilibrium level of non-compliance (because the enforcer as a leader can relatively inexpensively induce the offender to reduce the level of non-compliance). Similarly, the *less* responsive the enforcer-follower is to a change in the level of non-compliance, the lower is the equilibrium level of non-compliance (because the offender as a leader has to significantly lower the level of non-compliance to induce the enforcer to reduce the level of enforcement). It follows that the enforcer prefers to move first if both players are relatively responsive to a change in the other player's level of activity, and prefers to move second if both players are relatively unresponsive to a change in the other player's level of activity.

We show, in particular, that the enforcer is better off refraining from committing to an enforcement strategy if the sanction for non-compliance is sufficiently low. In this case, because the expected sanction for non-compliance does not vary much with the level of enforcement, the offender is relatively unresponsive to a change in the level of enforcement. As a result, the enforcer, as a leader, has little power to deter non-compliance. In contrast, if the offender moves first, he takes into account the fact that increasing the level of non-compliance increases the probability of forfeiting his gains. To maximize his expected payoff from non-compliance, the offender has an incentive to choose a lower level of non-compliance than in an enforcer-leadership game. The enforcer is consequently better off letting the offender move first.

However, the enforcer is better off committing to an enforcement strategy if the marginal enforcement cost increases at a slow rate with the probability of detection. The enforcer's best response in this case is to intensely investigate if the level of non-compliance is high (because the marginal benefit from enforcement is high), but to spend little on investigation if the level of non-compliance is low (because the marginal benefit from enforcement is low). Because the enforcer is relatively responsive to a change in the level of non-compliance, the offender as a leader can induce the enforcer to significantly ease up on enforcement by slightly reducing his level of non-compliance. If the enforcer moves first, however, she can choose a sufficiently high level of enforcement, thereby inducing a lower level of non-compliance than in an offender-leadership game. The enforcer's ability to commit to an enforcement strategy is thus highly valuable.

To further explore the strategic differences between the two leadership games, we compare the effect of an increase in the magnitude of the sanction or

the magnitude of the harm on the equilibrium strategies and payoffs. We show that the effect of an increase in the sanction or the harm on the enforcer's level of enforcement depends on whether the enforcer leads or follows, but always lowers the offender's level of non-compliance. We further show that, as the magnitude of the harm increases, the enforcer's equilibrium payoff as a leader may surprisingly increase.

Finally, to illustrate one of the paper's main insights, consider the enforcement crusade launched by Mexican President Felipe Calderón against drug trafficking at the end of 2006. This policy change of the Mexican government amounted to a commitment to a higher level of enforcement, intended to bring down the level of crime. However, in the years that followed the initiation of the enforcement campaign, the level of violence surprisingly increased. This paper sheds some light on the increase in the level of non-compliance which followed the intensified enforcement efforts. As we show, an offender-follower might choose a higher level of non-compliance than an offender-leader. Transforming an offender-leadership game into an enforcer-leadership game might, accordingly, result in a higher level of non-compliance.

The paper proceeds as follows. Section 2 surveys the related literature. Section 3 sets up the model. Section 4 characterizes the equilibrium outcomes in simultaneous and sequential games. Section 5 compares the enforcer's equilibrium payoff and the enforcer's and the offender's equilibrium strategies in the sequential games. Section 6 concludes.

## 2 Related Literature

This paper is related to the literature on the economics of crime, initiated by Becker (1968). Becker's paper has spawned a large literature which investigates a sequential game in which a benevolent enforcer acts as a Stackelberg leader by committing to an enforcement strategy. Underlying this literature is the assumption that non-compliance always results in harm and, accordingly, that enforcement aims solely at deterrence, rather than preventing or remedying harm from non-compliance (see, e.g., Polinsky and Shavell, 2007).<sup>3</sup> Here we adopt an alternative assumption—common in the literature on inspection games—that, in addition to detecting and punishing offenders, enforcement either remediates or prevents a portion of the harm from non-compliance. This assumption seems particularly plausible in property crimes such as theft and

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<sup>3</sup>Exceptions to this notion are papers that focus on forfeiture of offenders' gains (Bowles et al., 2000, Tabbach, 2009) and papers that study the effects of imprisonment—through rehabilitation and incapacitation—on social welfare (see Ehrlich, 1981, Shavell, 1987).



embezzlement. That enforcement may remediate harm from non-compliance gives rise to the possibility that the enforcer spends enforcement resources without resorting to commitment power.

Within the economics of crime literature, Mookherjee and Png (1992) and Shavell (1991) are particularly related to this paper. Mookherjee and Png consider the optimal mix of investigation and monitoring, whereas Shavell compares the optimal sanction under general versus specific enforcement. These papers stand out in that they explicitly model the method of enforcement. In contrast to this paper, however, both these papers assume that enforcement is aimed solely at being a deterrent against non-compliance, rather than at the prevention or remediation of harm. More important, both papers implicitly assume that the enforcer acts as a Stackelberg leader.<sup>4</sup> This paper also shares similar features with Reinganum (1993), who considers a two-stage model of enforcement and plea bargain. In contrast to Reinganum's model, however, this paper focuses on a sequential—rather than a simultaneous—enforcement game, and derives best-response functions from a general non-compliance technology.

This paper is also related to the literature on inspection games, which spans a wide range of applications (Graetz et al., 1986, Reinganum and Wilde, 1986, Chander and Wilde, 1992; Besanko and Spulber, 1989; Borch, 1990). This literature has considered a game in which the enforcer and the offender act simultaneously (see, e.g., Avenhaus et al., 2002). Enforcement in inspection games is designed to prevent ongoing harm from non-compliance or to avert harm from future non-compliance, rather than merely to detect and to sanction past non-compliance (as in the economics of crime literature).

This paper differs from the literature on inspection games in two main respects. First, this literature has largely dismissed the possibility of an offender-leadership game because an offender supposedly cannot credibly commit to performing an illegal act.<sup>5</sup> As we argued in the Introduction,<sup>6</sup> however, the offender need not commit to a non-compliance strategy to render his strategy irreversible; rather, the offender simply has to engage in non-compliance in

<sup>4</sup>Mookherjee and Png (1992) assume that the enforcer can commit to a schedule of enforcement strategies so that the level of enforcement is conditioned on the level of non-compliance (a similar assumption is made in the literature on marginal deterrence—see, for example, Polinsky and Shavell, 2007, pp. 432-34). Here, by contrast, we restrict attention to unconditional commitment to enforcement strategy, which requires a much weaker commitment power on the part of the enforcer.

<sup>5</sup>"The notion of leadership ... is particularly suitable for inspection games since an inspector can credibly announce his strategy and stick to it, *whereas the inspectee cannot do so if he intends to act illegally.*" (Avenhaus et al., 2002, p. 29; emphasis added).

<sup>6</sup>See f.n. 1.

an irreversible way. Second, whereas the literature on inspection games has assumed that enforcement is designed to detect non-compliance—we define such enforcement as *monitoring*—here we focus on the case in which enforcement is designed to detect and apprehend the offender as well as collect evidence that would facilitate his punishment—we define such enforcement as *investigation*.<sup>7</sup>

This paper shares similar features with papers that examine the order of play in duopoly games and contests.<sup>8</sup> These papers, as does this paper, compare the players' equilibrium payoffs under different timing schemes (see Gal-Or, 1985; Hamilton and Slutsky, 1990; Dixit, 1987; Baik and Shogren, 1992). Our results on the nature of the enforcer's first- and second-mover advantages, moreover, rest on considerations similar to those in Dixit's and Baik and Shogren's papers. However, the derivation and interpretation of the players' reaction curves, as well as their shapes, are different in this paper. In particular, the slopes of the reaction curves here have opposite signs and are globally monotone, whereas the slopes of the reaction curves are not predetermined in duopoly games and are not globally monotone in contests.<sup>9</sup> More important, the question of endogenous timing does not arise in our model because we assume that the enforcer can unilaterally dictate the sequence of moves in the game. In contrast, in duopoly games and contests, a firm or a contestant cannot unilaterally choose to be a leader or a follower.

### 3 Model

Consider two strategic, risk-neutral players: an offender (he) and an enforcer (she). The offender's strategy,  $q \in [0, 1]$ , is a level of non-compliance (or an offense seriousness). Non-compliance inflicts harm on the enforcer. The harm from non-compliance is proportional to the level of non-compliance and is given by  $qH$ , where  $H > 0$ .<sup>10</sup> The enforcer's strategy,  $p \in [0, 1]$ , is a probability of

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<sup>7</sup>*Cf.* Mookherjee and Png (1992) who define investigation as enforcement activity whose level depends on information on the severity of the offense.

<sup>8</sup>We use the notion of first- and second-mover advantage in a slightly different sense than the literature on duopoly games. According to this literature, a firm has a first-mover (second-mover) advantage if its Stackelberg-leader payoff is higher (lower) than its Stackelberg-follower payoff. Here, by contrast, we follow Turocy and von Stengel (2002; p. 26) in saying that a player has a first-mover (second-mover) advantage if his Stackelberg-leader (Stackelberg-follower) payoff is higher than his payoff in a simultaneous-move game.

<sup>9</sup>Profits in both inspection games and contests are monotone decreasing in a rival's action. Reaction curves, by contrast, need not be monotone in contests.

<sup>10</sup>We follow the literature on inspection games in assuming that the offender's gains from non-compliance do not affect the enforcer's payoff. Our results would continue to hold if we assumed instead, as the literature on the economics of crime generally does, that the

detection. Detection of the offender remediates or prevents a fraction  $\mu \in (0, 1]$  of the harm from non-compliance (for example, a detected thief can be forced to return a portion of his loot). Detection of the offender also subjects the offender to a sanction. The sanction for non-compliance is proportional to the level of non-compliance and is given by  $qS$ , where  $S > 0$ .<sup>11</sup> If the offender is detected, his illicit gains are forfeited. For example, detected polluters could be forced to disgorge their illegal profits from pollution. Similarly, convicted thieves put behind bars can no longer benefit from their loot, even if not all of it is returned. Our results would not change if we assumed instead that only a fraction of the offender's gains are forfeited.<sup>12</sup> For simplicity, we assume that the sanction and forfeiture are costless to the enforcer.<sup>13</sup>

The offender's gains from non-compliance are given by  $G(q)$ , where  $G'(q) \geq 0$  and  $G''(q) < 0$ ; that is, the gains from non-compliance are (weakly) increasing at a decreasing rate in the level of non-compliance. We normalize the maximum level of non-compliance to 1 by assuming that  $G'(1) = 0$ . The costs to the enforcer of detecting the offender with probability  $p$  are given by  $c(p)$ , where  $c'(p) > 0$  and  $c''(p) > 0$ ; that is, the costs of enforcement are increasing at an increasing rate with the probability of detection. We assume that the cost of detecting non-compliance is independent of the level of non-compliance (i.e.,  $\frac{\partial^2 c(p)}{\partial p \partial q} = 0$ ). This assumption allows us to define the enforcer's strategy as a probability of detection, rather than as a cost of enforcement. To ensure an interior solution for the enforcer's equilibrium strategy in a simultaneous-move game, we assume that  $c'(0) < \mu H$ . For simplicity, we assume that the costs of enforcement are independent of the enforcement technology—investigation or monitoring.<sup>14</sup>

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enforcer's payoff incorporates these gains.

<sup>11</sup>The assumption that the harm and sanction are linear in the level of non-compliance is made to simplify the analysis, but it suffices that they are convex in the level of non-compliance.

<sup>12</sup>The assumption that a detected offender forfeits his entire gains is made for simplicity, but there need not be any relationship between the fraction of the harm prevented or remediated by enforcement and the fraction of the forfeited gains. More generally, if a detected offender retained a portion  $1 - \gamma$  of his gains, then his gains from non-compliance would be  $G[(1-p)q + p(1-\gamma)q] = G[(1-p\gamma)q]$ , where  $G(\cdot)$  is the offender's concave utility function (see below).

<sup>13</sup>The sanction does not affect the enforcer's payoff if it is not paid to the enforcer. If, in contrast, the sanction is paid to the enforcer (e.g., a fine), then assume the enforcer internalizes the offender's burden of paying the fine.

<sup>14</sup>We make this assumption to illustrate the players' first- and second-mover advantage relative to a simultaneous-move game. The comparison between the different Stackelberg games, however, does not depend on this assumption.

The offender's and enforcer's payoff functions are given by

$$v(q, p) = (1 - p)G(q) - pqS, \quad (1)$$

and

$$u(q, p) = -[c(p) + (1 - p\mu)qH]. \quad (2)$$

The offender's payoff is equal to the expected gains from non-compliance less the expected sanction for non-compliance. The enforcer's payoff is equal to the expected harm from non-compliance less the costs of enforcement. We proceed by considering the players' reaction curves. Throughout the paper, we use subscripts to denote partial derivatives.

### 3.1 Best-Response Functions

Consider first the offender's best-response function,  $q_{br}(p)$ : the offender's optimal level of non-compliance as a function of the enforcer's probability of detection. The offender's problem is to choose  $q$  to maximize (1). Differentiating (1) with respect to  $q$  gives

$$v_q = (1 - p)G'(q) - pS. \quad (3)$$

The first term is the offender's marginal benefit from increasing the level of non-compliance discounted by the probability of non-detection. The second term is the offender's marginal cost from the increased expected punishment. If  $p = 0$  (i.e., no enforcement), the offender's level of non-compliance satisfies  $G'(q) = 0$ ; the offender's best response is therefore  $q = 1$  (recall that  $G'(1) = 0$ ). In contrast, if  $p$  is sufficiently high, the offender chooses  $q = 0$  (i.e., full compliance). Specifically, let  $\tilde{p}$  be the value of  $p$  that satisfies  $(1 - p)G'(0) = pS$  ( $\Rightarrow \tilde{p} = \frac{G'(0)}{G'(0) + S}$ ). Then, because  $v_q \leq 0$  for all  $p \in [\tilde{p}, 1]$ , the offender's best response to  $p \in [\tilde{p}, 1]$  is  $q = 0$ , and  $\tilde{p}$  is thus the minimal probability of detection that induces full compliance. For  $p \in [0, \tilde{p})$ , the offender's optimal choice of  $q$  satisfies  $v_q = 0$ .<sup>15</sup>

The offender's best-response function is therefore

$$q_{br}(p) = \begin{cases} 1 & \text{if } p = 0 \\ q : G'(q) = pS/(1 - p) & \text{if } 0 < p < \tilde{p} \\ 0 & \text{if } \tilde{p} \leq p \leq 1 \end{cases}. \quad (4)$$

<sup>15</sup>Observe that, since  $G''(q) < 0$ ,  $v_{qq} < 0$  for all  $q$ .

Implicitly differentiating  $v_q = 0$ , plugging  $G'(q)^{\frac{1-p}{p}}$  for  $S$  (from the offender's FOC) and rearranging we have

$$\left. \frac{dq_{br}(p)}{dp} \right|_{p \in [0, \widehat{p})} = -\frac{v_{qp}}{v_{qq}} = [p(1-p)]^{-1} \frac{G'(q)}{G''(q)} < 0. \tag{5}$$

(5) implies that, for  $p \in [0, \widehat{p})$ , the offender's best response is monotone decreasing with the enforcer's probability of detection. This reflects the notion that, from the offender's perspective, this is a game with *strategic substitutes*: the more aggressive the enforcer is, the less aggressive the offender will be.<sup>16</sup>

Next, consider the enforcer's best-response function,  $p_{br}(q)$ : the enforcer's optimal probability of detection as a function of the offender's level of non-compliance. The enforcer's problem is to choose  $p$  to maximize (2). Differentiating (2) with respect to  $p$  yields

$$u_p = -c'(p) + \mu q H. \tag{6}$$

The first term is the enforcer's marginal cost of increasing the probability of detection; the second term is the enforcer's marginal benefit from the prevention of harm. Let  $\bar{q}$  be the value of  $q$  that satisfies  $c'(0) = q\mu H$  ( $\Rightarrow \bar{q} = \frac{c'(0)}{\mu H}$ ). Then, the enforcer's best response to  $q \in [0, \bar{q}]$  is  $p = 0$  (i.e., no enforcement). This follows because, for a sufficiently low level of non-compliance, the enforcer's marginal benefit from the prevention or remediation of harm is lower than the marginal cost. For  $q \in (\bar{q}, 1]$ , the enforcer's optimal choice of  $p$  satisfies  $u_p = 0$ .<sup>17</sup>

The enforcer's best-response function is therefore

$$p_{br}(q) = \begin{cases} 0 & \text{if } 0 \leq q \leq \bar{q} \\ p : c'(p) = q\mu H & \text{if } \bar{q} \leq q \leq 1 \end{cases}. \tag{7}$$

Implicitly differentiating  $u_p = 0$ , plugging  $\mu H = \frac{c'(p)}{q}$ , and rearranging we have

$$\left. \frac{dp_{br}(q)}{dq} \right|_{q \in (\bar{q}, 1)} = -\frac{u_{pq}}{u_{pp}} = \frac{1}{q} \frac{c'(p)}{c''(p)} > 0. \tag{8}$$

(8) implies that, for  $q \in (\bar{q}, 1]$ , the enforcer's best response is monotone increasing with the offender's level of non-compliance. This reflects the notion that,

<sup>16</sup>The definitions of strategic substitutes and strategic complements (which we discuss below) was introduced in Bulow et al. (1985) in relation to duopoly games.

<sup>17</sup>Observe that, since  $c''(p) > 0$ ,  $u_{pp} < 0$  for all  $p$ .

from the enforcer's perspective, this is a game with *strategic complements*: the more aggressive the offender is, the more aggressive the enforcer will be.

Finally, by the envelope theorem:

$$\left. \frac{dv(q_{br}(p), p)}{dp} \right|_{p \in [0, \bar{p})} = \frac{\partial v}{\partial p} = -(G(q) + qS) < 0, \quad (9)$$

and

$$\left. \frac{du(q, p_{br}(q))}{dq} \right|_{q \in [\bar{q}, 1)} = \frac{\partial u}{\partial q} = -(1 - p\mu)H < 0. \quad (10)$$

That both players' expected payoffs decrease along their reaction curves (i.e., as the other player's strategy increases), reflects the notion that this is a game of competition. The following observation summarizes the essential features of the game.

**Observation:** *The game between the enforcer and the offender is a game of conflict in which, for strictly positive levels of non-compliance and enforcement, the players' strategies are complements from the perspective of the enforcer, and substitutes from the perspective of the offender.*

We proceed by comparing three game configurations: a simultaneous-move game, a sequential game with enforcer-leadership, and a sequential game with offender-leadership; we will use the superscripts n, e, and o, respectively, to denote these games. To facilitate the comparison between the different games, we assume that enforcement costs do not depend on the enforcement technology—monitoring or investigation—and therefore do not depend on the timing structure of the game. Our main interest, however, is comparing the equilibrium payoffs in the two Stackelberg games: an enforcer-leadership game and an offender-leadership game. In the former game, enforcement is usually assumed to take the form of investigation; in the latter game, enforcement can only take the form of investigation.

## 4 Equilibrium under Different Move-Sequences

### 4.1 Simultaneous-Move Game

In a simultaneous-move game, both the enforcer and the offender choose their strategies independently (i.e., without observing the other player's strategy choice). A simultaneous-move game reflects the players' inability to commit to a strategy or their preference to keep their strategy unknown to the other

player.<sup>18</sup> The solution concept is accordingly Nash Equilibrium (NE). The following Lemma is immediate (equilibrium strategies are marked with stars).

**Lemma 1 (equilibrium strategies in a simultaneous game)**

The unique Nash equilibrium strategies in a simultaneous-move game are  $q^{n*} \in (\bar{q}, 1)$  and  $p^{n*} \in (0, \tilde{p})$  satisfying  $v_q(q^{n*}, p^{n*}) = 0$  and  $u_p(q^{n*}, p^{n*}) = 0$ .  $\parallel$

Lemma 1 implies that, at the Nash equilibrium levels of non-compliance and enforcement, the marginal benefit of each player’s activity is equal to its marginal cost. Therefore, at the Nash equilibrium levels, both the offender and the enforcer choose an optimal response to the other player’s strategy.

**Remark:** Note that there is no equilibrium of no-compliance (since  $p_{br}(1) > 0$  and  $q_{br}(p : p > 0) < 1$ ),<sup>19</sup> or one in which the level of non-compliance is lower than or equal to  $\bar{q}$  (since  $p_{br}(q : q \leq \bar{q}) = 0$  or  $q_{br}(0) = 1$ ). Similarly, there is no equilibrium of no enforcement (since  $q_{br}(0) = 1$  and  $p_{br}(1) > 0$ ),<sup>20</sup> or one in which the level of enforcement is greater than  $\tilde{p}$  (since  $q_{br}(p : p > \tilde{p}) = 0$  and  $p_{br}(0) = 0$ ).

To illustrate the equilibrium outcome in a simultaneous-move game, consider the following example.

**Example 1 (iso-payoff and reaction curves)**

Suppose  $G(q) = -q^2 + 2q$  and  $c(p) = (0.1 + p)^3$ . Note that  $G(0) = 0$ ,  $G'(q) = -2q + 2 \geq 0$ ,  $G'(1) = 0$ , and  $G''(q) = -2 < 0$ ;  $c(0) = 0.1$ ,  $c'(p) = 3(0.1 + p)^2 > 0$ , and  $c''(p) = 6(0.1 + p) > 0$ .

(i) The enforcer’s and the offender’s best-response functions are:

$$p_{br}(q) = \begin{cases} 0 & \text{if } 0 \leq q \leq \frac{0.03}{\mu H} \\ \sqrt{\mu H q / 3} - 0.1 & \text{if } \frac{0.03}{\mu H} < q \leq 1 \end{cases}$$

and

$$q_{br}(p) = \begin{cases} 1 - \frac{pS}{2(1-p)} & \text{if } 0 < p < \frac{2}{2+S} \\ 0 & \text{if } \frac{2}{2+S} \leq p \leq 1 \end{cases}.$$

(ii) Assume that  $H = \mu = 1$  and  $S = 0.8$ . Then the Nash equilibrium strategies are  $q^{n*} = 0.73$  and  $p^{n*} = 0.40$  (see Figure 1).  $\parallel$

As Figure 1 shows, the enforcer’s reaction curve passes through the uppermost points of her iso-payoff curves. At these points, the marginal benefit

<sup>18</sup>For justifications of the notion that enforcement games should be analyzed as simultaneous-move games see Tsebelis (1993).

<sup>19</sup> $q_{br}(p : p > 0) < 1$  follows from the assumption that  $G'(1) = 0$ .

<sup>20</sup> $p_{br}(1) > 0$  follows from the assumption that  $c'(0) < \mu H$ .

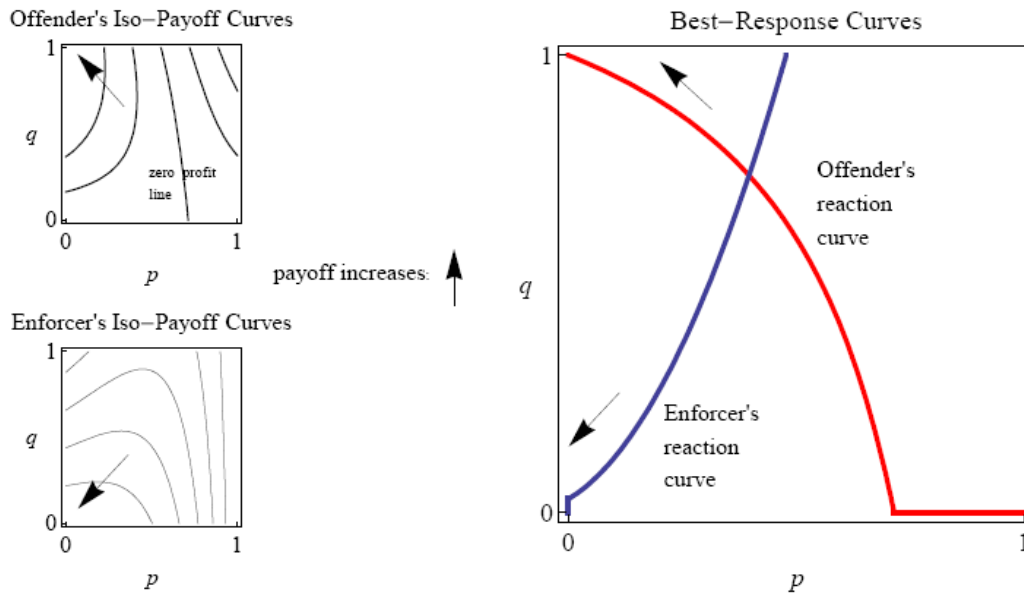


Figure 1: Iso-payoff and reaction curves (Example 1)

from enforcement is equal to its marginal cost. Similarly, for all enforcement levels left to the zero-profit curve, the offender's reaction curve passes through the rightmost points of his iso-payoff curves (the offender's best response is to fully comply for all other levels of enforcements). At these points, the marginal benefit from non-compliance is equal to its marginal cost.

Because the enforcer takes the level of non-compliance as given in a simultaneous-move game, compliance (deterrence) cannot be the aim of enforcement. Instead, the purpose of enforcement is the prevention or remediation of harm.<sup>21</sup> Compliance (deterrence) in a simultaneous-move game is a by-product of the enforcer's goal of preventing or remedying harm. It follows that the portion of preventable harm,  $\mu$ , and the magnitude of harm,  $H$ , but

<sup>21</sup>The literature on inspection games usually does not motivate the enforcer's incentive to incur enforcement costs. Here, we emphasize that the enforcer's incentive to do so results from her ability to prevent or rectify harm for non-compliance.



not the magnitude of the sanction,  $S$ , affect the enforcer's reaction curve.<sup>22,23</sup>

## 4.2 Enforcer-Leadership Game

We now turn to the case in which the enforcer acts as a Stackelberg leader. We assume that in an enforcer-leadership game, the enforcer commits to spend enforcement resources in an observable, irreversible and unconditional way; for example, the enforcer may enter into employment contracts which are costly to breach, or make irreversible investment in detection devices. The offender's level of non-compliance, in turn, constitutes a best response to the enforcer's strategy. In choosing a level of enforcement, therefore, the enforcer takes into account the effect of her strategy on the offender's strategy. The solution concept is Subgame Perfect Equilibrium (SPE).

An alternative modeling choice of the enforcer's commitment to an enforcement strategy is for the enforcer to commit to spending enforcement resources if and only if the offender chose not to comply. Specifically, the enforcer commits to  $p = 1$  if  $q > 0$ , and to  $p = 0$  otherwise. The offender's best response, in turn, is to fully comply. Besides the fact that this form of conditional commitment is of little theoretical interest, it is costly to establish and is not likely to be credible. We focus instead on the case in which the enforcer spends enforcement resources before the offender chooses a level of non-compliance, thereby avoiding the issue of the credibility of the enforcer's threat to investigate non-compliance.

In an enforcer-leadership game, the enforcer chooses  $p \in [0, 1]$  to solve (2), where  $q$  is substituted by the offender's best-response function,  $q_{br}(p)$ , given by (4). The enforcer's problem is thus to choose  $p \in [0, 1]$  to minimize:

$$u(q_{br}(p), p) = \begin{cases} -H & \text{if } p = 0 \\ -[(1 - p\mu)q_{br}(p)H + c(p)] & \text{if } 0 < p < \tilde{p} \\ -c(p) & \text{if } \tilde{p} \leq p \leq 1 \end{cases} \quad (11)$$

<sup>22</sup>For example, as  $\mu$ , the portion of preventable harm, approaches zero, the enforcer's expected payoff from harm prevention approaches zero; accordingly, the equilibrium level of non-compliance approaches 1 for any magnitude of sanction.

<sup>23</sup>Note that an increase in the sanction makes the offender's best-response curve steeper—i.e., more elastic—but does not affect the enforcer's reaction curve, as the sanction is costless to the enforcer. This leads to a new NE in which the levels of non-compliance and enforcement are lower as compared to the initial NE ( $\frac{dq^{n*}}{dS} < 0$ ,  $\frac{dp^{n*}}{dS} < 0$ ). This result contrasts Tsebelis' (1990) argument that in a game-theoretic model, the penalty has no effect on crime. See also, Hirshleifer and Rasmusen (1992).

The enforcer never chooses a level of enforcement greater than  $\tilde{p}$ , because the offender's best response to  $p > \tilde{p}$  is to fully comply. The enforcer's payoff from choosing  $\tilde{p}$  is consequently greater than her payoff from choosing  $p \in (\tilde{p}, 1]$ . The enforcer, moreover, never chooses a level of enforcement lower than the Nash equilibrium level,  $p^{n^*}$ . To see why, note that the offender's best response to  $p < p^{n^*}$  is  $q > q^{n^*}$ . But because the enforcer's best response to  $q > q^{n^*}$  is  $p > p^{n^*}$  and because the enforcer's payoff is decreasing along her reaction curve, the enforcer's Nash equilibrium payoff ( $u(q^{n^*}, p^{n^*})$ ) is higher than her equilibrium payoff from choosing, as a leader,  $p < p^{n^*}$ .

Because the enforcer's level of enforcement is smaller than  $\tilde{p}$  but higher than  $p^{n^*}$ , the enforcer chooses  $p \in [p^{n^*}, \tilde{p}]$ , where her optimal choice is either  $\tilde{p}$  (i.e., a corner solution) or  $p \equiv \hat{p}$  satisfying the following first-order condition:

$$\frac{du(q_{br}(p), p)}{dp} = -c'(p) + q_{br}(p)\mu H - \frac{dq_{br}(p)}{dp}(1 - p\mu)H = 0. \quad (12)$$

The first term is the enforcer's marginal cost from increasing the probability of detection. The second term is the enforcer's marginal benefit from preventing harm, taking the offender's level of non-compliance as given (direct effect). Observe that at  $p^{n^*}$  (i.e., the enforcer's NE strategy), the sum of the first two terms is zero. The third term is the enforcer's marginal benefit from inducing compliance (strategic/deterrence effect).

The enforcer's optimal probability of detection is given by<sup>24</sup>

$$p^{e^*} = \begin{cases} \hat{p} & \text{if } u_{p^-}^e(\tilde{p}) < 0 \\ \tilde{p} & \text{otherwise} \end{cases}. \quad (13)$$

The offender's equilibrium level of non-compliance is, accordingly,  $q^{e^*} = q_{br}(p^{e^*})$ .

**Lemma 2 (equilibrium strategies in an enforcer-leadership game)**

The SPE in an enforcer-leadership game is either  $q^{e^*} = 0$  and  $p^{e^*} = \tilde{p}$  (full-compliance equilibrium) or  $q^{e^*} > 0$  and  $p^{e^*} < \tilde{p}$  (partial-compliance equilibrium). ||

**Remark:** In an enforcer-leadership game, the role of enforcement is to induce compliance and to prevent or remediate harm from non-compliance (if  $q^{e^*} > 0$ ). The enforcer's choice of level of enforcement induces either full compliance or partial compliance. The equilibrium outcome—full compliance or partial compliance—depends on the elasticity of the offender's reaction curve:

<sup>24</sup> $u_{p^-}^e$  is the derivative from the left of  $u^e$  with respect to  $p$ .

If the offender's reaction curve is sufficiently elastic (inelastic), then the equilibrium is one of full compliance (partial compliance). For example, if the sanction is sufficiently low, the offender's optimal level of non-compliance is relatively insensitive to the enforcer's probability of detection. Because the enforcer's costs of inducing compliance are relatively high, the enforcer prefers to induce partial, rather than full, compliance.<sup>25</sup> If, in contrast, the sanction is sufficiently high, the offender's optimal level of non-compliance is relatively sensitive to the enforcer's probability of detection. Because the costs of inducing compliance are relatively low, the enforcer prefers to induce full compliance.<sup>26</sup>

**Proposition 1 (enforcer-leadership versus simultaneous game)**

*In comparison to a simultaneous-move game, an enforcer-leadership game is characterized by (1) more compliance ( $q^e < q^{n*}$ ), (2) more enforcement ( $p^e > p^{n*}$ ), (3) higher equilibrium payoff for the enforcer ( $u^e > u^{n*}$ ), and (4) lower equilibrium payoff for the offender ( $v^e < v^{n*}$ ). ||*

To see why the enforcer commits to a higher level of enforcement (part 2), recall that, in a simultaneous-move game, the enforcer equates the marginal benefit and cost from enforcement *given* the offender's equilibrium level of non-compliance (see (6)). In an enforcer-leadership game, in contrast, the enforcer takes into account the effect of her strategy on the offender's non-compliance choice (see the third term in (12)). This implies that at the enforcer's NE strategy,  $p^{n*}$ , there is an additional marginal benefit to the enforcer from increasing the probability of detection. Because the enforcer as a leader can choose her NE strategy and thereby guarantee her NE payoff, any deviation from her NE strategy must increase her equilibrium payoff (part 3).

To see why the level of non-compliance is lower relative to a simultaneous-move game (part 3), recall that the offender's best-response function is decreasing with  $p$  for  $p \in [0, \tilde{p}]$ . This, together with the fact that the level of enforcement in an enforcer-leadership game is higher than the Nash equilibrium level, implies that the level of non-compliance is lower than the Nash equilibrium level. Finally, because the level of enforcement is higher than the Nash equilibrium level and the offender's payoff is decreasing along her reaction curve, the offender's equilibrium payoff is lower than her payoff in a simultaneous-move game (part 4).

<sup>25</sup>If  $S = 0$ , then the offender's reaction curve is inelastic for all  $p \in [0, 1)$  (see Figure 2).

<sup>26</sup>In contrast to a simultaneous-move game, an increase in the sanction either increases or decreases the level of enforcement. To see this, note that in a full-compliance equilibrium, the level of enforcement decreases with  $S$ , whereas in a partial-compliance equilibrium, the level of enforcement may either increase or decrease with  $S$ .

Intuitively, because this is a *game of competition with strategic substitutes from the offender's perspective*, the enforcer enjoys a first-mover advantage relative to a simultaneous-move game: The enforcer gains from inducing the offender to decrease his level of non-compliance by committing to a higher level of enforcement. Because the offender's payoff is decreasing along his reaction curve, the offender's payoff is lower than in a simultaneous-move game. The offender thus suffers a second-mover disadvantage relative to a simultaneous-move game.

### 4.3 Offender-Leadership Game

We now consider the case in which the offender acts as a Stackelberg leader. In an offender-leadership game, the offender irrevocably chooses an observable level of non-compliance. The enforcer's level of enforcement, in turn, constitutes a best response to the offender's level of non-compliance. The offender therefore takes into account the effect of his strategy on the enforcer's strategy. As in an enforcer-leadership game, the solution concept is SPE.

As mentioned in the Introduction, an offender-leadership game underlies enforcement settings that involve the investigation of criminal or administrative offenses. In many of these settings, the offender cannot retroactively alter his level of non-compliance. The enforcer, on her part, chooses a level of enforcement (i.e., investigation) after having observed the offender's level of non-compliance.

In an offender-leadership game, the offender chooses  $q \in [0, 1]$  to maximize (1), where  $p$  is substituted by the enforcer's best-response function,  $p_{br}(q)$ , given by (7). The offender's problem is thus to choose  $q \in [0, 1]$  to maximize:

$$v(q, p_{br}(q)) = \begin{cases} G(q) & \text{if } 0 \leq q \leq \bar{q} \\ (1 - p_{br}(q))G(q) - qp_{br}(q)S & \text{if } \bar{q} < q \leq 1 \end{cases} \quad (14)$$

The offender never chooses a level of non-compliance smaller than  $\bar{q}$ , because the enforcer's best response to  $q < \bar{q}$  is to not enforce. The offender's payoff from choosing  $\bar{q}$  is consequently greater than his payoff from choosing  $q \in [0, \bar{q})$ . The offender, moreover, never chooses a level of non-compliance higher than the Nash equilibrium level,  $q^{n*}$ . To see why, note that the enforcer's best response to  $q > q^{n*}$  is  $p > p^{n*}$ . But because the offender's best response to  $p > p^{n*}$  is  $q < q^{n*}$  and because the offender's payoff is decreasing along his reaction curve, the offender's NE payoff ( $v(q^{n*}, p^{n*})$ ) is greater than his payoff from choosing as a leader  $q > q^{n*}$ . The offender's Nash equilibrium payoff is thus greater than his payoff from choosing, as a leader,  $q > q^{n*}$ .

Because the offender's level of non-compliance is smaller than  $q^{n^*}$  but higher than  $\bar{q}$ , the offender chooses  $q \in [\bar{q}, q^{n^*}]$ , where his optimal choice is either  $\bar{q}$  (a corner solution) or  $q \equiv \widehat{q}$ , satisfying the following first-order condition:<sup>27</sup>

$$\frac{dv(q, p_{br}(q))}{dq} = (1 - p_{br}(q))G'(q) - p_{br}(q)S - \frac{dp_{br}(q)}{dq}(G(q) + qS) = 0. \quad (15)$$

The first two terms in (15) represent the offender's marginal net benefit (or cost) from increasing the level of non-compliance, given the enforcer's probability of detection. Observe that at  $q^{n^*}$  (i.e., the offender's NE strategy), the sum of the first two terms is zero. The last term reflects the offender's additional marginal cost from increasing  $q$  stemming from the higher level of enforcement induced by a higher level of non-compliance (strategic/inducement effect).

The offender's optimal level of non-compliance is given by<sup>28</sup>

$$q^{o^*} = \begin{cases} \bar{q} & \text{if } v_{q^+}^o(\bar{q}) \leq 0 \\ \widehat{q} & \text{otherwise} \end{cases}. \quad (16)$$

The enforcer's equilibrium level of enforcement is, accordingly,  $p^{o^*} = p_{br}(q^{o^*})$ .

**Lemma 3 (equilibrium strategies in an offender-leadership game)**

*The SPE in an offender-leadership game is either  $q^{o^*} = \bar{q}$  and  $p^{o^*} = 0$  (no-enforcement equilibrium) or  $q^{o^*} > \bar{q}$  and  $p^{o^*} > 0$  (partial-enforcement equilibrium). ||*

**Remark:** As in a simultaneous-move game, the role of enforcement in an offender-leadership game is to prevent or remediate harm from non-compliance, rather than to induce compliance. In contrast to a simultaneous-move game, however, compliance (deterrence) also results from the offender's strategic response to the fact that a higher level of non-compliance induces more enforcement. The equilibrium outcome (partial enforcement or no enforcement) depends on the elasticity of the enforcer's reaction curve: if the enforcer's reaction curve is sufficiently elastic (inelastic), then the equilibrium is one of no enforcement (partial enforcement).<sup>29</sup>

<sup>27</sup>We assume that  $p'''(\cdot) > 0$  so that (15) has a unique solution.

<sup>28</sup> $v_{q^+}^o$  is the derivative from the right of  $v^o$  with respect to  $q$ .

<sup>29</sup>For example, if enforcement costs are linear in the probability of detection (i.e., if  $c(p) = cp$ , for some constant  $c > 0$ ), then the enforcer's best-response correspondence is perfectly elastic at  $q = \bar{q}$  (see Figure 5). The offender can therefore induce  $p = 0$  (i.e., no-enforcement) by choosing his NE strategy.

Proposition 2 compares the equilibrium outcomes in an offender-leadership game and a simultaneous-move game. Although the method of enforcement is different in these games (i.e., investigation versus monitoring), comparing them serves as an analytical step towards comparing the two Stackelberg games.<sup>30</sup>

**Proposition 2 (offender-leadership versus simultaneous game)**

*In comparison to a simultaneous-move game, an offender-leadership game is characterized by (1) more compliance ( $q^{o*} < q^{n*}$ ), (2) less enforcement ( $p^{o*} < p^{n*}$ ), (3) higher equilibrium payoff for the enforcer ( $u^{o*} > u^{n*}$ ), and (4) higher equilibrium payoff for the offender ( $v^{o*} > v^{n*}$ ). ||*

To see why the offender chooses a lower level of non-compliance (part 1), recall that in a simultaneous-move game the offender equates the marginal benefit and cost from non-compliance *given* the enforcer's equilibrium level of enforcement. In an offender-leadership game, in contrast, the offender takes into account the effect of his strategy on the enforcer's choice of level of enforcement (see the third term in (15)). This implies that at the offender's NE strategy,  $q^{n*}$ , there is an additional marginal cost to the offender from increasing the level of non-compliance. Because the offender as a leader can choose his NE strategy and thereby guarantee his NE payoff, any deviation from his NE strategy must increase his expected payoff (part 4).

To see why the level of enforcement is higher relative to a simultaneous-move game (part 2), recall that the enforcer's best-response function is increasing with  $q$ . This, together with the fact that the offender's level of non-compliance in an offender-leadership game is lower than the Nash equilibrium level, imply that the enforcer's level of enforcement is lower than the Nash equilibrium level. Finally, because the level of non-compliance is lower than the NE level and the enforcer's payoff is decreasing along her reaction curve, the enforcer's equilibrium payoff is higher in an offender-leadership versus a simultaneous-move game (part 3).

Intuitively, because this is a *game of competition with strategic complements from the enforcer's perspective*, the offender enjoys a first-mover advantage relative to a simultaneous-move game: The offender, as a leader, gains from inducing the enforcer to lessen her level of enforcement by choosing a lower level of non-compliance. Because the enforcer's payoff is decreasing along her reaction curve, the enforcer gains (relative to a simultaneous-move

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<sup>30</sup>Moreover, Proposition 2 always holds if enforcement is more efficient in a simultaneous-move than in an offender-leadership game—more precisely, if for any cost of enforcement, the probability of detecting non-compliance is higher in a simultaneous-move game than in an offender-leadership game.

game) from this lower level of non-compliance. The enforcer thus enjoys a second-mover advantage relative to a simultaneous-move game.

We conclude this section by illustrating the equilibrium outcomes in the different leadership games using the specific functions given in Example 1. As Figure 2 shows, in each Stackelberg equilibrium, the leader's iso-payoff curve is tangent to the follower's reaction curve.

**Example 2** (*equilibria in leadership games*)

Suppose  $G(q) = -q^2 + 2q$ ,  $c(p) = (0.1 + p)^3$ ,  $H = \mu = 1$  and  $S = 0.8$ . Then:  
 (i) The levels of non-compliance and enforcement in an enforcer-leadership game are  $q^e = 0.44$  and  $p^e = 0.58$ , respectively;  
 (ii) The levels of non-compliance and enforcement in an offender-leadership game are  $q^o = 0.49$  and  $p^o = 0.30$ , respectively; and  
 (iii) The enforcer's equilibrium payoff is higher as a follower ( $-0.4$ ) than as a leader ( $-0.5$ ). ||

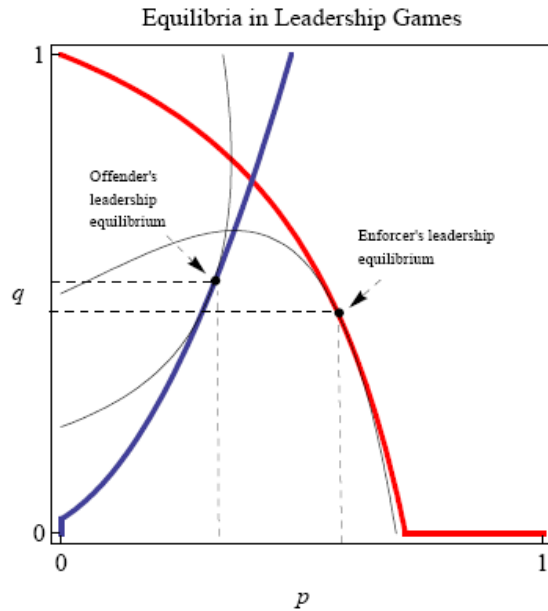


Figure 2: Enforcer- and offender-leadership equilibria (Example 2)

Example 2 illustrates that, although the level of non-compliance is lower in an enforcer-leadership game than in an offender-leadership game, the enforcer's equilibrium payoff is higher as a follower than as a leader. The next section more closely compares the equilibrium outcomes in the different leadership games.

## 5 Enforcer- versus Offender-Leadership Game

### 5.1 Enforcer's Equilibrium Payoff

We now turn to the question of whether the enforcer's equilibrium payoff is higher as a leader or as a follower; that is, whether the enforcer should commit to an enforcement strategy or let the offender move first. This question arises, in particular, if the enforcement technology in an enforcer-leadership game is based on investigation. Before proceeding to compare the equilibrium strategies and players' payoffs in the Stackelberg games, we summarize the results of Propositions 1 and 2 in Table 1:

	Enforcer-Leadership	Offender-Leadership
Non-Compliance [ $q$ ]	Lower	Lower
Enforcement [ $p$ ]	Higher	Lower
Enforcer-Payoff [ $u$ ]	Higher	Higher
Offender-Payoff [ $v$ ]	Lower	Higher

Table 1: Comparisons of equilibrium strategies in Stackelberg games versus a simultaneous game

**Proposition 3 (offender-leadership versus enforcer-leadership game)**

*In comparison to an enforcer-leadership game, an offender-leadership game is characterized by (1) more or less compliance ( $q^{o^*} \geq < q^{e^*}$ ), (2) less enforcement ( $p^{o^*} < p^{e^*}$ ), (3) higher or lower equilibrium payoff for the enforcer ( $u^{o^*} \geq < u^{e^*}$ ), and (4) higher equilibrium payoff for the offender ( $v^{o^*} > v^{e^*}$ ). ||*

Consider first part (2). Recall from Propositions 1(2) and 2(2), respectively, that the equilibrium level of enforcement is higher in an enforcer-leadership game than in a simultaneous-move game ( $p^{e^*} > p^{n^*}$ ) and is higher in a simultaneous-move game than in an offender-leadership game than ( $p^{n^*} > p^{o^*}$ ). It follows that the enforcer's level of enforcement is higher in an enforcer-leadership game than in an offender-leadership game.

Next, consider Part (4). Recall from Propositions 2(4) and 3(4), respectively, that the offender suffers a second-mover disadvantage and enjoys a first-mover advantage: The offender's equilibrium payoff is higher in a simultaneous-move game than in an enforcer-leadership game ( $v^{n^*} > v^{e^*}$ ) and is higher in an offender-leadership game than in a simultaneous-move game ( $v^{o^*} > v^{n^*}$ ). It



follows that the offender’s equilibrium payoff is higher in an offender-leadership game than in an enforcer-leadership game (that is, his equilibrium payoff is higher as a leader than as a follower).

To prove parts (1) and (3), we consider three examples that illustrate that the relation between the enforcer’s equilibrium payoff as a leader versus as a follower, depends on the relation between the enforcer’s first- and second-mover advantage. The first two examples consider cases in which the enforcer has either no first-mover advantage or no second-mover advantage. The third example considers a case in which the enforcer’s first- and second-mover advantages depend on the model parameters. These examples also show that the level of non-compliance may be either higher or lower in an enforcer-leadership game relative to an offender-leadership game.<sup>31</sup>

**Example 3** (*no punishment*)

Suppose  $S = 0$ . Then (i)  $q^{o*} < q^{e*} = q^{n*}$ , (ii)  $p^{o*} < p^{e*} = p^{n*}$ , and (iii)  $u^{o*} > u^{e*} = u^{n*}$ . ||

We proceed by showing that, if there is no sanction for non-compliance, the enforcer has no first-mover advantage but has a second-mover advantage. The enforcer’s equilibrium payoff, consequently, is higher as a follower than as a leader.

When there is no sanction for non-compliance, the offender’s best-response correspondence (see (6)) is

$$q_{br}(p) = \begin{cases} 1 & \text{if } 0 \leq p < 1 \\ [0, 1] & \text{if } p = 1 \end{cases} . \tag{17}$$

(17) implies that the offender chooses not to comply for  $p \in [0, 1)$  and is indifferent between any level of non-compliance for  $p = 1$  (see Figure 3(a)). To see why, note that if there is no sanction for non-compliance, the offender’s marginal cost of increasing the level of non-compliance is zero. Because the marginal benefit from non-compliance is always positive, the offender maximizes his payoff for any  $p \in [0, 1)$  by choosing the highest level of non-compliance.

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<sup>31</sup>Note that if  $q^{o*} \leq q^{e*}$  then  $u^{o*} > u^{e*}$ . To see this, observe that  $u(p_{br}(q^{o*}), q^{o*}) > u(p_{br}(q^{e*}), q^{e*}) > u(p^{e*}, q^{e*})$ , where the first inequality follows because  $\frac{du(p_{br}(q), q)}{dq} < 0$  and the second inequality follows because  $u(p_{br}(q'), q') > u(p', q')$  for  $p' \neq p_{br}(q')$ . In contrast,  $q^{e*} < q^{o*}$  does not imply that  $u^{e*} > u^{o*}$ . To see this, suppose an enforcer-leadership game is characterized by full compliance ( $q^{e*} = 0$ ) and an offender-leadership game is characterized by no-enforcement ( $p^{o*} = 0$ ). Then the enforcer’s equilibrium payoff is higher in an enforcer-leadership game than in an offender-leadership game iff  $c(\bar{p}) < \bar{q}H$ . In particular, if  $\bar{q}$  is sufficiently small, then  $u^{o*} > u^{e*}$ .

If  $p = 1$ , in contrast, the offender is indifferent between any level of non-compliance (because his payoff is zero for any level of non-compliance). It follows that the equilibrium strategies in an enforcer-leadership game are the NE strategies ( $q^{e^*} = q^{n^*}$ ,  $p^{e^*} = p^{n^*}$ ) and that the enforcer's equilibrium payoff as a leader is identical to her NE payoff (i.e.,  $u^{e^*} = u^{n^*}$ ); the enforcer thus has no first-mover advantage.

In contrast, if the offender acts as a leader, his problem is to maximize  $(1 - p_{br}(q))G(q)$ . This implies that the offender equates the marginal benefit from non-compliance and its marginal cost. Because a higher level of non-compliance induces a higher level of enforcement, the offender faces a positive marginal cost of increasing the level of non-compliance. The offender's optimal level of non-compliance must be lower than 1, because the offender's marginal benefit at  $q = 1$  is zero. Finally, because the offender chooses to comply with positive probability as a leader but not as a follower, the enforcer has a second-mover advantage but not a first-mover advantage.<sup>32</sup>

**Example 4** (*linear enforcement costs*)

Suppose that enforcement costs are linear in the probability of detection ( $\Rightarrow c(p) = \bar{c}p$  for some constant  $\bar{c} > 0$ ). Then (i)  $q^{e^*} < q^{o^*} = q^{n^*} = \bar{q}$ , (ii)  $p^{e^*} > p^{n^*} > p^{o^*} = 0$ , and (iii)  $u^{e^*} > u^{o^*} = u^{n^*}$ . ||

We proceed by showing that, if enforcement costs are linear in the probability of detection, the enforcer does not have a second-mover advantage, but has a first-mover advantage. The enforcer's equilibrium payoff, consequently, is higher as a leader than as a follower.

When enforcement costs are linear in the probability of detection, the enforcer's best-response correspondence (see (10)) is

$$p_{br}(q) = \begin{cases} 0 & \text{if } 0 \leq q < \bar{q} \\ [0, 1] & \text{if } \bar{q} = \frac{c}{\mu H} \\ 1 & \text{if } \bar{q} < q \leq 1 \end{cases} . \quad (18)$$

(18) implies that the enforcer chooses not to enforce for  $q < \bar{q}$ , to fully enforce for  $q > \bar{q}$ , and is indifferent about the level of enforcement for  $q = \bar{q}$  (see Figure 3(b)). This follows because, for  $q > (<)\bar{q}$ , the marginal benefit from enforcement is greater (smaller) than its marginal cost. The offender as a leader can therefore induce no-enforcement by choosing a level of non-compliance

<sup>32</sup>The case in which there is no sanction for non-compliance belongs to a broader class of cases in which the sanction for non-compliance is constant and sufficiently small so that the enforcer's and the offender's reaction curves intersect at  $q = 1$ .

infinitesimally smaller than the Nash equilibrium level. A limit argument suggests that the offender's equilibrium strategy as a leader is his NE strategy ( $q^{o*} = q^{n*}$ ) and that the enforcer's equilibrium strategy as a follower is to not enforce ( $p^{o*} = 0$ ). The enforcer's equilibrium payoff as a follower is consequently identical to his Nash equilibrium payoff (i.e.,  $u^{o*} = u^{n*}$ —recall that, for  $q = q^{n*}$ , the enforcer's equilibrium payoff is invariant to the probability of detection), and the enforcer has no second-mover advantage.

If the enforcer acts as a leader, in contrast, she faces the downward-sloping reaction curve of the offender. The enforcer can therefore induce a lower level of non-compliance as compared to a simultaneous-move game. This implies that the enforcer's equilibrium payoff as a leader is greater than her Nash equilibrium payoff (i.e.,  $u^{e*} > u^{n*}$ ). The enforcer then enjoys a first-mover advantage. Because the enforcer enjoys a first-mover advantage but not a second-mover advantage, the enforcer's equilibrium payoff is higher as a leader than as a follower. We would obtain the same result if the cost of enforcements were  $c(p) = \begin{cases} \bar{c} & \text{if } p > 0 \\ 0 & \text{if } p = 0 \end{cases}$  for some  $\bar{c} > 0$ , so that the enforcer can detect the offender with any probability if she spends a fixed cost of  $\bar{c}$ .

**Example 5** (*linear gains*)

Suppose the offender's gains are linear in the level of non-compliance (i.e.,  $G(q) = qG$  for some  $G > 0$ ), enforcement costs are given by  $c(p) = p^2$ , and  $\mu = 1$ . Then, letting  $\lambda = \frac{G}{G+S}$ : (i)  $p^{n*} = \lambda$ ,  $q^{n*} = \frac{2\lambda}{H}$ ,  $p^{e*} = \lambda$ ,  $q^{e*} = 0$ ,  $p^{o*} = \lambda/2$ , and  $q^{o*} = \frac{\lambda}{H}$ ; and (ii)  $u^{e*} > (\leq) u^{o*}$  iff  $\lambda < (\leq) \frac{4}{5}$  (that is, iff  $S > (\leq) \frac{G}{4}$ ). ||

We prove in Appendix A the more general case in which  $c(p) = p^k$ , where  $k > 1$ . If the offender's gains are linear in the level of non-compliance, the offender's best-response correspondence (see (6)) is

$$q_{br}(p) = \begin{cases} 1 & \text{if } 0 \leq p < \lambda \\ [0, 1] & \text{if } p = \lambda \\ 0 & \text{if } \lambda < p \leq 1 \end{cases} . \tag{19}$$

(19) implies that the offender chooses to fully not comply for  $p < \lambda$ , to fully comply for  $p > \lambda$ , and is indifferent about the level of non-compliance for  $p = \lambda$  (see Figure 3(c)). This follows because, for  $p > (<)\lambda$ , the marginal benefit from non-compliance is smaller (greater) than its marginal cost. The enforcer, as a leader, can therefore induce full compliance by choosing a level of enforcement infinitesimally higher than the Nash equilibrium level. A limit argument suggests that the enforcer's equilibrium strategy as a leader is her

Nash equilibrium strategy ( $p^{o*} = p^{n*}$ ), and that the offender's equilibrium strategy as a follower is to fully comply ( $q^{o*} = 0$ ). Given linear gains from non-compliance, therefore, the enforcer has the greatest first-mover advantage: she can induce full compliance by choosing her Nash equilibrium strategy. Whether the enforcer's equilibrium payoff is higher if she moves first or second depends on the magnitude of her first- versus second-mover advantage. This depends, in turn, on the relation between the magnitude of the sanction and the offender's gains from non-compliance.

To see this, note that the enforcer's Nash equilibrium payoff is  $-q^{n*}(1 - p^{n*})H - (p^{n*})^2$ , her equilibrium payoff as a leader is  $-(p^{n*})^2$ , and her equilibrium payoff as a follower is  $-q^{o*}(1 - p^{o*})H - (p^{o*})^2$ . The enforcer's first- and second-mover advantages are therefore

$$q^{n*}(1 - p^{n*})H$$

and

$$[q^{n*}(1 - p^{n*})H - q^{o*}(1 - p^{o*})H + (p^{n*})^2 - (p^{o*})^2],$$

respectively. The first expression is the difference between the enforcer's equilibrium payoff as a leader and her Nash equilibrium payoff; the second expression is the difference between the enforcer's equilibrium payoff as a follower and her Nash equilibrium payoff.

The enforcer's second-mover advantage is greater than her first-mover advantage iff

$$(p^{n*})^2 - (p^{o*})^2 > q^{o*}(1 - p^{o*})H.$$

That is, the enforcer prefers to move second if her gains from the lower enforcement costs as a follower are greater than her loss from the greater expected harm as a follower. Plugging  $q^{o*} = \frac{\lambda}{H}$ ,  $p^{o*} = \frac{1}{2}\lambda$ , and  $p^{n*} = \lambda$ , gives

$$\begin{aligned} (p^{n*})^2 - (p^{o*})^2 &= \\ \frac{3}{4}\lambda^2 &> \lambda - \frac{1}{2}\lambda^2 \\ &= q^{o*}(1 - p^{o*})H. \end{aligned}$$

This inequality implies that the enforcer prefers to move first if  $\lambda > \frac{4}{5}$ , to move second if  $\lambda < \frac{4}{5}$ , and is indifferent between moving first and second if  $\lambda = \frac{4}{5}$ . Thus, as the costs of enforcement in an enforcer-leadership game ( $(p^{e*})^2 = \lambda^2$ ) become sufficiently high, the enforcer's saving in enforcement costs as a follower outweighs her lower expected harm as a leader. It follows that if the sanction is sufficiently low relative to the gains from non-compliance,

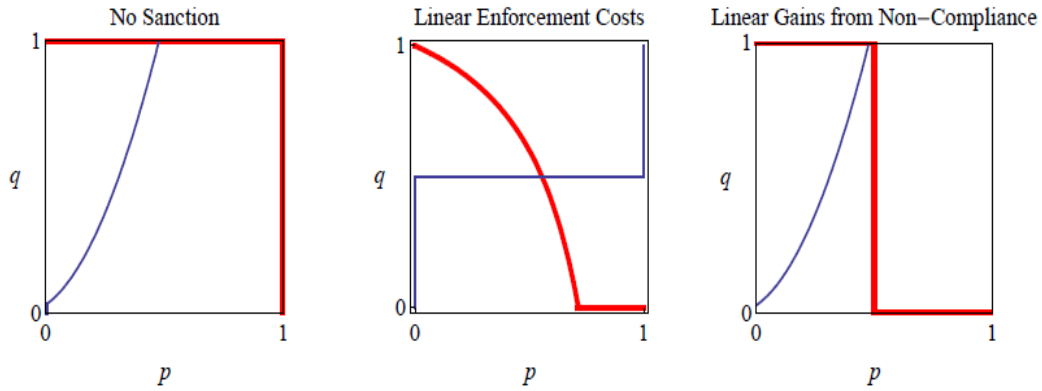


Figure 3: (a) no sanction (Example 3); (b) linear enforcement costs (Example 4); (c) linear gains from non-compliance (Example 5)

the enforcer's payoff is higher as a follower than as a leader.

More generally, whether the enforcer's payoff is higher as a leader or as a follower depends on the relation between her first- and second-mover advantage. Each advantage depends on the strategic effects in the different leadership games: the greater the deterrence effect in an enforcer-leadership game, the greater the enforcer's first-mover advantage; the smaller the inducement effect in an enforcer-leadership game, the greater the enforcer's second-mover advantage. More specifically, the more responsive an offender-follower is to a given increase in the level of enforcement, the greater the enforcer's first-mover advantage will be. Similarly, the less responsive an enforcer-follower is to a given decrease in the level of non-compliance, the greater the enforcer's second-mover advantage will be. As the previous examples showed, the magnitude of the enforcer's first- and second-mover advantages depend on the sanction for non-compliance, as well as the enforcement and non-compliance technologies.

## 5.2 Comparative Statics

In this section, to further explore the differences between an enforcer-leadership game and an offender-leadership game, we compare the effect of an increase in the magnitude of the sanction ( $S$ ) and the magnitude of the harm ( $H$ ) on the equilibrium strategies and payoffs in the two leadership games. This

comparison sheds additional light on the direct and strategic effects of an increase in the leader’s strategy on the follower’s response in the two games.

**Proposition 4 (effect of sanction and harm on equilibrium strategies)**

(1) In an enforcer-leadership game: (i) As the harm increases, the level of enforcement increases and the level of non-compliance decreases; (ii) As the sanction increases, the level of enforcement may either increase or decrease, whereas the level of non-compliance decreases.

(2) In an offender-leadership game: (i) As the sanction increases, the level of non-compliance and the level of enforcement decrease; (ii) As the harm increases, the level of non-compliance decreases, whereas the level of enforcement may either increase or decrease.<sup>33</sup> ||

**Proof.** See Appendix B.

The following Table presents the results of Proposition 4:

Game↓ / Change⇒	S↑	H↑
Enforcer-leadership:	q↓ p↑↓	q↓ p↑
Offender-leadership:	q↓ p↓	q↓ p↑↓

Table 2: Effect of an increase in  $S$  and  $H$  on equilibrium strategies

As the harm increases, the enforcer’s marginal benefit from detecting the offender, given a fixed level of non-compliance, increases, because enforcement now prevents or remediates a greater harm. For the same reason, the enforcer’s marginal benefit from deterring non-compliance also increases with the harm. The higher marginal benefit from enforcement thus causes the enforcer-leader to choose a higher level of enforcement. Because non-compliance and enforcement are strategic substitutes from the offender’s perspective, the offender as a follower chooses a lower level of non-compliance.<sup>34</sup>

A similar reasoning applies to the effect of an increase in the sanction on the offender-leader’s choice of a level of non-compliance. As the sanction increases, the offender’s marginal costs of increasing the level of non-compliance—given a fixed level of enforcement—increases, because a detected offender now faces a higher sanction. For the same reason, the offender’s marginal cost of increasing the level of non-compliance and thereby inducing a

<sup>33</sup>Assume that, from the offender’s perspective, non-compliance and enforcement in an offender-leadership game are strategic substitutes.

<sup>34</sup>An increase in the harm in an enforcer-leadership game has only an indirect effect on the level of non-compliance, through its (positive) effect on the level of enforcement.

higher level of enforcement also increases with the sanction. The higher marginal cost from non-compliance causes the offender-leader to choose a lower level of non-compliance. Because enforcement and non-compliance are strategic complements from the enforcer's perspective, the enforcer as a follower chooses a lower level of enforcement.<sup>35</sup>

The effect of an increase in the sanction on the enforcer-leader's choice of a level of enforcement is more subtle. The enforcer's marginal benefit from detecting the offender—given a fixed level of non-compliance—is not affected by the magnitude of the sanction. However, because an increase in the sanction induces the offender-follower to choose a lower level of non-compliance for any level of enforcement, the enforcer's marginal benefit from detecting non-compliance is now lower. The enforcer's marginal benefit from deterring non-compliance, in contrast, is greater because a higher sanction renders enforcement more effective: For each level of enforcement, the offender's best response involves a lower level of non-compliance. Thus, the enforcer's marginal benefit from enforcement either increases or decreases as the sanction increases. Accordingly, the level of enforcement and the magnitude of the sanction are either complementary or substitute enforcement instruments.<sup>36</sup>

The effect of an increase in the harm on the offender-leader's choice of a level of non-compliance is similarly ambiguous. The offender's marginal cost of increasing the level of non-compliance—given a fixed level of enforcement—increases with the harm, because the enforcer's best response now involves a higher level of enforcement for any level of non-compliance. However, the offender's marginal cost of increasing the level of non-compliance, and thereby inducing a higher level of enforcement, may either increase or decrease, depending on the elasticity of the enforcer's (shifted) best-response curve.

To resolve this indeterminacy, we assume that, as in the simultaneous-move game, non-compliance and enforcement are strategic substitutes from the offender's perspective.<sup>37</sup> This assumption implies that the offender-leader's marginal cost of increasing the level of non-compliance increases with the magnitude of the harm and therefore that the level of non-compliance decreases as

<sup>35</sup>An increase in the sanction in an offender-leadership game has only an indirect effect on the level of enforcement, through its (negative) effect on the level of non-compliance.

<sup>36</sup>The level of non-compliance necessarily decreases because both the direct effect (holding the level of enforcement fixed) and the indirect effect (through the (positive) direct effect on the level of enforcement) of an increase in the sanction on the level of non-compliance are negative.

<sup>37</sup>Specifically, we assume that the partial derivative of the offender's leadership-payoff with respect to the level of enforcement,  $p$ , is negative. A sufficient, although not necessary, condition for this partial derivative to be negative is that  $c'''(p) < 0$  (see (B16) in Appendix B).

the harm increases. The enforcer-follower's level of enforcement, in contrast, may either increase or decrease: Although the increase in the harm increases the marginal benefit from enforcement (direct effect), the offender's lower level of non-compliance decreases it (indirect effect).

The next proposition considers the effect of an increase in the magnitude of the sanction or the harm on the enforcer's and the offender's equilibrium payoffs:

**Proposition 5 (effect of sanction and harm on equilibrium payoffs)**

(1) *As the sanction increases, the enforcer's equilibrium payoff increases and the offender's equilibrium payoff decreases.*

(2) *As the harm increases, the offender's equilibrium payoff decreases, whereas the enforcer's equilibrium payoff decreases if she is the leader, but either increases or decreases if she is the follower. ||*

**Proof.** See Appendix B.

Interestingly, an increase in the harm does not necessarily decrease the enforcer's equilibrium payoff as a follower. On one hand, given a fixed level of non-compliance, the enforcer's payoff decreases with the magnitude of the harm. On the other hand, because the enforcer's level of enforcement increases as the harm increases, the offender is more reluctant as a leader to increase his level of non-compliance. If the effect of an increase in the harm on the offender-leader's choice of a level of non-compliance is sufficiently high, the enforcer's equilibrium payoff increases with the magnitude of the harm.

## Conclusion

This paper considers the role of commitment in enforcement games when enforcement can remediate or prevent harm from non-compliance. We examined a one-shot game in which an offender chooses a level of non-compliance and an enforcer chooses a level of enforcement. We showed that the enforcer enjoys both a first-mover and a second-mover advantage relative to a simultaneous-move game. The enforcer realizes her first-mover advantage by choosing a higher level of investigation as compared to a simultaneous-move game, thereby inducing a lower level of non-compliance. The enforcer's second-mover advantage stems from the fact that the offender exploits his leadership position by choosing a lower level of non-compliance as compared to a simultaneous-move game. Whether the enforcer prefers to move first by committing to an enforcement strategy or rather let the offender move first thus depends on the relation between her first- versus second-mover advantage.



We further showed that the enforcer’s first-mover advantage depends on the responsiveness of the offender as a follower to a change in the level of enforcement. The enforcer’s second-mover advantage, in contrast, depends on her own responsiveness as a follower to a change in the level of non-compliance. The value of commitment to the enforcer thus hinges on the relative responsiveness of each player—the offender and the enforcer—to the other player’s strategy. In particular, the enforcer prefers to be a follower if the offender is relatively unresponsive to a change in the level of enforcement; for example, if the sanction for non-compliance is sufficiently low. In contrast, the enforcer prefers to be a leader if she is relatively responsive to a change in the level of non-compliance; for example, if the enforcement technology exhibits constant marginal returns.

## Appendix A

In this Appendix, we prove a generalized version of example 4.

*Suppose the offender’s gains are linear in the level of non-compliance (i.e.,  $G(q) = qG$  for some  $G > 0$ ), enforcement costs are given by  $c(p) = p^k$ , where  $k > 1$ , and  $\mu = 1$ . Then:*  
*(i)  $0 = q^{e^*} < \bar{q} = q^{o^*} < q^{n^*}$ , (ii)  $0 = p^{o^*} < p^{n^*} = p^{e^*}$ , and (iii)  $u^{e^*} > (\leq) u^{o^*}$  if and only if  $\lambda < (\geq) \frac{k\left(\frac{k-1}{k}\right)^{k-1}}{(1+(k-1)\left(\frac{k-1}{k}\right)^k)}$ , where  $\lambda \equiv \frac{G}{G+S}$  reflects the relation between the gains from and sanction for non-compliance.*

**Proof.** First, note that  $c(0) = 0$ ,  $c'(p) = kp^{k-1} > 0$ ,  $c'(0) = 0$ , and  $c''(p) = k(k-1)p^{k-2} > 0$ .

From (6) and (10), the offender’s best response correspondence and the enforcer’s best response functions, respectively, are

$$q_{br}(p) = \begin{cases} 1 & \text{if } 0 \leq p < \tilde{p} \\ [0, 1] & \text{if } p = \tilde{p} \\ 0 & \text{if } \tilde{p} < p \leq 1 \end{cases} \quad (\text{A1})$$

and

$$p_{br}(q) = \left(\frac{qH}{k}\right)^{\frac{1}{k-1}}, \quad (\text{A2})$$

where  $\tilde{p} \equiv \lambda$ . From Lemma 1 we have the the following Lemma:

**Lemma A1 (equilibrium strategies in a simultaneous game)**

The unique equilibrium strategies in a simultaneous-move game are  $q^{n^*} = \lambda^{k-1} \frac{k}{H}$  and  $p^{n^*} = \lambda$ .

Next, consider an enforcer-leadership game. Since the enforcer as a leader can induce full compliance by choosing  $p = \tilde{p} + \varepsilon$  for some  $\varepsilon > 0$ , it follows that the offender's best response function in an enforcer-leadership game is

$$q_{br}(p) = \begin{cases} 1 & \text{if } 0 \leq p < \tilde{p} \\ 0 & \text{if } p \geq \tilde{p} \end{cases}. \quad (\text{A3})$$

Now, since  $u(\tilde{p}, 0) > u(p', 0)$  for  $p' \in (\tilde{p}, 1]$ , the enforcer's equilibrium strategy is  $p = \tilde{p}$

**Lemma A2 (equilibrium strategies in an enforcer-leadership game)**

(i) The unique SPE strategies in an enforcer-leadership game are  $q^{e^*} = 0$  and  $p^{n^*} = \lambda$ .

(ii) The enforcer's equilibrium payoff is  $u^{e^*} = -\lambda^k$ .

Consider now an offender-leadership game. From (15) we have

$$(1 - p_{br}(q))G'(q) - p_{br}(q)S - \frac{dp_{br}(q)}{dq}(G(q) + qS) = 0. \quad (\text{A4})$$

Plugging  $p_{br}(q) = \left(\frac{qH}{k}\right)^{\frac{1}{k-1}}$ ,  $\frac{dp_{br}(q)}{dq} = \frac{1}{k-1} \left(\frac{H}{k}\right)^{\frac{1}{k-1}} q^{\left(\frac{1}{k-1}-1\right)}$ , and rearranging gives

$$\lambda = \left(\frac{qH}{k}\right)^{1/(k-1)} \left(\frac{k}{k-1}\right). \quad (\text{A5})$$

Solving for  $q^{o^*}$  gives

$$q^{o^*} = \lambda^{k-1} \left(\frac{k-1}{k}\right)^{k-1} \frac{k}{H}. \quad (\text{A6})$$

The enforcer's equilibrium strategy,  $p_{br}(q) = \left(\frac{q^{o^*}H}{k}\right)^{\frac{1}{k-1}}$ , is therefore

$$p^{o^*} = \lambda \frac{k-1}{k}. \quad (\text{A7})$$

**Lemma A3 (equilibrium strategies in an offender-leadership game)**

(i) The unique SPE strategies in an offender-leadership game are

$q^{o^*} = \lambda^{k-1} \left(\frac{k-1}{k}\right)^{k-1} \frac{k}{H}$  and  $p^{n^*} = \lambda \frac{k-1}{k}$ . (ii) The enforcer's equilibrium payoff is  $u^{o^*} = -\lambda^k \left(\frac{k-1}{k}\right)^k (1-k) - k\lambda^{k-1} \left(\frac{k-1}{k}\right)^{k-1}$ .

Finally, comparing the enforcer's equilibrium payoff as a leader versus as a follower shows that  $u^{e*} > (\leq) u^{o*}$  if and only if  $\lambda < (\geq) \frac{k\left(\frac{k-1}{k}\right)^{k-1}}{1+(k-1)\left(\frac{k-1}{k}\right)^k}$ . In particular, for  $k = 2$ , the enforcer prefers to move first if and only if  $\lambda < \frac{4}{5}$ .

## Appendix B

This Appendix restates and proves Propositions 4 and 5.

### Proposition 4 (*effect of sanction and harm on equilibrium strategies*)

(1) *In an enforcer-leadership game: (i) As the harm increases, the level of enforcement increases and the level of non-compliance decreases; (ii) As the sanction increases, the level of enforcement may either increase or decrease, whereas the level of non-compliance decreases.*

(2) *In an offender-leadership game: (i) As the sanction increases, the level of non-compliance and the level of enforcement decrease; (ii) As the harm increases, the level of non-compliance decreases, whereas the level of enforcement may either increase or decrease.*<sup>38</sup>

**Proof.** Consider the following system:

$$\begin{cases} F_q(q(t), p(t), t) = 0 \\ G_p(q(t), p(t), t) = 0 \end{cases} \quad (\text{B1})$$

Totally differentiating each equation with respect to  $t$  gives:

$$\begin{cases} F_{qq} \frac{dq}{dt} + F_{qp} \frac{dp}{dt} + F_{qt} = 0 \\ G_{pq} \frac{dq}{dt} + G_{pp} \frac{dp}{dt} + G_{pt} = 0 \end{cases} \quad (\text{B2})$$

Solving for  $\frac{dq}{dt}$  and  $\frac{dp}{dt}$  gives

$$\frac{dq}{dt} = \frac{1}{\Delta} [F_{qp} G_{pt} - G_{pp} F_{qt}], \quad (\text{B3})$$

and

$$\frac{dp}{dt} = \frac{1}{\Delta} [G_{pq} F_{qt} - F_{qq} G_{pt}], \quad (\text{B4})$$

where  $\Delta = F_{qq} G_{pp} - F_{qp} G_{pq} > 0$ .

<sup>38</sup> Assuming that, from the offender's perspective, enforcement and non-compliance are strategic substitutes.

We will now apply these general results in the proof of Proposition 4.

(1) In an enforcer-leadership game, the enforcer's problem is

$$\begin{aligned} \max_p u(p, q) &= -c(p) - (1 - p\mu)qH \\ \text{s.t. } v_q &= (1 - p)G'(q) - pS = 0. \end{aligned} \quad (\text{B5})$$

The maximand is the enforcer's payoff (see (2)). The constraint ensures that the offender's level of non-compliance is a best response to the enforcer's level of enforcement.

The Lagrangian is

$$L(p, q, \lambda) = -c(p) - (1 - \mu p)qH - \lambda[(1 - p)G'(q) - pS]. \quad (\text{B6})$$

The enforcer's optimal level of enforcement and the offender's corresponding level of non-compliance are obtained by solving

$$\begin{cases} \frac{\partial L(p, q, \lambda)}{\partial p} = -c'(p) + q\mu H + \lambda(G'(q) + S) = 0 \\ \frac{\partial L(p, q, \lambda)}{\partial q} = -(1 - \mu p)H - \lambda(1 - p)G''(q) = 0 \\ \frac{\partial L(p, q, \lambda)}{\partial \lambda} = v_q = (1 - p)G'(q) - pS = 0. \end{cases} \quad (\text{B7})$$

From the second equality, we have  $\lambda = \frac{-(1-p)\mu H}{(1-p)G''(q)}$ . Plugging in the first equality, the level of enforcement and the level of non-compliance,  $p^{e^*}$  and  $q^{e^*}$ , are implicitly defined by the following system:

$$\begin{cases} L_p = -c'(p) + q\mu H - \frac{dq}{dp}(1 - p\mu)H = 0 \\ v_q = (1 - p)G'(q) - pS = 0, \end{cases} \quad (\text{B8})$$

where  $\frac{dq}{dp} = -\frac{v_{qp}}{v_{qq}} = \frac{(G'(q)+S)}{(1-p)G''(q)}$ .

The partial derivatives of  $L_p$  and  $v_q$  with respect to  $p$  and  $q$  are:

$$\begin{aligned} L_{pp} &= -c''(p) - \frac{(G'(q) + S)H}{G''(q)} \frac{1 - \mu}{(1 - p)^2} < 0; \\ L_{pq} &= \mu H - \frac{(1 - p\mu)H}{(1 - p)} \frac{[G''(q)]^2 - (G''(q) + S)G'''(q)}{[G''(q)]^2} > 0 \text{ (by assumption);} \\ v_{qq} &= (1 - p)G''(q) < 0; \\ v_{qp} &= -(G'(q) + S) < 0; \\ \Delta &= L_{pp}v_{qq} - L_{pq}v_{qp} > 0. \end{aligned} \quad (\text{B9})$$

The partial derivatives of  $L_p$  and  $v_q$  with respect to  $S$  and  $H$  are:

$$\begin{aligned} L_{ps} &= -\frac{(1-p\mu)H}{(1-p)G''(q)} > 0; \\ L_{ph} &= q\mu - \frac{(G'(q) + S)}{(1-p)G''(q)}(1-p\mu) > 0; \\ v_{qs} &= -p < 0; \\ v_{qh} &= 0. \end{aligned} \tag{B10}$$

We thus have

$$\frac{dp^{e^*}}{dS} = \frac{1}{\Delta} \left( \overbrace{L_{pq}}^+ \overbrace{v_{qs}}^- - \overbrace{v_{qq}}^- \overbrace{L_{ps}}^+ \right) <> 0; \tag{B11}$$

$$\frac{dq^{e^*}}{dS} = \frac{1}{\Delta} \left( \overbrace{v_{qp}}^- \overbrace{L_{ps}}^+ - \overbrace{L_{pp}}^- \overbrace{v_{qs}}^- \right) < 0; \tag{B12}$$

$$\frac{dp^{e^*}}{dH} = \frac{1}{\Delta} \left( \overbrace{L_{pq}}^- \overbrace{v_{qh}}^0 - \overbrace{v_{qq}}^- \overbrace{L_{ph}}^+ \right) > 0; \tag{B13}$$

$$\frac{dq^{e^*}}{dH} = \frac{1}{\Delta} \left( \overbrace{v_{qp}}^- \overbrace{L_{ph}}^+ - \overbrace{L_{pp}}^- \overbrace{v_{qh}}^0 \right) < 0. \tag{B14}$$

An increase in the sanction thus either increases or decreases the enforcer's level of enforcement, but decreases the offender's level of non compliance. An increase in the harm increases the enforcer's level of enforcement, but decreases the offender's level of non-compliance. ■

(2) In an offender-leadership game, the level of enforcement and the level of non-compliance,  $p^{o^*}$  and  $q^{o^*}$ , are implicitly defined by the following system:

$$\begin{cases} L_q = (1-p)G'(q) - pS - \frac{dp}{dq}(G(q) + qS) = 0 \\ u_p = -c'(p) + q\mu H = 0, \end{cases} \tag{B15}$$

where  $L(p, q, \lambda) = (1-p)G'(q) - pqS - \lambda[-c'(p) + q\mu H]$  and  $\frac{dp}{dq} = -\frac{u_{pq}}{u_{pp}} = \frac{\mu H}{c''(p)}$ .

The partial derivatives of  $L_q$  and  $u_p$  with respect to  $q$  and  $p$  are:

$$\begin{aligned}
 L_{qq} &= (1-p)G''(q) - \frac{\mu H}{c''(p)}(G'(q) + S) < 0; \\
 L_{qp} &= -(G'(q) + S) + \frac{\mu H(G(q) + qS)}{c''(p)^2}c'''(p) < 0 \text{ (by assumption);} \\
 u_{pp} &= -c''(p) < 0; \\
 u_{pq} &= \mu H > 0; \\
 \Delta &= L_{qq}u_{pp} - L_{qp}u_{pq} > 0.
 \end{aligned}
 \tag{B16}$$

The partial derivatives of  $L_q$  and  $u_p$  with respect to  $S$  and  $H$  are:

$$\begin{aligned}
 L_{qs} &= -p - \frac{q\mu H}{c''(p)} < 0; \\
 L_{qh} &= -\frac{\mu(G(q) + qS)}{c''(p)} < 0; \\
 u_{ps} &= 0; \\
 u_{ph} &= q\mu > 0.
 \end{aligned}
 \tag{B17}$$

We thus have

$$\frac{dp^{o*}}{dS} = \frac{1}{\Delta} \left( \overbrace{u_{pq}}^+ \overbrace{L_{qs}}^- - \overbrace{L_{qq}}^- \overbrace{u_{ps}}^0 \right) < 0;
 \tag{B18}$$

$$\frac{dq^{o*}}{dS} = \frac{1}{\Delta} \left( \overbrace{L_{qp}}^- \overbrace{u_{ps}}^0 - \overbrace{u_{pp}}^- \overbrace{L_{qs}}^- \right) < 0;
 \tag{B19}$$

$$\frac{dp^{o*}}{dH} = \frac{1}{\Delta} \left( \overbrace{u_{pq}}^+ \overbrace{L_{qh}}^- - \overbrace{L_{qq}}^- \overbrace{u_{ph}}^+ \right) < > 0;
 \tag{B20}$$

$$\frac{dq^{o*}}{dH} = \frac{1}{\Delta} \left( \overbrace{L_{qp}}^- \overbrace{u_{ph}}^+ - \overbrace{u_{pp}}^- \overbrace{L_{ph}}^+ \right) < 0.
 \tag{B21}$$

An increase in the sanction thus decreases the enforcer's level of enforcement and the offender's level of non compliance. An increase in the harm either increases or decreases the enforcer's level of enforcement, but decreases the

offender's level of non-compliance. ■

**Proposition 5 (effect of sanction and harm on equilibrium payoffs)**

(1) As the sanction increases, the enforcer's equilibrium payoff increases and the offender's equilibrium payoff decreases.

(2) As the harm increases, the offender's equilibrium payoff decreases, the enforcer's equilibrium payoff as a leader decreases, and her equilibrium payoff as a follower either increases or decreases.

**Proof.** We will prove the parts of Proposition 5 which pertain to the enforcer. A similar proof applies to the parts which pertain to the offender.

Consider an enforcer-leadership game. Recall from (B6) that the Lagrangian is

$$L(p, q, \lambda) = -c(p) - (1 - \mu p)qH - \lambda[(1 - p)G'(q) - pS].$$

By the Envelope Theorem for constrained optimization, the effect of an increase in  $S$  and  $H$  on the enforcer's equilibrium payoff is given by the partial derivatives of the Lagrangian with respect to  $S$  and  $H$ :

$$\begin{aligned} \frac{dL(p, q, \lambda)}{dS} \Big|_{p=p^{e*}, q=q^{e*}} &= \frac{\partial L(p, q, \lambda)}{\partial S} \Big|_{p=p^{e*}, q=q^{e*}} \\ &= \lambda p^{e*} > 0, \end{aligned} \tag{B22}$$

and

$$\begin{aligned} \frac{dL(p, q, \lambda)}{dH} \Big|_{p=p^{e*}, q=q^{e*}} &= \frac{\partial L(p, q, \lambda)}{\partial H} \Big|_{p=p^{e*}, q=q^{e*}} \\ &= -(1 - \mu p^{e*})q^{e*} > 0. \end{aligned} \tag{B23}$$

An increase in the sanction thus increases the enforcer's equilibrium payoff as a leader and an increase in the harm decreases it.

Consider next an offender-leadership game. To find the effect of an increase in the sanction or the harm on the enforcer's equilibrium payoff as a follower, observe that the enforcer's problem in an offender-leadership game is

$$\begin{aligned} \max_p u(p, q) &= -c(p) - (1 - \mu p)qH \\ \text{s.t. } v_q &= (1 - p)G'(q) - pS - \frac{dp}{dq}((G(q) + qS)) = 0, \end{aligned} \tag{B24}$$

where  $\frac{dp}{dq} = -\frac{u_{pq}}{u_{pp}} = \frac{\mu H}{c''(p)}$ . That is, the enforcer's problem is to choose an optimal level of enforcement as a follower subject to the offender choosing an optimal level of non-compliance as a leader.

The Lagrangian is

$$L(p, q, \lambda) = -c(p) - (1 - p\mu)qH - \lambda \left( (1 - p)G'(q) - pS - \frac{\mu H}{c''(p)}(G(q) + qS) \right). \quad (\text{B25})$$

By the Envelope Theorem, the effect of an increase in  $S$  and  $H$  on the enforcer's equilibrium payoff is given by the partial derivative of the Lagrangian with respect to  $S$  and  $H$ :

$$\begin{aligned} \frac{dL(p, q, \lambda)}{dS} \Big|_{p=p^{o*}, q=q^{o*}} &= \frac{\partial L(p, q, \lambda)}{\partial S} \Big|_{p=p^{o*}, q=q^{o*}} \\ &= \lambda p^{o*} > 0, \end{aligned} \quad (\text{B26})$$

and

$$\begin{aligned} \frac{dL(p, q, \lambda)}{dH} \Big|_{p=p^{o*}, q=q^{o*}} &= \frac{\partial L(p, q, \lambda)}{\partial H} \Big|_{p=p^{o*}, q=q^{o*}} \\ &= -(1 - \mu p^{o*})q^{o*} + \frac{\mu \lambda}{c''(p^{o*})}(G(q^{o*}) + q^{o*}S) \\ &< > 0. \end{aligned} \quad (\text{B27})$$

An increase in the sanction thus increases the enforcer's equilibrium payoff, but an increase in the harm either increases or decreases it. ■

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