# Litigation and Selection with Correlated Two-Sided Incomplete Information

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This article explores the selection of disputes for litigation in a setting with two-sided incomplete information and correlated signals. The models analyzed here suggest that Priest and Klein's conclusion that close cases are more likely to go to trial than extreme cases remains largely valid when their model is interpreted as involving correlated, two-sided incomplete information and is updated (i) to incorporate take-it-or-leave-it offers or the Chatterjee–Samuelson mechanism, (ii) to take into account the credibility of the plaintiff's threat to go to trial, and (iii) to allow parties to make sophisticated, Bayesian inferences based on knowledge of the distribution of disputes. On the other hand, Priest and Klein's prediction that the plaintiff will win 50% of litigated cases is sensitive to bargaining and parameter assumptions. (*JEL*: K40, K41)

## 1. Introduction

This article examines the selection of disputes for litigation in a setting where the litigants have correlated two-sided incomplete information regarding the merit of their dispute. Although legal scholars have been

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studying litigation, settlement, and selection for over three decades, settings involving two-sided incomplete information and correlated signals have not been analyzed rigorously.

The vast majority of the existing litigation and settlement models involve one-sided asymmetric information. In these models, one party—which may be either the plaintiff or the defendant—is fully informed about the probability that the plaintiff will prevail if the case were to go to trial, and the other party knows only the distribution of suits. Ordinarily, one party makes a take-it-or-leave-it settlement offer. Depending on the model, the party making the offer can be either the uninformed party (screening) or the informed party (signaling). P'ng (1983), Bebchuk (1984), Reinganum and Wilde (1986), Nalebuff (1987), Reinganum (1988), Shavell (1996), and Spier (1992). Under these models, cases that favor the informed party are more likely to be litigated. So, for example, if the defendant has private information about liability and hence the likelihood that he will lose at trial, cases the defendant is more likely to win are more likely to be litigated.

The informational disparity assumed in one-sided asymmetric information models is extreme and unrealistic. In most suits, both parties will possess some information relevant to the likelihood that the plaintiff will win at trial. For example, in a suit for discrimination by a terminated employee, the plaintiff might have better information about how she was treated by her supervisor, while the employer might have better information about her performance relative to other workers. Both treatment and performance will be relevant to the employer's liability.

A small number of models have analyzed two-sided incomplete information. Most of them assume that the plaintiff's and defendant's information are independent from one another.[1] See Schweizer (1989),

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1. In most of the literature, the signals are independent *unconditionally*, but correlated *conditional* on the underlying state. For example, in Friedman and Wittman (2007), the signals are independent draws from the same distribution, but the "true" state is the average of the signals, so the signals are correlated conditional on the underlying true

Daughety and Reinganum (1994), and Friedman and Wittman (2007). But see Yousefi and Black (2016). In reality, however, there is likely to be a high correlation between the parties' information. In any dispute, the plaintiff's and the defendant's information will frequently overlap to a considerable degree. While each side may have better information about some things, the other side usually has some relevant knowledge of those topics. In the employment discrimination example, the plaintiff likely knows something about her performance relative to other workers, and the defendant can interview the supervisor and thereby learn about how the plaintiff was treated. While there may still be much that one party knows that the other does not, their overall assessments of case merit are likely to be highly correlated. As discussed in the next section, the selection implications of two-sided incomplete information models have not been rigorously explored.

An early model that can be interpreted as featuring two-sided correlated signals is Priest and Klein (1984), which famously hypothesizes that the plaintiff trial win rate will have a tendency to approach 50%.[2] In their model, case merit is represented as a real number, and the plaintiff prevails at trial if and only if this value is above some threshold. Each litigant, however, observes case merit with a normally distributed error and must make a prediction about the true case merit based on her signal. Although Priest and Klein's model provides an information structure that has intuitive appeal, the model has been criticized for lacking rigorous treatment of information and bargaining. Indeed, the model is silent about how the parties bargain and thus about whether and how each party learns about the other's signal. Furthermore, the Priest–Klein model assumes that the plaintiff always has a credible threat to take a case to trial, even when its probability of prevailing is extremely low.

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state. Conversely, in this paper, the signals are the underlying state plus an error term which is independent for each party. Thus, the signals are correlated unconditionally, but independent conditional on the underlying state. When the signals are correlated unconditionally, each party can make inferences about the other's signal based on its own signal. That is not possible when the signals are independent unconditionally.

2. Whether Priest and Klein (1984) is best interpreted as involving a common prior or inconsistent priors (divergent expectations) is discussed extensively in Section 2 of Lee and Klerman (2016). In this article, we adopt the common prior approach, but, as pointed out in Lee and Klerman (2016), results are very similar when one adopts the inconsistent prior interpretation.

This article attempts to fill the gap in the literature and explores the selection implications of a rigorous model with correlated two-sided incomplete information.[3] Our strategy is to begin with the correlated two-sided information structure from Lee and Klerman (2016) and then model bargaining with (i) take-it-or-leave-it offers or (ii) the Chatterjee-Samuelson mechanism. In addition, we improve Priest and Klein's model in two ways: (i) we consider the credibility of the plaintiff's threat to go to trial and (ii) we allow litigants to be sophisticated Bayesians who make inferences by taking the underlying distribution of disputes into account. See Lee and Klerman (2016).

A key result in most equilibria we identify and analyze is that extreme cases—those where the defendant's conduct is much better or worse than legally required—are more likely to settle and close cases are more likely to go to trial.[4] This result contrasts with the findings under one-sided asymmetric information models, where extreme cases that favor the informed party are more likely to litigate. Part of the reason for observing this difference is that our model uses fact or case space—rather than probability space[5]—and that the parties' signals are correlated. In Bebchuk (1984) and Reinganum and Wilde (1986) case type is the probability that the plaintiff will prevail.

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3. Yousefi and Black (2016) set out a simple model with correlated, two-sided information, but their model lacks features that modern models of litigation are expected to have. Most notably, the party receiving an offer does not update its beliefs based on the offer, even though the offer conveys information about the offeror's beliefs. See p. 186, n. 8. Wittman (1985) considers a similar model, but his model, like Priest and Klein (1984) and Lee and Klerman (2016) does not assume a particular bargaining protocol, but instead assumes parties settle whenever the plaintiff's expected gain from trial exceeds defendant's expected loss.

4. The exception is asymmetric Nash equilibria under the Chatterjee–Samuelson mechanism. See Section 4.2.

5. In fact space, disputes are distributed over the entire real line, and case merit—the factual strength of each dispute for the plaintiff—is represented by a real number between negative infinity and positive infinity. In the Priest–Klein model and the updated version used in this article, the plaintiff wins with certainty or probability $\theta_H$ if the case merit is greater than or equal to some threshold value and loses with certainty or probability $1 - \theta_L$ otherwise. Although the parties' estimates of the plaintiff's probability of prevailing vary continuously with case strength, the objective probability of prevailing takes only two values. In probability space, case merit is represented by a real number between 0 and 1 (inclusively), where the number represents the probability that the plaintiff will prevail at trial. Because of the binary nature of fact space translation of the model into probability space would require drastic changes. For an attempt to analyze the Priest–Klein model in probability space, see Hylton and Lin (2012). If the probability of plaintiff win at trial varied continuously over fact space, translation between the two spaces might be possible.

In contrast, in this article, as in Priest and Klein (1984), case type is facts that indicate how close the defendant's conduct was to the legal standard. This approach is consistent also with Kornhauser (2008), in which a legal rule maps a vector of characteristics into liability or no liability. An advantage of this approach is that it makes possible unbiased errors of constant magnitude. When cases are represented in probability space, it is not possible to assume unbiased and positive errors at or near zero and one, because then parties' estimates of the probability that the plaintiff would prevail might fall below zero or exceed one, which is impossible.[6] In contrast, a case in fact space can take on any real value, so errors can be positive and unbiased.

A simple example illustrates why selection is toward close cases in a case-space model with correlated information. In a traffic accident case, case type might be how fast the defendant was driving, and the legal standard might be the speed limit. If the speed limit was 30 miles per hour, the driver would be liable if her speed exceeded that amount. The plaintiff and defendant each receive correlated signals of how fast the defendant was going. The defendant's signal might reflect her own recollection of the speed on her speedometer the last time she looked before the crash as well as other information gathered from witnesses and physical evidence. The plaintiff's signal might reflect his visual assessment of the defendant's speed as well as witness and physical evidence. Because all the evidence reflects the common underlying reality (the true speed of the car), and because much evidence is common, the signals are correlated. Of course, neither signal is likely to be completely accurate, so both signals reflect the true speed plus some error, which might usually be within 5 miles per hour of the true speed. In this scenario, it is easy to see that extreme cases—cases where the defendant's speed was well above or below the 30-mile-per-hour speed limit—will rarely litigate. Suppose, for example, that the defendant's true speed was 50 miles per hour, an extreme case where defendant's conduct was far from the legal standard. Even with errors in their favor, the parties are likely to agree that the defendant is almost certain to be held liable. Even if the defendant's signal is 5 miles per hour too low, the defendant estimates her speed to be 45 miles per hour, so the defendant still thinks that

6. Others have tried to work around this problem by assuming heteroscedastic errors across probability space. See, for example, Hylton and Lin (2012).

she violated the speed limit by 15 miles per hour and thus estimates that she will be held liable with very high probability. Even if the plaintiff's signal is 5 miles per hour too high, the error will not result in a significantly higher estimate of liability, because the defendant has significantly exceeded the speed limit whether she was going 45 miles per hour or 55 miles per hour. Thus, the plaintiff and the defendant are likely to agree that plaintiff will prevail with high probability and thus be able to negotiate a settlement. Similarly, if both parties receive signals that suggest that the defendant was traveling well below the speed limit, they will agree that liability is unlikely, and will negotiate a low settlement. Only when the defendant's speed is close to the speed limit (30 miles per hour) is significant disagreement and thus failure to settle likely. For example, if defendant's true speed is 30 miles per hour, the plaintiff might receive a signal suggesting that defendant's speed was 35 miles per hour, while defendant might receive a signal suggesting that she was traveling 25 miles per hour. In that situation, the plaintiff would think his case is strong, while the defendant would think she was likely to be exonerated at trial. In such a close case, settlement negotiations would likely fail, and the case would go to trial. Thus, litigation is only likely in close cases (where actual speed is close to the speed limit), and settlement is more likely in extreme cases (where speed is much higher or lower than the speed limit).

Although our conclusion that close cases will tend to go to trial and extreme cases will tend to settle is similar to Priest and Klein's original conclusion, the results of the models explored here also differ from theirs in important ways. In the take-it-or-leave it model, even though only close cases are likely to go to trial, we find that the plaintiff trial win rate can deviate significantly from 50%. This is because the party making the offer is able to use its bargaining power to settle favorably nearly all cases it is likely to lose and litigate only cases it is more likely to win. For example, if the plaintiff makes the take-it-or-leave-it offer, nearly all cases the plaintiff is likely to lose settle, as do cases that the plaintiff is very likely to win. But an intermediate range of cases—where the plaintiff is likely, but not highly likely to prevail—will go to trial. As a result, the plaintiff trial win rate is over 50%. More generally, the trial win rate will favor the party making the offer. When the defendant is making the offer, the plaintiff trial win rate will be less than 50%.

One take-away of our results is that one should be cautious about making inferences about the strength of the cases selected to go to trial based on observed win rates. Even if one side wins most of the cases, this does not mean that the evidence or law in litigated cases strongly favors that side. Instead, it could mean that close cases litigate, but that the evidence or law in those cases usually slightly favors one side.

In the model using the Chatterjee–Samuelson mechanism, we establish the existence of a number of different classes of Nash equilibria. The presence of multiple equilibria with contrasting selection implications makes predictions about plaintiff trial win rates difficult. Nonetheless, if we assume the plaintiff and the defendant face identical litigation costs and employ symmetric strategies—in other words, if the plaintiff and the defendant are assumed to be equally aggressive in their settlement demands when they are equally confident that the outcome will favor their sides, respectively—we find that close cases are more likely to go to trial, and, under some parameter assumptions, that the plaintiff trial win rate will be 50%. In contrast, in asymmetric equilibria, where one party is more aggressive in its settlement demands, plaintiff trial win rates can deviate substantially from 50%.

The rest of this paper is organized as follows. Part 2 provides an overview of the relevant literature on litigation and settlement as it bears on the issue of selection. Part 3 presents a model with take-it-or-leave-it offers, and Part 4 explores the implications of using the Chatterjee–Samuelson mechanism. Part 5 concludes. An Appendix includes the proofs of all propositions as well as the technical expositions of the models, and an Online Appendix includes proofs of lemmas necessary to establish Proposition 1.

## 2. Related Literature

Legal scholars have been studying the selection implications of litigation for over three decades. Priest and Klein (1984) argued that the disputes selected for trial will not be a random set, but will tend to be close cases. Relying on a graphical argument and simulations, they also hypothesized that there will be a "tendency toward 50 percent plaintiff victories" among litigated cases (p. 20). Lee and Klerman (2016) formalize Priest and Klein's model and prove that the 50% prediction and a number of other hypotheses derived from Priest and Klein (1984) are mathematically valid given the model's set-up. But see Klerman and Lee (2014).

Priest and Klein's prediction that close cases will tend to go to trial contrasts with the selection implications of the canonical one-sided screening and signaling models. See Waldfogel (1998). In one-sided asymmetric information models, cases the informed party is more likely to win are more likely to be litigated. See Wickelgren (2013) and Klerman and Lee (2014). Under Bebchuk's screening model, the uninformed party makes a take-it-or-leave-it offer, and there is a cutoff that divides cases according to the informed party's type, which is the probability that the plaintiff will prevail at trial. All cases on one side of the cutoff settle, and all cases on the other side litigate. In Bebchuk's original model, the defendant is the informed party, and thus, only the cases in which the plaintiff's probability of prevailing is less than a certain threshold go to trial. Bebchuk's model can be modified for a setting in which the plaintiff is the informed party, and in that case, all cases in which the plaintiff's probability of prevailing is greater than a certain threshold go to trial. See Klerman and Lee (2014). Thus, under the screening model, it is not close cases, but rather cases that favor the informed party—including extreme cases—that go to trial.

Under Reinganum and Wilde's (1986) signaling model, there is no litigation/settlement cutoff, but the probability that a case will be litigated varies continuously and monotonically with the strength of the plaintiff's case. If the plaintiff is the informed party, cases with higher expected value are more likely to be litigated. Conversely, if the defendant is the informed party, cases with lower expected value are more likely to be litigated. In Reinganum and Wilde's model, cases vary in strength based on damages. Klerman and Lee (2014) modify this model so that cases vary in the likelihood that plaintiff will prevail. Under this modified model, the implication is not that close cases are litigated, but rather that the closer the case is to one extreme—the extreme in which the informed party prevails 100% of the time—the more likely the case will go to trial.

The selection implications of two-sided incomplete information have not been systematically explored. In Schweizer (1989), the parties receive independent, binary signals about case merit (good news or bad news), and the defendant makes a take-it-or-leave-it offer. Schweizer (1989) does not discuss the selection implications of his model, and even under the separating equilibrium it is not clear whether close cases or cases favoring the defendant will be litigated more often. Daughety and Reinganum (1994) analyze a model in which the plaintiff has private information about damages, while the defendant has private information about liability, where damages and liability are uncorrelated, and either party makes a take-it-or-leave-it offer. Although their original article did not explore the selection implications of the model, it can be shown that, in both the plaintiff-offer model and the defendant-offer model, the more likely the defendant is to be found liable, the more likely the case is to be settled. So, like Reinganum and Wilde (1986)'s signaling model, cases that are extreme—in that the plaintiff has a very low probability of prevailing—are more likely to be litigated. On the other hand, the plaintiff is more likely to settle cases where damages are likely to be low, and thus, the cases selected for trial will tend to have higher damages but lower probability of defendant liability.[7]

In Friedman and Wittman (2007), the litigants receive continuous but independent signals of case strength and then employ the Chatterjee–Samuelson bargaining mechanism to negotiate. Although case strength refers to judgment amount rather than the probability that the defendant will be found liable—the defendant's liability is assumed in the model—their model can be modified to analyze a situation involving uncertainty over defendant liability (Friedman and Wittman, 2007, pp. 109–110). They restrict their attention to symmetric Nash equilibria and find that the selection implications of their model are similar to Priest and Klein. Converted to a model based on defendant liability, Friedman and Wittman's model

7. In their 2012 survey article, Daughety and Reinganum noted that, under their 1994 model, weak plaintiffs and weak defendants "will be more likely to settle, trimming the distribution from both ends—again resembling the primary characteristics of the Priest–Klein approach." (2012, p. 440). While it is true that both high probability of liability cases and low damage cases are more likely to be settled, this does not mean that cases that are middling in terms of expected value are more likely to be litigated. In the model where the plaintiff makes the offers, the lowest expected value cases (those where both the plaintiff's probability of prevailing and damages are low) are litigated and those with the highest expected value (where both plaintiff's probability of prevailing and damages are high) are settled. So, cases that litigate include all the extremely low expected liability cases. Conversely, when defendant makes the offer, the lowest expected value cases all settle, and cases with the highest expected liability litigate, again meaning the extreme cases (now extremely high expected value cases) are more likely to litigate.

suggests that plaintiffs will prevail 50% of the time in litigated cases. Their model's findings, however, depart from Priest and Klein's prediction because they find that close cases are more likely to be litigated only when litigation costs are assumed to be high. When litigation costs are low, they find that extreme cases are more likely to be litigated. Nevertheless, even when litigation costs are low, plaintiffs still prevail 50% of the time, because cases in which the plaintiff is very likely to prevail exactly offset cases in which the plaintiff is very unlikely to prevail.

We believe combining the correlated information structure from Priest and Klein's original model with the more modern take-it-or-leave-it or Chatterjee–Samuelson bargaining protocols makes for a worthwhile inquiry for several reasons. Waldfogel (1998) documents that data are more consistent with Priest and Klein's model than with the screening or signaling models, so it is worthwhile to see whether Priest and Klein's results hold up when their model is made more rigorous. In addition, our models show how the main results of Priest and Klein (1984) will change if we were to take into account the possibility of negotiation failure and *ex post* inefficiency. See Myerson and Satterthwaite (1983).

All in all, the models explored in this article share some similar results with many of the models discussed above. The result, under most versions of our model, that close cases are more likely to go to trial is consistent with Priest and Klein's model. Likewise, as with Bebchuk (1984) and Reinganum and Wilde (1986), when one party makes a take-it-or-leave-it offer, litigated cases favor one side, although, in their models, litigated cases favor the informed party, whereas in our model litigated cases favor the party making the offer. The result in the Chatterjee–Samuelson version of our model that the trial win rate may be 50% and neither side enjoys a first-mover advantage is consistent with Friedman and Wittman (2007) and Priest and Klein (1984).

## 3. The Model with Take-It-or-Leave-It Offers

We begin with the model in Lee and Klerman (2016), which formalizes Priest and Klein (1984). Without loss of generality, we normalize the judgment to 1. That is, if the plaintiff prevails, defendant pays the plaintiff 1. If the case goes to trial, the plaintiff and the defendant incur litigation costs,

$C_p > 0$ and $C_d > 0$, respectively. Settlement is assumed to be a costless transfer of the settlement amount from the defendant to the plaintiff.[8] Because the parties are assumed to agree on damages, this model is most applicable to disputes where the parties disagree primarily about liability. For a model with disagreement about damages, see Helland et al. (2018).

The merit of a dispute is represented by a random variable, $Y$, which takes on a real number. The real number can be interpreted as factual information pertaining to the defendant's liability, such as the speed of defendant's car in an accident case. The court observes $y$, the realization of $Y$, without error. Under the original Priest–Klein model, the plaintiff wins if $y > 0$ and the defendant wins if $y \leqslant 0$. Thus, $y = 0$ is the threshold factual disposition for finding the defendant liable. The threshold could be $y = a$, where $a$ is any real number (such as the speed limit), but, without loss of generality, we normalize to $y = 0$.

We generalize this set-up slightly by assuming that the plaintiff wins with probability $\theta_H$ if $y > 0$ and with probability $\theta_L$ if $y \leqslant 0$, where $0 \leqslant \theta_L < \theta_H \leqslant 1$.[9] Therefore, we will say the case is of high merit when $y > 0$ and of low merit when $y \leqslant 0$. Note that if $(\theta_L, \theta_H) = (0, 1)$, then the set-up is identical to the original Priest–Klein model. The reason we consider $\theta_L > 0$ is that Priest and Klein's original specification raises a concern that the plaintiff who is reasonably certain that $y \leqslant 0$ will lack a credible threat to take the case to trial. See Lee and Klerman (2016). One possible

8. One can equally assume settlement to entail costs as well, $0 < S_p < C_p$ and $0 < S_d < C_d$, and we would obtain essentially the same results.

9. We note that assuming $\theta_L > 0$ and $\theta_H < 1$ presents an interpretation of the dispute space that is different from Priest and Klein's original model. Priest and Klein's original set-up (under which $\theta_L = 0$ and $\theta_H = 1$) represents a fact space, in which the fact patterns of disputes can range from those in which the defendant's conduct is far from problematic ($y \ll 0$) to those in which the defendant's conduct is egregiously bad ($y \gg 0$). The threshold $y = 0$ in that case would represent the line the court would draw in finding the defendant liable. This interpretation would need to be modified when we assume $\theta_L > 0$ and $\theta_H < 1$. One interpretation is that $\theta_L > 0$ and $\theta_H < 1$ reflect court error. Even when the defendant's culpability is insufficient for liability, courts find the defendant liable with probability $\theta_L$. Even when the defendant is more culpable than required for liability, courts sometimes exonerate the defendant, so the probability of liability is $\theta_H < 1$. Under this interpretation, Priest and Klein's model can be seen as assuming courts make no errors, and this paper can be seen as incorporating the possibility of court error. Another interpretation is that there are really only two types of cases—low-merit cases and high-merit cases—and $y_p$ and $y_d$ represent the extent to which the parties can be sure that the dispute is of one type or the other.

approach to address this concern is to assume $\theta_L \geqslant C_p$.[10] Such an approach of assuming that all plaintiffs have a sufficiently high probability of prevailing at trial is similar to the approaches taken by a number of other models to avoid the problem of non-credible threats. See Bebchuk (1984), Reinganum and Wilde (1986), and Daughety and Reinganum (1994). But see Nalebuff (1987) and Baker and Mezzetti (2001). Hubbard (2016) points out that there is a similar problem with the credibility of the defendant's threat to defend. If $\theta_H > 1 - C_d$, then the defendant would be better off just paying the full damage without going to trial. Thus, we will also consider the case where $\theta_H \leqslant 1 - C_d$.

We assume that $Y$ is distributed according to $g_Y(x)$, which is bounded above, continuous, and strictly positive. In Section 3.3 we discuss the general case, but we start in Sections 3.1 and 3.2 by assuming $g_Y(x)$ is an improper uniform distribution over the entire real line. In other words, $g_Y(x) = 1$ for all $x$. While this violates the condition that a probability density function must integrate to 1, its mathematical properties have been worked out by Hartigan (1983) and DeGroot (2004), and such a distribution has previously been used in the global games literature as well as in the context of Priest and Klein's model. See Morris and Shin (2003) and Lee and Klerman (2016). There are two reasons why it will be useful to work with an improper uniform distribution rather than a normal distribution. First, this simple distribution allows us to construct an explicit first-order differential equation that will provide insight as to how the litigants bargain and respond. Second, when we discuss the case of general distributions in Section 3.3, we find that the plaintiff trial win rate in the limit—as parties become more accurate in discerning case merit—approaches the win rate that would obtain if the distribution were improper uniform. For this reason, we believe our results in Section 3.3 provide strong reasons to take seriously the case of the improper uniform distribution.

10. Although this approach to non-credible threats is somewhat arbitrary, it is closely related to the approaches taken by most other studies in the literature. The more elegant approach in Nalebuff (1987) would not work here, because, in our model, both parties receive informative signals. Thus, even with semi-pooling, a plaintiff with a very weak signal would not have a credible threat to take the case to trial if $\theta_L < C_p$. While other approaches could be devised to address this problem, we believe they would further complicate the model and would distract from the key point of the analysis here, which is selection, not nuisance suits.

The litigants, who are risk-neutral, do not observe $y$. Instead, the plaintiff receives a private signal $Y_p = Y + \epsilon_p$, and the defendant receives $Y_d = Y + \epsilon_d$, where $\epsilon_p, \epsilon_d \sim N(0, \sigma^2)$. Thus, for $i = p, d$, $f_{Y_i|Y=y}(x) = \varphi_\sigma(y - x)$, where $\varphi_\sigma(\cdot)$ is the normal distribution with mean zero and standard deviation $\sigma$.[11] We assume that $\epsilon_p$, $\epsilon_d$, and $Y$ are independently distributed. Although the errors are independent of each other, the observed signals will be correlated, because they both depend on $Y$. We shall refer to a plaintiff that observes $y_p$ (as the realized value of $Y_p$) as type $y_p$, and a defendant that observes $y_d$ as type $y_d$.

Given signal $y_i$, each litigant can construct the distribution of $Y$ conditional on $Y_i = y_i$ using Bayes' rule. Since the distribution of disputes is uniform over $\boldsymbol{R}$, it is easy to show that $f_{Y|Y_i=y_i}(x) = \varphi_\sigma(y_i - x)$ for $i =$ p, d. See DeGroot (2004, p. 191). Note that this implies that each party will estimate the probability that the case has high merit ($y > 0$) as greater than zero but less than one. No party has absolute confidence in the outcome.

The plaintiff can also estimate the expected distribution of $Y_d$ conditional on his observed $y_p$, $f_{Y_d|Y_p=y_p}(x)$. This is a compound distribution: the plaintiff first estimates the distribution of $Y$ given his $y_p$, and for each $y$ value estimates the distribution of $Y_d$ and compounds the distributions. Therefore, we have

$$f_{Y_d|Y_p=y_p}(x) = \int_{-\infty}^{\infty} f_{Y_d|Y=y}(x) f_{Y=y|Y_p=y_p}(y)\, \mathrm{d}y.$$

Similarly, the defendant can estimate the distribution of true case merit and of the plaintiff's signal.

## 3.1. The Plaintiff-Offer Model under the Improper Uniform Distribution

We begin with the model in which the plaintiff makes a take-it-or-leave-it offer to the defendant (the $P$-model). The implications of having the defendant make a take-it-or-leave-it offer (the $D$-model) are explored in Section 3.2. In the $P$-model, the plaintiff's pure strategy, $s(\cdot)$, is a function that maps her signal $y_p$ to a settlement demand, $S \in \boldsymbol{R}$. The

11. Because the context will make it clear, we will not include any subscript in $f_{Y_p|Y=y}(x)$ to specify the standard deviation, $\sigma$.

basic idea, as in Daughety and Reinganum (1994), is that the plaintiff's demand signals information about its type. The defendant in turn accepts or rejects the demand based on its own signal, $y_d$, and information about the plaintiff's type revealed by the plaintiff's demand. Thus, the plaintiff signals, and the defendant gets screened. The derivation of the perfect Bayesian equilibria is quite complicated and is set out in detail in the Appendix. In the main text, we provide only the intuitions and results.

Note first that a defendant, regardless of his type, should always accept a settlement demand of $S = C_d + \theta_L$. Because the defendant will always place *some* positive probability that the case will be of high merit, he should expect to pay at least that much if the case were to go to trial. Thus, for the defendant, accepting the settlement demand of $S = C_d + \theta_L$ will always outperform going to trial in expectation. This indicates two things. First, from the plaintiff's perspective, any demand $S < C_d + \theta_L$ will be strictly dominated by $S = C_d + \theta_L$. Second, in a fully separating equilibrium that is everywhere differentiable, we should not observe $S = C_d + \theta_L$ since the plaintiff's settlement demand must change (strictly) monotonically with her type. Because plaintiff types span the real line, there is no minimum plaintiff type that could offer $S = C_d + \theta_L$. By a similar argument, we can show that no defendant will accept a settlement demand greater than $C_d + \theta_H$—the highest possible cost the defendant would face—but will prefer to go to trial. We show in the Appendix that as a result no plaintiff has an incentive to demand greater than or equal to this amount (see Lemma A1).

As shown in the Appendix, there is neither a complete pooling equilibrium nor a fully separating one. Instead, her offer will be semi-pooling in the following sense. Each plaintiff that receives a sufficiently weak signal (i.e., below some threshold $y_p = y^0$) will demand $S = \theta_L + C_d$, which will be accepted by *every* defendant type. Since the plaintiff is assumed to have the bargaining power to make a take-it-or-leave-it offer, she can always extract at least this much in the $P$-model. In fact, the main reason why a fully separating equilibrium does not exist in the $P$-model is that $\theta_L + C_d$ (the corner solution) turns out to be more lucrative to a weak plaintiff than the interior solution, which would fully reveal her type to the defendant. When the plaintiff receives a signal above $y^0$, by contrast, her settlement demand increases monotonically with her signal, and as a result, the plaintiff's type

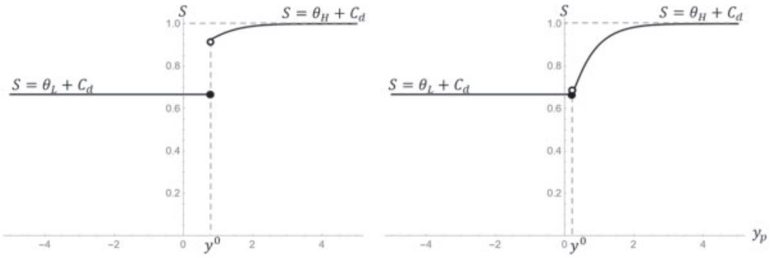


**Figure 1.** The Plaintiff's Optimal Settlement Demand Given the Plaintiff's Signal, $y_p$ ($\sigma = 1$, $C_p = C_d = 1/3 = \theta_L = 1 - \theta_H$, and $c = 0.5$, $\frac{\text{erf}(-4)+1}{2}$).

will be fully revealed to the defendant. Because it is never rational for any plaintiff type to demand more than $S = \theta_H + C_d$, the settlement demands approach $\theta_H + C_d$ asymptotically as $y_p$ increases—as the plaintiff becomes more and more confident—but will never attain that value.

The monotonically increasing portion of the settlement demand is determined by an ordinary differential equation stemming from the first-order condition. As is often the case, the differential equation admits a one-parameter family of solutions, depending on boundary values.

Figure 1 includes two representative graphs for the plaintiff's optimal settlement demand under two different boundary values.[12] Both graphs exhibit a jump discontinuity between the flat portion and the monotonically increasing portion, and that will remain true regardless of the boundary value. The boundary value, $s_0$, can be assigned by specifying what amount the plaintiff *would have* demanded under the interior solution[13] if she were to observe $y_p = 0$. Note that when the plaintiff observes $y_p = 0$, she believes her probability of victory at trial is exactly 50%. $s_0$ can take on any value strictly between $\theta_L + C_d$ and $\theta_H + C_d$. Let $c = \frac{s_0 - (\theta_L + C_d)}{\theta_H - \theta_L}$ indicate where in this range $s_0$ falls. The value of $c$ will in turn determine how aggressively

12. Figure 1 plots the plaintiff's optimal demand strategy under the assumption that $\theta_L = \frac{1}{3}$ and $C_d = \frac{1}{3}$. This specification allows each plaintiff to walk away with at least $\frac{2}{3}$. However, one can also reasonably assume much smaller values of $\theta_L$ and $C_d$. Although the flat portion will be lowered in that case, the graphs will remain otherwise very similar.

13. Note that in equilibrium, for most parameters we consider, a plaintiff who observes $y_p = 0$ will demand $\theta_L + C_d$, the corner solution, rather than an interior solution.

each plaintiff type will make her demand in the separating portion of the equilibrium and the threshold $y^0$. A low $c$ indicates that the plaintiff, at each signal, will make relatively low settlement demands, and a high $c$ indicates that the plaintiff will make relatively aggressive demands. Nevertheless, as Lemma A5 indicates in the Appendix, the expected value of the plaintiff's recovery increases as $c$ decreases. In other words, the plaintiff does better with less aggressive demands on the whole. (More precisely, the plaintiff does better with less aggressive demands when the defendant is expecting less aggressive demands from a plaintiff of a given type.) The defendant is more likely to accept lower offers, and the saving in litigation costs offsets the fact that settlements are lower.

The graph in the left panel of Figure 1 assumes $c = 0.5$, which is halfway between the two end points. The graph in the right panel assumes $c = \frac{\text{erf}(-4)+1}{2}$, which is very close to zero.[14] As explained in the Appendix, the range of feasible values of $c$ varies with litigation costs and other parameters (see Lemma A4). More specifically, we find that when litigation is expensive (i.e., $C_d + C_p \geqslant \theta_H - \theta_L$ so that the parties would collectively spend at least as much as the total amount at stake in going to trial), then $c$ can be as close to zero as possible ($c$, of course, cannot equal zero in a monotonically increasing equilibrium). Thus, when litigation is expensive, there is no minimum possible $c$ value. On the other hand, when litigation is inexpensive (i.e., $C_d + C_p < \theta_H - \theta_L$), then $c$ has to exceed zero by some positive amount, and there exists a minimum $c$ value. This analytical result makes intuitive sense: when litigation is inexpensive, there is a greater value to going to trial, and therefore, the plaintiff will prefer to go to trial than be timid in her settlement demand.

We state the main results regarding the perfect Bayesian equilibria.

Proposition 1. Perfect Bayesian Equilibria under the $P$-Model. *In the* P*-model, for each* $\left(C_p, C_d, \theta_L, \theta_H\right) \in (0,1)^2 \times [0,1) \times (0,1]$ *such that* $0 \leqslant \theta_L < \theta_H \leqslant 1$, *there is a one-parameter class of semi-pooling perfect*

14. We chose $c = \frac{\text{erf}(-4)+1}{2}$ to illustrate plaintiff behavior when $c$ is close to zero because for this value of $c$ the plaintiff's optimal settlement demand curve is well-defined for every plaintiff type for all $(\theta_L, \theta_H)$ we consider in Table 1. See Lemma A4(b). To the extent that a smaller value of $c$ can lead to a fully-defined plaintiff demand curve for some $(\theta_L, \theta_H)$, the curve will look substantively similar to the case we plotted.

*Bayesian equilibria, each of which satisfies the following. First, the plaintiff demands* $C_d + \theta_L$ *for all* $y_p \leqslant y^0$ *(where* $y^0$ *is the unique* $y_p$ *value at which the expected utility of the plaintiff observing* $y_p$ *and demanding according to the interior solution would equal* $C_d + \theta_L$*).*[15] *Second, the plaintiff's demand is characterized by a jump discontinuity at* $y_p = y^0$ *and increases continuously for* $y_p > y^0$*. The defendant holds the following beliefs: for* $S \in (C_d+\theta_L, C_d+\theta_H)$*, the defendant has a point belief that is consistent with the interior optimal demand strategy; for* $S = C_d+\theta_L$ *the defendant's belief is a truncated normal probability distribution, which is zero for* $y_p > y^0$ *but takes on* $\frac{\varphi_1\left(\frac{y_p-y_d}{\sqrt{2}\sigma}\right)}{\sqrt{2}\sigma\,\Phi\left(\frac{y^0-y_d}{\sqrt{2}\sigma}\right)}$ *for* $y_p \leqslant y^0$*, where* $\Phi(\cdot)$ *is the standard normal cumulative distribution function; for* $S \in (-\infty, C_d + \theta_L) \cup [C_d + \theta_H, \infty)$*, virtually any belief is possible because this strategy is strictly dominated for all plaintiff types.*

The presence of multiple equilibria poses a challenge in terms of using this model to predict equilibrium behavior. In litigation and settlement models with signaling, a plausible assumption regarding the sender's behavior at an end point of type space (e.g., the lowest sender type) can provide an argument for pinning down a specific boundary condition. See Reinganum and Wilde (1986) and Daughety and Reinganum (1994). This line of argument, however, is unavailable under our set-up because there are no end points to the real line.

One possible approach is to see whether some of the equilibrium refinement criteria may be used to eliminate certain equilibria. As we show in the Appendix (Lemma A7), the two most frequently invoked criteria—Cho and Kreps (1987)'s "intuitive" criterion and Banks and Sobel (1987)'s D1—are not helpful in this regard: every boundary value equilibrium survives the "intuitive" criterion but fails the D1 criterion.[16] On the other hand, we also

15. We assume that if the plaintiff is indifferent between demanding $\theta_L + C_d$ or some amount greater than this value, then she will demand $\theta_L + C_d$.

16. It is unclear whether a perfect Bayesian equilibrium which survives D1 will exist in our game. The general existence result established by Banks and Sobel (1987) does not extend to our game for two reasons: (i) type spaces are unbounded in our game and (ii) we restrict our attention to pure-strategy equilibria. These two factors imply that the convexity condition necessary to establish the existence result will be violated. We are

show that the equilibrium corresponding to the minimum possible boundary value is the unique "undefeated" equilibrium in the sense similar to one defined in Mailath et al. (1993).[17] In particular, we show that given two boundary values $s_0 < s_0'$, the $s_0$-equilibrium "defeats" the $s_0'$-equilibrium. Relatedly, we also noted that the expected utility of every plaintiff type decreases in $s_0$. In other words, from the plaintiffs' (e.g., senders') perspective, given $s_0 < s_0'$, the $s_0$-equilibrium will dominate the $s_0'$-equilibrium.[18] For these reasons, in the case of inexpensive litigation, it may make sense to focus on the equilibrium corresponding to the lowest $c$. This will also correspond to the *maximally* separating equilibrium. In the case of expensive litigation (which has no lowest possible $c$), it may make sense to study how the equilibrium behavior changes as we let $c$ approach 0.

It is also helpful to examine the probability of rejection a plaintiff (of a certain type) faces in making a particular settlement demand. In the canonical one-sided signaling game by Reinganum and Wilde (1986), the plaintiff possesses private information regarding the extent of damages and makes a settlement demand that is monotonically increasing. The defendant observes the settlement demand and randomizes between accepting the offer and rejecting the offer. Because the probability of the defendant's rejection increases in the plaintiff's settlement demand amount, a low-damage plaintiff (who has less to gain from going to trial) is not incentivized to mimic a high-damage plaintiff, and there exists a fully separating equilibrium.

In our model, given the plaintiff's equilibrium strategy, each defendant will accept all offers below a certain threshold, according to his type, and reject those above it. In calculating this optimal threshold, the defendant takes into account both his own signal and the information conveyed by the plaintiff's offer. As a result, for each settlement demand amount,

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unaware of any theorem extending Banks and Sobel (1987)'s result to games featuring these two factors.

17. As we discuss in the Appendix, the definition of the "undefeated" refinement has to be modified to fit our game because, once the respective signals are observed, the parties no longer have a common prior regarding the plaintiff type distribution.

18. Given that the plaintiff is the one making the offer, there is an argument for choosing the value of $s_0$ giving the plaintiff the maximum expected payoff. If the plaintiff has sufficient bargaining power that it can make take-it-or-leave-it offers, it seems reasonable to assume that it has the bargaining power to select the most favorable equilibrium as well. See Bernardo and Talley (1996).

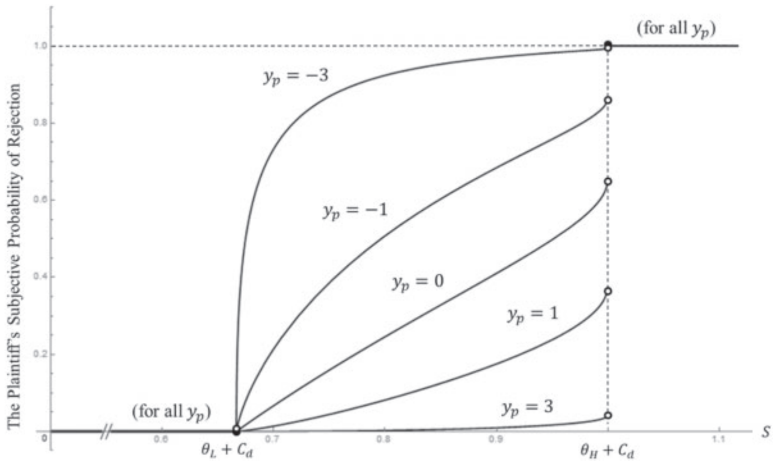


**Figure 2.** The Probability of Rejection Given Demand $S$ ($\sigma = 1$, $C_p = C_d = 1/3 = \theta_L = 1 - \theta_H$, and $c = \frac{\text{erf}(-4)+1}{2}$).

some defendants (below a certain threshold $y_d$) will accept, and others (above the threshold $y_d$) will choose to go to trial. Therefore, although the defendant's strategy is pure and involves no mixing or random element, because the defendant bases his decision not only on the plaintiff's settlement demand but also on his own information about the case merit, the defendant's response will appear to the plaintiff as if the defendant were relying on a mixed strategy: for every settlement demand greater than $\theta_L + C_d$, the probability that the defendant will accept lies strictly between zero and one. Furthermore, because this probability of rejection will depend on the plaintiff's estimate of the defendant type distribution given the plaintiff's own signal, each plaintiff type faces a different probability of rejection for her settlement demand. For this reason, we will call it the plaintiff's subjective probability of rejection for each settlement demand $S$.

Figure 2 graphs the subjective probability of rejection each plaintiff calculates for her settlement demand, $S$, and signal, $y_p$. Regardless of plaintiff type, a plaintiff who demands less than or equal to $\theta_L + C_d$ can expect this probability to be zero. Likewise, a plaintiff who demands $\theta_H + C_d$ or higher can expect this probability to be one. For intermediate demands, as one would expect, the probability that defendant will reject a settlement

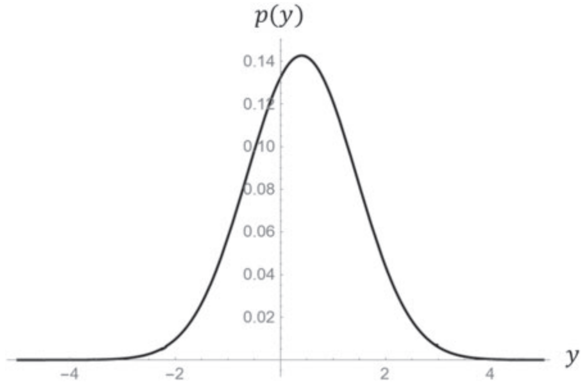


**Figure 3.** Litigation Probability Function for the $P$-Model ($\sigma = 1$, $C_p = C_d = 1/3 = \theta_L = 1 - \theta_H$, and $c = \frac{\mathrm{erf}(-4)+1}{2}$).

demand increases with the amount demanded.[19] See Reinganum and Wilde (1986). The subjective probability of rejection also varies across the plaintiff's signals because the plaintiff's and defendant's signals are correlated. When the plaintiff receives a high signal, the defendant is also likely to receive a high signal, which makes the defendant more likely to accept the offer, because a high signal means that the defendant is more likely to be liable.

We now consider the selection implications of the $P$-model. The combination of the plaintiff and the defendant strategies, sketched above, can be used to calculate the objective probability that a case of given merit, $y \in \boldsymbol{R}$, will go to trial. Given a case of merit $y$, we can calculate the probability distribution of the pair of signals $(y_p, y_d) \in \boldsymbol{R}^2$. Given the plaintiff's demand strategy and the defendant's response function, it is then possible to determine whether each pair of signals will lead to a trial or not. By aggregating this analysis across all possible pairs according to the framework provided in Lee and Klerman (2016), we can calculate the objective probability of going to trial for each $y$.

The result is shown in Figure 3. The horizontal axis is case merit, and the vertical axis is the probability of litigation. Since the threshold for liability

19. As we show in the Appendix (Lemma A9), the graphs are discontinuous at $S = \theta_L + C_d$ and at $S = \theta_H + C_d$.

was (without loss of generality) assumed to be $y = 0$, all cases to the right of the vertical axis are high merit cases and will result in judgments for the plaintiff with probability $\theta_H$. Conversely, all cases to the left of the vertical axis are low merit cases and will result in judgments for the plaintiff with probability $\theta_L$. The probability of litigation peaks for cases where case merit is close to zero. Since the threshold for liability is $y = 0$, this means that close cases (cases where the true case merit is close to the threshold for liability) are more likely to be litigated. Nevertheless, the probability of litigation does not peak exactly at $y = 0$. In the aggregate, high merit cases are more likely to result in litigation. This results in selection that favors the plaintiff, the party who makes the offer.

Figure 3 raises two questions. First, why do cases with extreme $y$ values—high or low—have such a low probability of going to trial (or equivalently, such a high probability of settling)? Second, why is the likelihood of going to trial asymmetric and greater on the right (meritorious) side?

For the first question, we mentioned in Part 1 that part of the reason for getting this result was due to the use of fact or case space (rather than probability space) and the fact that the parties' signals are correlated. We provide a fuller explanation here. When case merit is high, due to correlation with the original $y$ value, both parties' signals are likely to be above the threshold for liability ($y = 0$) and both are likely to think the case is meritorious. Thus, the plaintiff can demand a settlement amount that is very close to the limit value, $\theta_H + C_d$. Suppose, for example, that true case merit is $y = 6$ and that the standard deviation of signal error is $\sigma = 1$. Then 99.7% of parties will receive signals between 3 and 9. Even if the defendant's signal is at the low end of that range, $y_d = 3$, he will initially estimate the probability the plaintiff prevails at 99.8%, because his signal is three standard deviations above the threshold for liability. Thus, the defendant will accept nearly any amount the plaintiff will demand as long as it is below $\theta_H + C_d$. Similarly, even if the plaintiff receives a signal at the high end of the range (e.g., $y_p = 9$), it is rational for her to make a demand that reflects the possibility that the defendant estimates that the plaintiff's probability of prevailing is "only" 99.8% (corresponding to a signal of $y_d = y_p = 3$). To do so, the plaintiff need only reduce her settlement demand

**Table 1**. The Plaintiff Trial Win Rate ($\sigma = 1$, $C_p = C_d = 1/3$)

| | Fraction of litigated cases such that $Y > 0$ | | The plaintiff trial win rate | |
|---|---|---|---|---|
| Parameters | $c = 0.5$ | $c = \frac{\mathrm{erf}(-4)+1}{2}$ | $c = 0.5$ | $c = \frac{\mathrm{erf}(-4)+1}{2}$ |
| $\theta_H = 1, \theta_L = 0$ | 0.68 | N/A | 68% | N/A |
| $\theta_H = 1, \theta_L = \frac{1}{3}$ | 0.70 | 0.67 | 80% | 78% |
| $\theta_H = \frac{2}{3}, \theta_L = \frac{1}{3}$ | 0.76 | 0.65 | 59% | 55% |

by less than 0.2%, but she gets the benefit of settling and thus saving litigation costs in many more cases, including cases where defendant's signal is $y_d = 3$.

The answer to the second question—why the probability of trial is asymmetric and greater on the right—hinges partly on the fact that we are working with a plaintiff-offer model. In this model, a low-merit plaintiff can capitalize on her option of walking away with the defendant's litigation cost. Consider cases with $y$ values that are far below zero. These are low-merit cases, and will in most instances generate small $y_p$ values—more specifically, $y_p$ values below $y^0$. Under the semi-pooling equilibrium, we observed that all plaintiffs observing $y_p \leqslant y^0$ will make a settlement demand $\theta_L + C_d$, which will be accepted by *every* defendant type.[20] It stands to reason that because $y_p$ is correlated to the true $y$, the more negative $y$ gets, the more likely $y_p$ will fall below $y^0$. For this reason, this threshold behavior will have the effect of disproportionately settling low-merit cases and can lead to a bias toward high-merit cases in terms of cases that go to trial.

The implications of the asymmetry observed in Figure 3 are illustrated in Table 1, which lists plaintiff trial win rates under various parametric specifications when $C_p = C_d = 1/3$. As before, we ran the simulations with two different boundary values, $s_0 = (\theta_L + C_d) + 0.5\,(\theta_H - \theta_L)$ and

20. Although the threshold $y_p$ value (i.e., $y_p = y^0$) might suggest a sharp discontinuity in the probability of litigation, there is no discontinuity in Figure 3 because the horizontal axis reflects the true value of $y$, not the plaintiff's signal, $y_p$.

$s_0 = (\theta_L + C_d) + \left(\frac{\text{erf}(-4)+1}{2}\right)(\theta_H - \theta_L)$. The second and third columns indicate the fraction of litigated cases where case merit is above the threshold for liability ($Y > 0$), and the last two columns indicate the corresponding plaintiff trial win rates. When the law is precise so that $\theta_L = 0$ and $\theta_H = 1$, the win rate and the fraction of litigated cases with $Y > 0$ will coincide. Otherwise there will be some divergence, because even cases below the threshold may result in plaintiff victories and/or even cases above the threshold may result in plaintiff losses. Note that higher initial conditions will lead to higher plaintiff trial win rates.

In the first row, the parameter values are the same as under Priest and Klein's original model. When $c = 0.5$, the plaintiff trial win rate is 68%, well over 50%. On the other hand, for reasons explained in the Appendix, when the initial condition is really small, the equilibrium fails. In the second row, we fixed $\theta_L = \frac{1}{3}$ to ensure that the plaintiff would have a credible threat to go to trial. In this case, the plaintiff trial win rate ranged from 78% to 80% depending on the initial condition. This increase is expected since we are raising the probability with which low merit plaintiffs will win at trial. In the third row, we further fixed $\theta_H = \frac{2}{3}$ to ensure that the defendant would also have a credible threat to defend. In this case, the plaintiff trial win rates ranged from 55% to 59%. The rates in this row are much closer to 50%, but the results here are driven by the fact that the plaintiff trial win rate must be strictly lower than 67%—the probability with which high merit plaintiffs will prevail at trial. We obtained qualitatively similar results when we ran the calculations with other assumptions about litigation costs.

Although plaintiff trial win rates of well over 50% are inconsistent with Priest and Klein's famous conjecture of 50% win rates, our model confirms a key implication of the Priest–Klein model, that close cases will tend to go to trial and that extreme cases will tend to settle. The results of our model are also, in a more subtle way, consistent with one-sided asymmetric information models. As in Reinganum and Wilde (1986)'s signaling model, the plaintiff in our model makes offers that ensure that cases that are weak from the plaintiff's perspective settle. That is, when the plaintiff has received a weak signal, the plaintiff uses her information and bargaining power to make offers in the pooling portion of the equilibrium, which are always accepted. On the other hand, when the plaintiff's signal is high enough that her offer is in the separating part of the equilibrium, once the plaintiff has made her offer, the game involves screening, as in Bebchuk (1984). Defendants who received high signals (indicating that the plaintiff is likely to win) settle, while defendants who received low signals (indicating that the plaintiff is likely to lose) litigate. As a result, as in Daughety and Reinganum (1994), where there is both signaling and screening, extreme cases are trimmed from both sides. See Daughety and Reinganum (2012, p. 44).

## 3.2. The Defendant-Offer and Random-Offer Models under the Improper Uniform Distribution

So far, we have considered the *P*-model, in which the plaintiff makes a take-it-or-leave-it offer. The *D*-model, in which the defendant makes the take-it-or-leave-it offer, is essentially a mirror image of the *P*-model. The two models exhibit what Daughety and Reinganum (1994) call "label duality." The analysis is very similar, and we show the following result in the Appendix.

Proposition 2. The Symmetry between the *P*-Model and the *D*-Model. *When the plaintiff and the defendant face identical litigation costs, the litigation probability function of the* D*-model will be the reflection of the litigation probability function from the* P*-model around* $y = 0$. *In other words, for each* $y \in \boldsymbol{R}$, *the probability that a case of merit* $y$ *will go to trial in the* P*-model is the same as the probability that a case of merit* $-y$ *will go to trial in the* D*-model. Furthermore, if, in addition,* $\theta_H = 1 - \theta_L$, *then the plaintiff trial win rate in the* D*-model is one minus the plaintiff trial win rate from the* P*-model.*

One corollary of Proposition 2 is that, in the *D*-model, we will observe a plaintiff trial win rate that is less than 50%. If we assume $C_p = C_d$ and $\theta_L = 1 - \theta_H$, then the plaintiff trial win rate is simply 1 minus the win rate from the *P*-model.

It is, of course, unrealistic to assume that one side has all the bargaining power. Instead, bargaining power is likely to be shared between plaintiffs and defendants. We can model equal bargaining power between the parties by assuming that plaintiff and defendant each makes the take-it-or-leave-it offer with probability 50%. One can analyze that situation by randomizing with equal probability between the $P$-model and the $D$-model. When $C_p = C_d$ and $\theta_H = 1 - \theta_L$, the plaintiff trial win rate will be 50%.[21] Under this scenario, the results mimic the Priest–Klein model: close cases are more likely to go to trial, and the plaintiff trial win rate is 50%.

## 3.3. Extension to a General Distribution of Disputes

The previous sections have assumed that $\sigma = 1$ and $g_Y(x) = 1$. In this section, we ask what happens when we extend the set-up to work with a more general probability density function. In general, for any fixed level of signal accuracy, $\sigma > 0$, the plaintiff trial win rate will depend on the shape of $g_Y(x)$. For example, if a disproportionately high number of disputes exist just to the right of 0 rather than to the left of 0, then the plaintiff trial win rate will be higher than one obtained under the assumption that $g_Y(x)$ is flat. Nevertheless, Priest and Klein (1984) argued that, regardless of the shape of $g_Y(x)$, the plaintiff trial win rate will approach 50% *in the limit as $\sigma$ goes to zero*, and Lee and Klerman (2016) proved the result under the assumption that $g_Y(x)$ be continuous, strictly positive, and bounded above. That is, as the signals both parties receive about case merit become more and more accurate, the plaintiff trial win rate converges to 50%. In this section, we likewise ask whether, as $\sigma$ approaches zero, the plaintiff trial win rate, for $g_Y(x)$ that is continuous, strictly positive, and bounded above, will approach the win rate obtained in Table 1.

We begin by considering the manner in which the litigants make inferences about case merit. As noted already, given his signal, $Y_p = y_p$, the plaintiff can construct the distribution of $Y$ conditional on $Y_p = y_p$ using Bayes' rule, and the defendant likewise. At this point, we discuss two different types of inferences for the litigants. A *naïve* litigant constructs the conditional distribution without knowledge of $g_Y(x)$, or alternatively, as if $g_Y(x)$ is flat. A *sophisticated* litigant, however, is aware of $g_Y(x)$ and will take the shape of $g_Y(x)$ into account in constructing the conditional

21. The plaintiff trial win rate will also be 50% even if $\theta_H \neq 1 - \theta_L$, if the randomization between the plaintiff-offer and defendant-offer models occurs with exactly the right proportions.

distribution. Thus, for $i = p, d$ we have

$$f_{Y|Y_i=y_i}(x) = \begin{cases} \varphi_\sigma\,(y_i - x) & \text{for a naïve litigant} \\ \frac{g_Y(x)\varphi_\sigma(y_i-x)}{\int_{-\infty}^{\infty} g_Y(z)\varphi_\sigma(y_i-z)dz} & \text{for a sophisticated litigant.} \end{cases}$$

One possible justification for naïve inferences is that the litigants are simply uninformed about $g_Y(x)$. In this case, naïve inference may be rationalized according to Laplace's suggestion that one should apply a uniform distribution to unknown events according to the "principle of insufficient reason." See Hartigan (1983, p. 2) and Lee and Klerman (2016). In their model, Priest and Klein (1984) implicitly assume that the litigants are naïve and state their 50% limit result under this assumption. Lee and Klerman (2016), however, prove the 50% limit hypothesis for both naïve and sophisticated litigants. In this section, we prove the corresponding result for the take-it-or-leave-it model when the litigants are naïve. We have been unable to prove the same result when the litigants are sophisticated. Nevertheless, we can think of no reason why it should not be true, and simulations suggest it remains true.

Note that when the litigants make their utility calculations based on naïve inferences, the set-up of the problem remains unaltered through Proposition 1. The only difference is that calculation of the plaintiff trial win rate will initially depend on the shape of the distribution. Therefore, when the litigants are naïve, the arguments used in Proposition 3 of Lee and Klerman (2016) can apply to show that the plaintiff trial win rate in the limit will indeed equal to the one calculated under the assumption that $g_Y(x) = 1$ for all $x$ and $\sigma = 1$. In the Appendix, we prove the following result.

PROPOSITION 3. The Irrelevance of the Dispute Distribution for the Limit Results in the Take-It-or-Leave-It Offer Models. *Under both the* P-*model and the* D-*model, given a distribution of disputes that is strictly positive, bounded above, and continuous, if the litigants make naïve inferences, the plaintiff trial win rate in the limit as* $\sigma$ *approaches zero will be equal to the plaintiff trial win rate obtained when* $\sigma = 1$ *and the distribution of disputes was assumed to be improper uniform.*

The intuition behind Proposition 3 is as follows. When the litigants are naïve, the litigation probability function will not depend on $g_Y(x)$. When

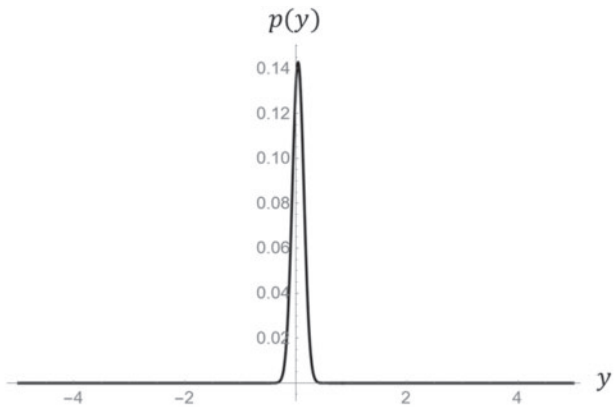


**Figure 4.** Litigation Probability Function for the $P$-Model ($\sigma = 0.1$, $C_p = C_d = 1/3$, and $(\theta_L, \theta_H) = \left(\frac{1}{3}, \frac{2}{3}\right), c = \frac{\mathrm{erf}(-4)+1}{2}$).

$\sigma = 1$, the litigant probability function will remain identical to Figure 3. But as $\sigma$ approaches zero, the shape of the litigation probability function will get progressively sharper, and the peak, which lies to the right of zero, will get arbitrarily close to zero (see Figure 4). This means that as parties get more and more accurate signals, only cases really close to zero will likely get litigated, so the distribution of disputes away from zero will be irrelevant to the plaintiff trial win rate. Note that varying $\sigma$ will not vary the *proportion* of the graph of the litigation probability function that lies to the right of 0. Meanwhile, because $g_Y(x)$ is continuous and strictly positive, as $\sigma$ becomes smaller, the cases going to trial that lie just to the right of zero as a fraction of the total number of cases going to trial will approximately equal this proportion.

Notice also that the same intuition should carry over even when litigants are sophisticated. As $\sigma$ approaches zero, the correlation between the parties' signals will become stronger, and the shape of $g_Y(x)$ will have less and less of an impact on litigants' inferences. In the limit, litigants' sophisticated inferences will coincide with their naïve inferences. In other words, as $\sigma$ approaches zero, it will be as if the litigants are making inferences as if $g_Y(x)$ were flat. Another way of explaining the intuition is that, with a smooth symmetrical distribution, naïve and sophisticated beliefs approach each other near zero. On the other hand, when the signal is far from zero, there is more likely to be divergence between naïve and sophisticated beliefs. Nevertheless, when the signal is far from zero, almost all cases settle anyway, so there is little effect on the selection of disputes for litigation. As $\sigma$ approaches zero, nearly all cases are "far" from zero and settle, so results with naïve and sophisticated beliefs coincide. A *Mathematica* simulation[22] using normal distributions for $g_Y(x)$ confirms that, when litigants are sophisticated, the plaintiff trial win rate will converge, as $\sigma$ approaches zero, to the same value as when litigants are naïve, although we have been unable to establish this result analytically.

## 4. The Model with the Chatterjee–Samuelson Mechanism

In this Part, we investigate the implications of the Chatterjee–Samuelson mechanism under the set-up discussed in Part 3. As before, the plaintiff receives a signal $Y_p = Y + \epsilon_p$ and the defendant receives a signal $Y_d = Y + \epsilon_d$, where $\epsilon_p, \epsilon_d \sim N(0, \sigma^2)$. We further assume that $C_p = C_d = C$, so that the litigants face the same litigation costs. Likewise, the plaintiff wins with probability $\theta_H$ if $y > 0$ and with probability $\theta_L$ if $y \leqslant 0$, where $0 \leqslant \theta_L < \theta_H \leqslant 1$.

Instead of having one party make a take-it-or-leave-it offer, we assume that each litigant submits a secret demand or offer to a neutral party (or computer). If the plaintiff's demand is greater than the defendant's offer, the case goes to trial. If the plaintiff's offer is less than or equal to the defendant's offer, then the case settles for the average of the two offers. The plaintiff makes her secret settlement demand, $p(y_p)$, based on the signal she observes and inferences about the distribution of the likely defendant signals. The defendant likewise makes a secret settlement offer, $d(y_d)$, based on his signal and similar inferences. Because the demands and offers are secret, neither party learns about or from the other party's signal. One advantage of the

22. For our simulations, we started with $g_Y(x)$ as a normal distribution with standard deviation equal to 1 and mean equal to 1. We then plotted $n(u)$, which is the plaintiff's offer curve in normalized variables (see Appendix), for $\kappa = 2$ and for progressively decreasing $\sigma$ values. We observed that the corresponding $n(u)$ curves converge toward the $n(u)$ curve we obtained when the distribution of disputes was assumed to be improper uniform. We then repeated the simulations for other standard deviation and mean values as well as for different $\kappa$ values.

Chatterjee–Samuelson mechanism is that it does not require an assumption that one side has all the bargaining power. As a result, there is no first-mover advantage and likewise, the mechanism does not allow one party to unilaterally extract the other party's litigation costs.

Friedman and Wittman (2007) first explored the implications of this bargaining mechanism for litigation in the case where the litigants' signals are distributed according to two independent uniform distributions over [0,1]. In their model, if the case goes to trial, the true case strength (the judgment amount) is the average of the two signals. They found that when trial cost is high, the selection result is consistent with the Priest–Klein result in that disputes that are close cases—that is, the *ex ante* probability that the plaintiff will prevail at trial is about 50%—are more likely to lead to larger gaps of signals and thus, are more likely to go to trial. See Friedman and Wittman (2007, pp. 108–110).

Although we follow their pioneering work, we also diverge from their model in a number of important ways. First, in our model, the support for the parties' signals is the entire real line. Second, the signals $Y_p$ and $Y_d$ are correlated, and as such, the plaintiff makes inferences about the distribution of $Y_d$ based on $y_p$, and likewise for the defendant. Third, if the parties cannot settle, the parties' respective payoffs at trial are determined by the underlying true case merit $y$, which does not depend on either party's signal. Fourth, we take into account plaintiff and defendant credibility by considering $\theta_L > 0$ and $\theta_H < 1$.

Like Friedman and Wittman (2007), we consider pure strategies contingent on the realized signal. Thus, a plaintiff's strategy is a measurable function $p(\cdot) : \boldsymbol{R} \to \boldsymbol{R}^+$ that assigns the demand $p = p(y_p) \in [0, \infty)$ when it observes signal $y_p$. Similarly, a defendant's strategy is a measurable function $d(\cdot) : \boldsymbol{R} \to \boldsymbol{R}$ that assigns the offer $d = d(y_d) \in (-\infty, \infty)$ when it observes signal $y_d$. The objective of the plaintiff is to maximize expected net payments, conditioned on its realized signal $y_p$ and the defendant's strategy $d(\cdot)$. The defendant's object is to minimize expected net payments.

The payoff function for the plaintiff is:

$$
\begin{aligned}
&U_p\left(p, y_p, d(y_d;\sigma);\sigma\right) \\
&\quad = \int_{\{y_d \mid p \leqslant d(y_d;\sigma)\}} \left(\frac{d(y_d;\sigma)+p}{2}\right) f_{Y_d \mid Y_p = y_p}(y_d)\, \mathrm{d}y_d
\end{aligned}
$$

$$+ \int_{\{y_d \mid p > d(y_d;\sigma)\}} \Big( (\theta_H - \theta_L) \Pr\big(Y \geqslant 0 | Y_p = y_p, Y_d = y_d\big) + (\theta_L - C) \Big) f_{Y_d|Y_p=y_p}(y_d)\, \mathrm{d}y_d.$$

The first-term in the right-hand side is the expected value of settling, and the second-term is the expected value of going to trial. Likewise, the payoff for the defendant is:

$$\begin{aligned} &U_d\left(d, y_d, p(y_p;\sigma);\sigma\right) \\ &= \int_{\{y_p | p(y_p;\sigma) \leqslant d\}} \left( \frac{d + p\left(y_p;\sigma\right)}{2} \right) f_{Y_p|Y_d=y_d}(y_p)\mathrm{d}y_p \\ &\quad + \int_{\{y_p | p(y_p;\sigma) > d\}} \Big( (\theta_H - \theta_L) \Pr\big(Y \geqslant 0 | Y_p = y_p, Y_d = y_d\big) \\ &\quad + (\theta_L + C) \Big) f_{Y_p|Y_d=y_d}(y_p)\mathrm{d}y_p. \end{aligned}$$

A Nash equilibrium (NE) of this game is defined as follows:

Definition 1. *A Nash equilibrium (NE) is a strategy pair* $(p(y_p;\sigma), d(y_d;\sigma))$ *such that*

- $p(y_p;\sigma) = \operatorname{argmax}_p U_p(p, y_p, d(y_d;\sigma);\sigma)$, and
- $d(y_d;\sigma) = \operatorname{argmin}_d U_d(d, y_d, p(y_p;\sigma);\sigma)$.

Friedman and Wittman (2007) show that in their game any best response can be represented by a function that is everywhere weakly increasing. Their argument can be adopted to show the same result in our game, and we include this result in the Appendix (Lemma B1). Note, however, that, as Friedman and Wittman (2007) observed, existence of a NE is not guaranteed in this game because we are restricting our analysis to pure strategy Nash equilibria. We have been unable to identify any pure strategy NE in which both parties employ fully continuous strategies.[23] But if we allow jump

23. Although Friedman and Wittman (2007) analyze continuous piecewise linear symmetric Nash equilibria, in our game, it can be shown that no such Nash equilibria exist. Because the compound distributions $f_{Y_p|Y_d=y_d}(y_p)$ and $f_{Y_d|Y_p=y_p}(y_d)$ are normal distributions, the best response function to a continuous piecewise linear function strategy fails to be a linear strategy. It is our hypothesis that all symmetric Nash equilibria will take

discontinuities, we are able to show that there are an infinite number of pure strategy Nash equilibria. Our analysis will focus on the properties of symmetric and asymmetric Nash equilibria and their existence in the domain of all possible strategies.

## 4.1. Symmetric Bargaining under the Improper Uniform Distribution

As before, we begin our analysis by considering the case where the distribution of disputes is assumed to be improper uniform. Friedman and Wittman (2007) limit their substantive analysis to *symmetric* Nash equilibria. These are Nash equilibria in which the plaintiff's strategy and the defendant's strategy are "symmetric" with respect to one another. Their definition of symmetry translates to our set-up as follows:

Definition 2. *The strategies $p(y_p;\sigma)$ and $d(y_d;\sigma)$ are symmetric if for all $x \in \boldsymbol{R}, p(x;\sigma) + d(-x;\sigma) = \theta_H + \theta_L$.*[24]

The main intuition behind symmetric strategies is that when two litigants are equally confident (or equally unconfident) that the outcome will favor their side, they will be equally aggressive (or timid) in their settlement terms. For example, suppose that under $p(y_p;\sigma)$, if the plaintiff receives a signal that leads her to believe with probability 90% (based on her private signal) that the case will be of high merit, she will make a settlement demand in the amount of $\theta_H - a$. Meanwhile, suppose that under $d(y_p;\sigma)$, if the defendant receives the mirror image of the plaintiff's signal and believes with probability 90% (based on his private signal) that the case will be of low merit, he will make a settlement offer in the amount of $\theta_L - b$. When $a = b$, we will say their strategies are symmetric. When such conditions are not satisfied in a NE, we will call it an *asymmetric* NE.

on the form of step functions. The highly restrictive conditions of correlated inferences together with equilibria symmetry lead us to surmise that there cannot be any Nash equilibrium in which even a portion of a litigant's strategy is continuously increasing or decreasing.

24. Note that when $\theta_H = 1$ and $\theta_L = 0$, then we have $p(x;\sigma) = 1 - d(-x;\sigma)$, which is equivalent to the definition from Friedman and Wittman (2007).

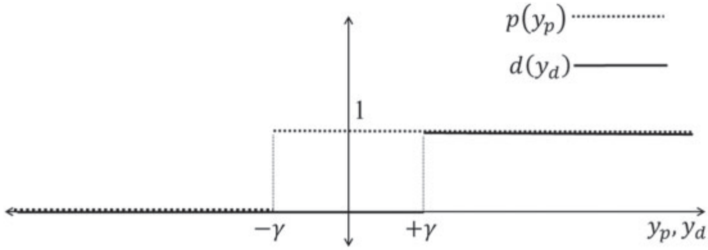


**Figure 5.** Symmetric NE Strategies.

When $g_Y(x) = 1$, notice that if a symmetric NE were to exist, the plaintiff trial win rate will necessarily be 50%. This can be seen as follows. The litigation condition set in this game is $R_\sigma(y_p, y_d) = \{(y_p, y_d) | p(y_p; \sigma) > d(y_d; \sigma)\}$. Note that if $p(y_p; \sigma)$ and $d(y_d; \sigma)$ are symmetric, $p(y_p; \sigma) > d(y_d; \sigma)$ if and only if $1 - d(-y_p; \sigma) > 1 - p(-y_d; \sigma)$, or $p(-y_d; \sigma) > d(-y_p; \sigma)$. Therefore, $(y_p, y_d) \in R_\sigma(y_p, y_d)$ if and only if $(-y_d, -y_p) \in R_\sigma(y_p, y_d)$ and the plaintiff trial win rate will necessarily be 50%.

Moreover, it can also be shown that there is at least one class of symmetric Nash equilibria. Consider a strategy profile in which the defendant offers $\theta_L$ up until some threshold $y_d$ value (say $y_d = \gamma(\sigma)$) and offers $\theta_H$ afterward. Likewise, suppose the plaintiff demands $\theta_L$ up until $y_p = -\gamma(\sigma)$ and $\theta_H$ afterward. The strategies are symmetric by construction. Then as long as we can find a suitable $\gamma(\sigma) \in \boldsymbol{R}$ such that these strategies constitute a NE, we will have a symmetric NE. In the Appendix, we show such $\gamma(\sigma) \in \boldsymbol{R}$ always exists. In addition, because the same argument applies for $\theta_L + \epsilon$ and $\theta_H - \epsilon$, when $\epsilon > 0$ is sufficiently small, there is a family of symmetric Nash equilibria.[25]

Figure 5 illustrates one such equilibrium when $\theta_H = 1$ and $\theta_L = 0$.[26] The plaintiff demands zero up to $-\gamma$ and then demands one. Defendant offers zero up to $\gamma$, and then offers one. It is relatively easy to see that this is a NE. Where both parties receive favorable signals (i.e., high $y_p$ and high $y_d$), a court judgment in the amount of $\theta_H = 1$ is highly likely, and therefore, it

25. Note that there are also other types of symmetric equilibria. For example, there is the trivial class of equilibria in which all cases go to trial because the plaintiff's demand is absurdly high and defendant's offer is unreasonably low (Friedman and Wittman, 2007).
26. Cf. Friedman and Wittman (2007, p. 105, Figure 1).

is in their mutual interest to save litigation costs and settle at 1. Similarly, when both parties receive weak signals (i.e., low $y_p$ and $y_d$), it is in the mutual interest to settle at zero.[27] This saves both parties litigation costs, and the plaintiff cannot expect to do better by litigating. Only when both of their signals are close to zero, the threshold for liability, is it rational for the parties to diverge in their settlement demands. The plaintiff observing a signal close to zero will demand one, because there is a good chance that defendant receives a signal greater than $\gamma$. Thus, by demanding one, the plaintiff has a good chance of settling at one. Similarly, the defendant observing a signal close to zero will offer zero and may very well be able to settle for zero.

Because the symmetry of the strategies leads to an equal number of litigated cases to the left and right of the decision standard, and because the distribution of disputes is assumed to be improper uniform, the plaintiff trial win rate will be $\frac{\theta_H+\theta_L}{2}$. If $\theta_H = 1 - \theta_L$, then the plaintiff trial win rate will be 50%. We summarize these results in Proposition 4.

PROPOSITION 4. Symmetric Nash Equilibria under the Chatterjee–Samuelson Bargaining Model. *Under the Chatterjee–Samuelson bargaining model with* $g_Y(x) = 1$, *the following is true.*

(a) *There exists a continuous family of symmetric Nash equilibria,*
(b) *the plaintiff trial win rate is* $\frac{\theta_H+\theta_L}{2}$ *for any symmetric* NE, *and*
(c) *a sufficient condition that ensures that extreme cases on both ends will be more likely to settle is that both the plaintiff's strategy and the defendant's strategy eventually coincide at a fixed value in each direction (as* $y_p$ *and* $y_d$ *approach positive infinity and negative infinity).*

A proof can be found in the Appendix.

27. By assuming $\theta_L = 0$, we are implicitly assuming that plaintiff threat credibility is not an issue and that plaintiff would go to trial even though litigation costs surely exceed expected recoveries for low signals. As a result, plaintiff is better off settling for zero than litigating when its signal is low.

## 4.2. Asymmetric Bargaining under the Improper Uniform Distribution

We now consider asymmetric Nash equilibria. For these equilibria, the plaintiff trial win rate can be greater or less than 50%. The easiest way to construct an example of an asymmetric equilibrium is to consider the case where one party offers (or demands) the same settlement amount regardless of his or her type and the other party employs a two-step strategy with a threshold type. For example, consider an equilibrium involving an "obstinate" plaintiff who chooses to demand a fixed settlement amount of $s$ regardless of her type, where $s \in (\max\{\theta_L + C, \theta_H - C\}, \theta_H + C)$, which is non-empty. The defendant's best response strategy is to offer either $s$ or $\theta_L - C$ based on his type. That is, the defendant offers $s$ (which will always be accepted), if his signal is above a certain threshold, $y_d^*$, but otherwise makes a very low offer, $\theta_L - C$, (which will always be rejected). Define the threshold according to the following equation:

$$(\theta_H - \theta_L)\,\Pr\left(Y \geqslant 0 | Y_d = y_d^*\right) + (\theta_L + C) = s.$$

The defendant type $y_d^*$ will be indifferent between settling at $s$ or going to trial. It follows that, given the plaintiff's strategy, all defendant types less than $y_d^*$ will prefer to litigate and all defendant types greater than $y_d^*$ will prefer to settle. Thus, a two-step strategy will be optimal for the defendant. Note that as long as $s \in (\max\{\theta_L + C, \theta_H - C\}, \theta_H + C)$, such $y_d^*$ will always exist. Moreover, the plaintiff in turn has no reason to deviate. Once she considers deviating below $s$, no deviation will be optimal unless she matches the defendant's offer of $\theta_L - C$. But she will have no reason to match $\theta_L - C$, given that this value is the minimum guaranteed value of going to trial for any plaintiff. On the other hand, as long as $s > \theta_H - C$, settling at $s$ is better than going to trial for any plaintiff type. In this equilibrium, disputes will go to trial whenever $y_d < y_d^*$, and we can show that the plaintiff trial win rate will be $\theta_L$ (see Appendix). Similarly, note that we can construct a class of "obstinate" defendant equilibria, in which disputes will go to trial whenever $y_p > y_p^*$ for some $y_p^*$, and the resulting plaintiff trial win rate will be $\theta_H$. We summarize the results in Proposition 5.

Proposition 5. Asymmetric Nash Equilibria under the Chatterjee–Samuelson Bargaining Model. *Under the Chatterjee–Samuelson bargaining model with* $g_Y(x) = 1$, *the following is true.*

(a) *There exists a continuous family of asymmetric Nash equilibria;*
(b) *the plaintiff trial win rate will be* $\theta_L$ *under the obstinate plaintiff equilibrium; and*
(c) *the plaintiff trial win rate will be* $\theta_H$ *under the obstinate defendant equilibrium.*

The proof is included in the Appendix. Note that if we assume $\theta_H + \theta_L = 1$ and if obstinate plaintiff and obstinate defendant equilibria occur with equal probability, then the plaintiff trial win rate will be 50%. Similarly, if $\theta_H + \theta_L \neq 1$, but obstinate plaintiff and obstinate defendant equilibria occur in exactly the right proportions, the plaintiff trial win rate will also be 50%. Nevertheless, because there is little reason to believe these assumptions to be true, we have reason to doubt that the aggregate plaintiff trial win rate will be 50%.

## 4.3. Extension to a General Distribution of Disputes

When we consider a general distribution of disputes, a few important differences emerge. First, because the selection of the disputes that go to trial is sensitive to the choice of the distribution, we state general results regarding the plaintiff trial win rate in the limit as $\sigma$ goes to zero. Second, as mentioned in Section 3.3, there are two ways in which the litigants make inferences regarding the conditional distribution of the opponent types—naïve and sophisticated. Third, unless the distribution of disputes is symmetric around 0, the symmetric step-function strategies, such as the one discussed in the previous section, will not constitute a NE.

For this reason, in this section, we consider a one-parameter family of Nash equilibria $\left(p(y_p;\sigma), d(y_d;\sigma)\right)$, where $\sigma$ is the parameter, and then analyze the equilibrium behavior as $\sigma$ approaches zero.

Definition 3. *Given a distribution of disputes,* $g_Y(x)$, *that is continuous, strictly positive, and bounded above, a one-parameter family of Nash equilibria is a set of NE strategy pairs* $\left(p(y_p;\sigma), d(y_d;\sigma)\right)$ *defined for each*

*$\sigma \in (0, \bar{\sigma})$ for some $\bar{\sigma} > 0$ such that $p(y_p; \sigma)$ and $d(y_d; \sigma)$ are both continuous in $\sigma$. Given a continuous family, a symmetric limit equilibrium is a pair of strategies $\big(p(y_p; 0), d(y_d; 0)\big)$, such that the following conditions hold true*:

- $p(y_p; 0) = \lim_{\sigma \to 0} p(y_p; \sigma)$;
- $d(y_d; 0) = \lim_{\sigma \to 0} d(y_d; \sigma)$;
- $p(y_p; 0) = \text{argmax}_p \lim_{\sigma \to 0} U_p(p, y_p, d(y_d; \sigma); \sigma) = \text{argmax}_p \lim_{\sigma \to 0} U_p(p, y_p, d(y_d; 0); \sigma)$;
- $d(y_d; 0) = \text{argmin}_d \lim_{\sigma \to 0} U_d(d, y_d, p(y_p; \sigma); \sigma) = \text{argmin}_d \lim_{\sigma \to 0} U_d(d, y_d, p(y_p; 0); \sigma)$; and
- $p(x; 0) = 1 - d(-x; 0)$ for all $x \in \boldsymbol{R}$.

Under this definition, a symmetric limit equilibrium is the limit (as $\sigma$ approaches 0) of a family of Nash equilibria defined over $\sigma \in (0, \bar{\sigma})$, and is itself a symmetric NE of the $\sigma$-game in the limit. Note, however, that given a symmetric limit equilibrium, it will not necessarily be the case that each NE $\big(p(y_p; \sigma), d(y_d; \sigma)\big)$ for $\sigma \in (0, \bar{\sigma})$ will itself be symmetric. The conditions require only that the equilibrium is symmetric *in the limit*. When the last condition is not satisfied, we will say the equilibrium is asymmetric in the limit.

By the analogous reasoning as before, given a symmetric limit equilibrium, it follows that the plaintiff trial win rate will necessarily be $\frac{\theta_H + \theta_L}{2}$ in the limit. In the Appendix, we show the following.

Proposition 6. Symmetric and Asymmetric Limit Equilibria under the Chatterjee–Samuelson Bargaining Model. *Suppose the distribution of disputes is strictly positive, bounded above, and continuous. Then under the Chatterjee–Samuelson bargaining mechanism, whether the litigants make naïve inferences or sophisticated inferences, the following is true.*

(a) *There exists at least one class of symmetric limit equilibria, and for all symmetric limit equilibria, the plaintiff trial win rate is $\frac{\theta_H + \theta_L}{2}$ as $\sigma$ approaches zero; and*

(b) *there exist classes of obstinate limit equilibria (which are asymmetric in the limit), and the plaintiff trial win rate is $\theta_L$ for* *obstinate plaintiff limit equilibria and $\theta_H$ for obstinate defendant limit equilibria as $\sigma$ approaches zero.*

A proof can be found in the Appendix.

## 5. Conclusion

This paper analyzes selection in a model with two-sided incomplete information and correlated signals under two bargaining protocols. Under the take-it-or-leave-it offer model, we identify a class of semi-pooling equilibria in which most disputes settle. We find that extreme cases (those in which case merit is far from the liability threshold) are more likely to settle, and close cases are more likely to go to trial. Nevertheless, unlike Priest and Klein (1984)'s prediction, cases that go to trial are biased in favor of the offeror, and the plaintiff trial win rate deviates systematically from 50% unless the party that makes the offer is randomized. Under the Chatterjee–Samuelson mechanism, we examine the litigants' behavior under symmetric and asymmetric Nash equilibria. Under symmetric equilibria, close cases litigate, and, depending on litigation costs and the modeling of plaintiff threat credibility, the plaintiff trial win rate may be 50%. Under asymmetric equilibria, extreme cases are more likely to go to trial and the plaintiff trial win rate will deviate significantly from 50%.

## Supplementary Material

Supplementary material is available at *American Law and Economics Review* online.

This Appendix derives the equilibria that are described more informally in the body of the paper. It also proves the propositions.

## Appendix

## A. Model with Take-It-or-Leave-It Offers

### The Plaintiff-Offer Model

We begin our analysis by considering the possibility of fully separating equilibria that are everywhere differentiable. Unfortunately, we conclude that they do not exist. In addition, we also find that—under a reasonable assumption regarding the off-the-equilibrium-path belief—there are no complete pooling equilibria. Instead, we identify a class of semi-pooling equilibria in which all plaintiff types below some threshold pool and only those plaintiff types above that threshold separate.

Our analysis proceeds in eight steps. First, we consider the defendant's optimal response to the plaintiff's settlement demand under the assumption that the plaintiff's settlement demand fully reveals her type. Second, we construct the plaintiff's expected utility function given the defendant's expected response and take the first-order condition to derive the ordinary differential equation giving rise to the plaintiff's (interior) optimal settlement demand function. Third, we derive the properties of the interior solution to the ordinary differential equation. Fourth, we return to the plaintiff's expected utility function to establish that a fully separating equilibrium does not exist because we find that weak plaintiffs do better with corner solutions. Fifth, we characterize a class of semi-pooling equilibria and also eliminate completely pooling equilibria. Sixth, we consider issues pertaining to equilibrium refinement. We examine various equilibrium refinement criteria and suggest that it makes sense to focus on the solution with the smallest feasible boundary value (if such a value exists) or to consider the behavior of the solution in the limit as the boundary value approaches the infimum (if the smallest feasible value does not exist). Seventh, we analyze the probability of rejection function for each defendant type under the semi-pooling equilibria. Eighth, we consider the selection implications.

*The Defendant's Strategy.* Suppose the plaintiff makes a settlement demand and the defendant can fully infer the plaintiff type from the demand. We normalize the settlement demand as $\tau = \frac{S - C_d - \theta_L}{\theta_H - \theta_L}$. Since there is a one-to-one correspondence between $\tau$ and $S$, we can assume without loss of generality that the plaintiff makes her settlement demand by simply specifying $\tau$. We have already shown that a defendant, regardless of his type, should always accept a settlement demand of $\tau = 0$. Similarly, a risk-neutral defendant should never accept any settlement demand $\tau \geqslant 1$. This would amount to a settlement demand greater than or equal to $C_d + \theta_H$. As before, because the defendant will always place *some* positive probability that the case may be of low merit, going to trial will always be better off (in expectation) than paying a sum greater than or equal to $C_d + \theta_H$. It also follows that the strategy of demanding $\tau \geqslant 1$ is strictly dominated for any plaintiff type. To see this, consider a plaintiff offer of $\tau = 1-\epsilon$ for small $\epsilon$. Since the defendants who would have rejected $\tau = 1-\epsilon$ will also reject $\tau \geqslant 1$, against those defendants, the payoff is equal whether the plaintiff offers $\tau = 1-\epsilon$ or $\tau \geqslant 1$. On the other hand, for those defendants who would have accepted $\tau = 1-\epsilon$, the plaintiff would get $(\theta_H - \theta_L)(1-\epsilon) + C_d + \theta_L$ if she offered $\tau = 1-\epsilon$, but if she offers $\tau \geqslant 1$, trial is guaranteed and the best possible outcome is $\theta_H - C_p$, which will be less than $(\theta_H - \theta_L)(1-\epsilon) + C_d + \theta_L$ as long as $\epsilon < \frac{C_p + C_d}{\theta_H - \theta_L}$. Such $\epsilon$ will always exist since litigation costs are positive. In short, regardless of her type, the plaintiff can always find a settlement demand that will dominate $\tau \geqslant 1$. Therefore, we have proved the following lemma.

LEMMA A1. *In a perfect Bayesian equilibrium, we must have* $\tau \in [0, 1)$. *Furthermore, if a fully separating equilibrium were to exist, then* $\tau \in (0, 1)$ *in equilibrium.*

In deciding whether to accept or reject, what matters to the defendant is the probability with which the defendant believes the case will be of high merit. The defendant can estimate this subjective conditional probability $P_d$ based on his own signal and the plaintiff's signal. In a fully separating equilibrium, the defendant can infer the plaintiff's exact type from her demand. Using Bayes' Rule, we have:

*The defendant's own estimate of the probability that the case is of high merit given his own signal* $y_d$ *and after inferring the plaintiff's type to be* $y_p$ *when* $g_Y(x) = 1$

$$P_d = \Pr(Y > 0 | Y_d = y_d, Y_p = y_p)$$
$$= \frac{\int_0^\infty \varphi_\sigma(x - y_d)\,\varphi_\sigma(x - y_p)dx}{\int_{-\infty}^\infty \varphi_\sigma(x - y_d)\,\varphi_\sigma(x - y_p)dx} = \Phi\left(\frac{y_p + y_d}{\sqrt{2}\sigma}\right), \qquad \text{(A1)}$$

where $\Phi(\cdot)$ is the cumulative distribution function corresponding to $\varphi(\cdot) \equiv \varphi_1(\cdot)$.

The defendant's strategy is to observe $y_d$ and $\tau$ and decide whether to accept or reject. The defendant has an initial conditional posterior about $y$ based on the signal he observed, $y_d$, but that will be updated because the plaintiff's settlement demand will reveal additional information about the case merit. If the defendant observes $y_d = -5$, initially he might be inclined to think the true $y$ is close to $-5$, indicating a strong case for the defendant. But if the plaintiff makes a settlement demand from which the defendant can validly infer that the plaintiff has observed $y_p = 5$, then the defendant will now update his posterior and believe that the true $y$ must be closer to zero, indicating that the case is actually a close call. The defendant will then take this new information into account in deciding whether to accept or reject the plaintiff's demand.

Let $b(\tau)$ denote the defendant's belief as to the plaintiff type when the plaintiff demands $\tau$. A rational defendant will accept the plaintiff's settlement demand if and only if the demand is less than or equal to the defendant's expected loss were the case to go to trial.[28] Where $b(\tau)$ is a point belief, the defendant will accept if and only if

$$S \leqslant \Phi\left(\frac{b(\tau)+y_d}{\sqrt{2}\sigma}\right)\theta_H + \left(1 - \Phi\left(\frac{b(\tau)+y_d}{\sqrt{2}\sigma}\right)\right)\theta_L + C_d.$$

This condition is equivalent to the following.

*The defendant's condition for accepting the plaintiff's settlement demand under a point belief*

$$\tau \leqslant \Phi\left(\frac{b(\tau)+y_d}{\sqrt{2}\sigma}\right). \tag{A2}$$

Define $\gamma_{y_p}(z)$ as the $y_d$ value, such that given the defendant's inference of the plaintiff's signal to be $y_p$, the probability that the case is of high merit is exactly $z$. In other words, $\gamma_{y_p}(z)$ is defined implicitly as follows: $\Pr(Y > 0 | Y_d = \gamma_{y_p}(z), Y_p = y_p) = \Phi\left(\frac{y_p + \gamma_{y_p}(z)}{\sqrt{2}\sigma}\right) = z$. We can then rewrite the defendant's strategy from Equation (A2) as follows: where $b(\tau)$ is a point belief, the defendant will accept $\tau$ if and only if $y_d \geqslant \gamma_{b(\tau)}(\tau)$. In short, $\gamma_{b(\tau)}(\tau)$ is the minimum $y_d$ that will persuade the defendant to accept the settlement demand of $\tau$ when he is acting under the belief function $b(\tau)$. For values of $\tau$ that correspond to a unique $y_p$, we can define $\gamma_{b(\tau)}(\tau)$ implicitly using Equation (A1)

$$\tau = \Phi\left(\frac{\gamma_{b(\tau)}(\tau)+b(\tau)}{\sqrt{2}\sigma}\right). \tag{A3}$$

28. We assume that if the defendant is indifferent between settling and going to trial, he will settle.

It follows that $\gamma_{b(\tau)}(\tau) = \sqrt{2}\sigma\Phi^{-1}(\tau) - b(\tau)$, where $\Phi^{-1}(\cdot)$ is the inverse cumulative distribution function. Differentiating this last expression with respect to $\tau$, we get $\frac{d\gamma_{b(\tau)}(\tau)}{d\tau} = \frac{\sqrt{2}\sigma}{\varphi(\Phi^{-1}(\tau))} - b'(\tau)$. Therefore, we have shown the following relationship between the defendant's belief function and the lowest defendant type that would accept a settlement demand of $\tau$.

LEMMA A2. *When $g_Y(x) = 1$ and for values of $\tau$ that correspond to a unique $y_p$, $\gamma_{b(\tau)}(\tau) = \sqrt{2}\sigma\Phi^{-1}(\tau) - b(\tau)$ and $\frac{d\gamma_{b(\tau)}(\tau)}{d\tau} = \frac{\sqrt{2}\sigma}{\varphi(\Phi^{-1}(\tau))} - b'(\tau)$.*

*The Plaintiff's Expected Utility Function and the First-Order Condition.* We now consider the plaintiff's expected utility function given her type, her settlement demand, and the defendant's belief function, $b(\tau)$. Let $F_{Y_d|Y_p=y_p}$ be the cumulative conditional distribution of $Y_d$ given $Y_p = y_p$. The expected payoff to the plaintiff who makes a settlement demand $\tau$ is given by the following equation.

*The plaintiff's expected utility function*

$$\begin{aligned} &U_p\left(y_p, \tau; b(\tau)\right) \\ &\quad = F_{Y_d|Y_p=y_p}\left(\gamma_{b(\tau)}(\tau)\right)\left[\Pr(Y > 0 \mid Y_p = y_p,\, Y_d < \gamma_{b(\tau)}(\tau))\theta_H \right. \\ &\qquad \left. + \left(1 - \Pr\left(Y > 0 \mid Y_p = y_p,\, Y_d < \gamma_{b(\tau)}(\tau)\right)\right)\theta_L - C_p\right] \\ &\qquad + \left(1 - F_{Y_d|Y_p=y_p}\left(\gamma_{b(\tau)}(\tau)\right)\right)S. \end{aligned} \tag{A4}$$

In the first line of Equation (A4), the first term on the right-hand side represents the plaintiff's expectation if the case goes to trial (because the defendant rejected the plaintiff's settlement demand) and the second term represents the plaintiff's expectation if the case settles. After some algebra, Equation (A4) can be rewritten as:

$$\begin{aligned} &U_p\left(y_p, \tau; b(\tau)\right) \\ &\quad = (\theta_H - \theta_L)\left[\left(\int_{-\infty}^{\gamma_{b(\tau)}(\tau)} \Phi\left(\frac{y_p + y_d}{\sqrt{2}\sigma}\right) f_{Y_d|Y_p=y_p}(y_d)\mathrm{d}y_d\right)\right. \\ &\qquad \left. + \left(1 - F_{Y_d|Y_p=y_p}\left(\gamma_{b(\tau)}(\tau)\right)\right)(\tau + \kappa)\right] + \left(\theta_L - C_p\right), \end{aligned} \tag{A5}$$

where $\kappa \equiv \frac{C_p + C_d}{\theta_H - \theta_L}$ is the ratio between the aggregate litigation costs and the difference in expected damages for the high merit case and the low merit case for the plaintiff. The denominator of $\kappa$ indicates the true amount at stake for the parties in going to trial. If $\kappa \geqslant 1$, we shall say litigation is *expensive* and if $\kappa < 1$, we shall say litigation is *inexpensive*.

Let $\zeta^*(y_p)$ denote an interior solution maximizing the plaintiff's expected utility function under Equation (A5). Because the plaintiff's expected payoff under $\zeta^*(y_p)$ may be lower than one under the corner solution ($\tau = 0$), we reserve $s^*(y_p)$ to denote the plaintiff's optimal strategy that accounts for this corner solution. Likewise, we let $\beta^*(\cdot)$ be the belief consistent with $\zeta^*(y_p)$, and use $b^*(\cdot)$ to denote the belief consistent with $s^*(y_p)$. The first-order condition derived from the plaintiff's expected utility must equal zero when $\tau = \zeta^*(y_p)$. We differentiate Equation (A5) with respect to $\tau$ and plug in $\Phi\left(\frac{\gamma_{b(\tau)}(\tau)+b(\tau)}{\sqrt{2}\sigma}\right) = \tau$ and $b(\tau) = \beta^*(\tau)$. Evaluating the expression at $\tau = \zeta^*(y_p)$ and setting it equal to zero, we obtain the following condition[29]:

$$1 - F_{Y_d|Y_p=y_p}\left(\gamma_{\beta^*(\zeta^*(y_p))}\left(\zeta^*(y_p)\right)\right)$$
$$= \kappa f_{Y_d|Y_p=y_p}\left(\gamma_{\beta^*(\zeta^*(y_p))}\left(\zeta^*\left(y_p\right)\right)\right)\left(\frac{\mathrm{d}\gamma_{\beta^*(\tau)}(\tau)}{\mathrm{d}\tau}|_{\tau=\zeta^*(y_p)}\right). \quad \text{(A6)}$$

The following lemma derives the differential equation defining the solution to Equation (A6).

LEMMA A3. *The solution $\zeta^*(y_p)$ to Equation (A6) satisfies the following differential equation:*

$$\frac{\mathrm{d}\zeta^*(y_p)}{\mathrm{d}y_p} = \left(\frac{\varphi\left(\Phi^{-1}\left(\zeta^*(y_p)\right)\right)}{\sqrt{2}\sigma}\right)\left[1 - \left(\frac{\varphi\left(\Phi^{-1}\left(\zeta^*(y_p)\right)\right)}{\kappa h\left(\Phi^{-1}\left(\zeta^*(y_p)\right) - \frac{\sqrt{2}y_p}{\sigma}\right)}\right)\right]^{-1}, \quad \text{(A7)}$$

*where $h(\cdot)$ is the hazard rate for the standard normal distribution.*

*Proof of Lemma A3.* We begin with a change of variables. Define $Y_1 = \frac{Y}{\sigma}$, $y_1 = \frac{y}{\sigma}$, $U = \frac{Y_p}{\sigma}$, $u = \frac{y_p}{\sigma}$, $V = \frac{Y_d}{\sigma}$, $v = \frac{y_d}{\sigma}$, $u^0 = \frac{y^0}{\sigma}$, $b_1(\tau) = \frac{b(\tau)}{\sigma}$,

29. We verified the second-order condition using *Mathematica* simulations. In all cases we checked, the second-order condition was locally satisfied, and the stationary point proved to be unique, indicating that the solution we found is the global interior maximum—that is, away from the corner solution.

and $\eta_{b_1(\tau)}(\tau) = \frac{\gamma_{b(\tau)}(\tau)}{\sigma}$. In addition, define $n(u) = \frac{\Phi^{-1}(\zeta^*(\sigma u))}{\sqrt{2}}$. This means $\zeta^*(y_p) = \zeta^*(\sigma u) = \Phi\big(\sqrt{2}n(u)\big)$ and $\zeta^{*\prime}(y_p) = \frac{d\zeta^*(y_p)}{\mathrm{d}y_p} = \Big(\frac{\sqrt{2}\varphi(\sqrt{2}n(u))}{\sigma}\Big)\, n'(u)$. Note also that if $g_Y(x) = 1, f_{Y_d|Y_p=y_p}(x) = \int_{-\infty}^{\infty} f_{Y_d|Y=y}(x) f_{Y=y|Y_p=y_p}(y)\mathrm{d}y = \frac{1}{\sqrt{2}\sigma}\varphi\left(\frac{x-y_p}{\sqrt{2}\sigma}\right)$ and $F_{Y_d|Y_p=y_p}(x) = \Phi\left(\frac{x-y_p}{\sqrt{2}\sigma}\right)$. Since $\frac{d\zeta^*(y_p)}{\mathrm{d}y_p} = \left(\frac{\sqrt{2}\varphi(\sqrt{2}n(u))}{\sigma}\right) n'(u)$, $\varphi\left(\Phi^{-1}(\zeta^*(y_p))\right) = \varphi\left(\sqrt{2}n(u)\right)$, and $h\left(\Phi^{-1}(\zeta^*(y_p)) - \frac{\sqrt{2}y_p}{\sigma}\right) = h\left(\sqrt{2}(n(u)-u)\right)$, in order to establish Equation (A7), it suffices to show that $n(u)$ is defined by the following $\sigma$-independent differential equation:

$$\begin{aligned} n'(u) &= \frac{1}{2}\left(1 - \frac{\varphi\left(\sqrt{2}n(u)\right)}{\kappa h\left(\sqrt{2}\,(n(u)-u)\right)}\right)^{-1} \\ &= \frac{1}{2} + \frac{1}{2}\left(\frac{\Phi(\sqrt{2}\,(u-n(u)))}{\kappa e^{2un(u)-u^2} - \Phi(\sqrt{2}\,(u-n(u)))}\right). \end{aligned} \tag{A8}$$

To show this, we begin with Equation (A6). Note that in equilibrium, beliefs must be consistent. Thus, if the equilibrium were to be fully separating, $b(\zeta^*(y_p)) = y_p$. This means

$$\begin{aligned} &\Pr(Y > 0|Y_p = y_p, Y_d = \gamma_{b(\zeta^*(y_p))}\left(\zeta^*\left(y_p\right)\right)) \\ &= \Pr(Y > 0|Y_p = y_p, Y_d = \gamma_{y_p}\left(\zeta^*\left(y_p\right)\right)) \\ &= \zeta^*\left(y_p\right) \end{aligned}$$

by Equation (A3). Thus, we have

$$\begin{aligned} \left(\kappa\left(\frac{\mathrm{d}\gamma_{b(\tau)}(\tau)}{\mathrm{d}\tau}|_{\tau=\zeta^*(y_p)}\right)\right)^{-1} &= \frac{f_{Y_d|Y_p=y_p}\left(\gamma_{y_p}\left(\zeta^*\left(y_p\right)\right)\right)}{1 - F_{Y_d|Y_p=y_p}\left(\gamma_{y_p}\left(\zeta^*\left(y_p\right)\right)\right)} \\ &= \frac{1}{\sqrt{2}\sigma}\left[\frac{\varphi\left(\frac{\gamma_{y_p}(\zeta^*(y_p))-y_p}{\sqrt{2}\sigma}\right)}{1-\Phi\left(\frac{\gamma_{y_p}(\zeta^*(y_p))-y_p}{\sqrt{2}\sigma}\right)}\right] \\ &= \frac{1}{\sqrt{2}\sigma} h\left(\frac{\gamma_{y_p}\left(\zeta^*\left(y_p\right)\right) - y_p}{\sqrt{2}\sigma}\right). \end{aligned} \tag{A9}$$

By Lemma A2, $\frac{d\gamma_{b(\tau)}(\tau)}{d\tau}|_{\tau=\zeta^*(y_p)} = \frac{\sqrt{2}\sigma}{\varphi(\Phi^{-1}(\zeta^*(y_p)))} - b'(\tau)|_{\tau=\zeta^*(y_p)}$. Note that $b'(\tau)|_{\tau=\zeta^*(y_p)} = \frac{1}{\zeta^{*\prime}(y_p)}$ at equilibrium as long as there is a unique $y_p$

whose equilibrium demand is $\zeta^*\left(y_p\right)$. This is because at equilibrium, we have $b(\zeta^*\left(y_p\right)) = y_p$ by belief consistency. Differentiating both sides by $y_p$, we get $b'(\zeta^*\left(y_p\right))\left(\zeta^{*\prime}\left(y_p\right)\right) = 1$. Therefore,

$$\left(\frac{\mathrm{d}\gamma_{b(\tau)}(\tau)}{\mathrm{d}\tau}|_{\tau=\zeta^*(y_p)}\right) = \frac{\sqrt{2}\sigma}{\varphi\left(\Phi^{-1}\left(\zeta^*\left(y_p\right)\right)\right)} - \frac{1}{\zeta^{*\prime}\left(y_p\right)}. \tag{A10}$$

Equation (A10) can be rewritten as

$$\left(\frac{\mathrm{d}\gamma_{b(\tau)}(\tau)}{\mathrm{d}\tau}|_{\tau=\zeta^*(y_p)}\right) = \frac{\sqrt{2}\sigma}{\varphi\left(\sqrt{2}n(u)\right)}\left(1 - \frac{1}{2n'(u)}\right). \tag{A11}$$

Similarly,

$$\begin{aligned}\frac{\gamma_{y_p}\left(\zeta^*(y_p)\right) - y_p}{\sqrt{2}\sigma} &= \frac{\left(\sqrt{2}\sigma\,\Phi^{-1}\left(\zeta^*\left(y_p\right)\right) - b\left(\zeta^*(y_p)\right)\right) - y_p}{\sqrt{2}\sigma}\\ &= \frac{\sqrt{2}\sigma\,\Phi^{-1}\left(\zeta^*\left(y_p\right)\right) - 2y_p}{\sqrt{2}\sigma}\\ &= \sqrt{2}(n(u) - u).\end{aligned}$$

Therefore, Equation (A9) becomes

$$\left(\frac{\sqrt{2}\kappa\sigma}{\varphi\left(\sqrt{2}n(u)\right)}\left(1 - \frac{1}{2n'(u)}\right)\right)^{-1} = \frac{1}{\sqrt{2}\sigma}h\left(\sqrt{2}(n(u) - u)\right),$$

which simplifies to Equation (A8). The lemma is proved. □

*The Plaintiff's Optimal Settlement Demand Strategy (Interior Solutions).* Equation (A7) admits a family of solutions depending on the boundary value $\zeta^*(0) = c$. For example, $c = \frac{1}{2}$ means that the plaintiff who observes 0 asks for $\frac{1}{2}(\theta_H - \theta_L) + (C_d + \theta_L)$. We will use $\zeta^*_{\kappa,c}\left(y_p\right)$ to denote the solution to Equation (A7) when $\zeta^*(0) = c$ and given $\kappa$. Likewise, let $\beta^*_{\kappa,c}(\tau)$ be the consistent belief function. (Without any confusion, we will continue to use $\zeta^*\left(y_p\right)$ when the specific parameter value and the initial condition are not germane to the discussion.) Although there is no closed-form solution for $\zeta^*_{\kappa,c}\left(y_p\right)$, we can show the following properties about $\zeta^*_{\kappa,c}\left(y_p\right)$.

LEMMA A4. The Plaintiff's Optimal Settlement Demand Strategy (Interior Solutions). *For all positive parameter values of* $\kappa$ *and suitable boundary values, there exists a unique solution to Equation (A7), which is continuously differentiable and strictly increasing in the plaintiff type. Specifically, the following statements are true.*

(a) *When litigation is expensive* ($\kappa \geqslant 1$), *for each* $\kappa \in [1, \infty)$ *and each* $c \in (0, 1)$, *there exists a unique and well-defined* $\zeta^*_{\kappa,c}\left(y_p\right)$;
(b) *when litigation is inexpensive* ($\kappa < 1$), *for each* $\kappa \in (0, 1)$, *there exists* $c_\kappa \in (0, 1)$ *such that there exists a unique and well-defined* $\zeta^*_{\kappa,c}\left(y_p\right)$ *for each* $c \in [c_\kappa, 1)$, *but not for* $c \in (0, c_\kappa)$;
(c) *each* $\zeta^*_{\kappa,c}\left(y_p\right)$ *is continuously differentiable and strictly increasing in* $y_p$; *and*
(d) $\lim_{y_p \to \infty} \zeta^*_{\kappa,c}\left(y_p\right) = 1$ *and* $\lim_{y_p \to -\infty} \zeta^*_{\kappa,c}\left(y_p\right) = 0$.

Lemma A4 states that when litigation is expensive, a solution to Equation (A7) exists for any $c \in (0, 1)$. By contrast, when litigation is inexpensive, $c$ cannot be too small (too close to 0).[30] Thus, if we let $\Gamma_1 \equiv \{(\kappa, c) \mid 0 < \kappa < 1,\ c_\kappa \leqslant c < 1\}$ and $\Gamma_2 \equiv \{(\kappa, c) \mid 1 \leqslant \kappa,\ 0 < c < 1\}$, then $\Gamma_1 \cup \Gamma_2$ is the set of $(\kappa, c)$ such that a unique and well-defined $\zeta^*_{\kappa,c}\left(y_p\right)$ exists. $\Gamma_1$ contains the inexpensive litigation cases and $\Gamma_2$ contains the expensive litigation cases. We will restrict our attention to $\Gamma_1 \cup \Gamma_2$. We include the proof of Lemma A4 in the Online Appendix.

Figure A1 plots $\zeta^*_{\kappa,c}\left(y_p\right)$ under two different boundary values. In the left panel, the plaintiff with type 0 demands $\tau = 0.5$ and in the right panel, she demands $\tau$ that is very close to zero ($\frac{\mathrm{erf}(-4)+1}{2}$, which is less than $10^{-8}$). Both graphs are strictly increasing even though the graph in the right panel looks flat to the left.

*The Plaintiff's Expected Utility under the (Interior) Optimal Settlement Demand Strategy.* Given $\zeta^*_{\kappa,c}\left(y_p\right)$, the interior optimal settlement demand for the plaintiff, we can now calculate the plaintiff's expected utility by

30. If the initial condition is too small, then the differential equation will not have a uniquely determined solution because of singularities.

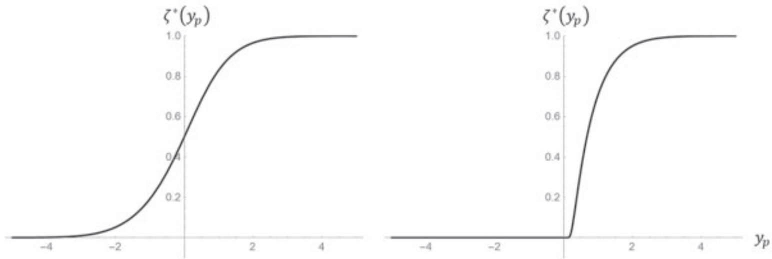


**Figure A1.** The Plaintiff's Optimal Settlement Demand $\zeta^*(y_p)$ (Interior Solution) as a Function of the Plaintiff's Signal ($\sigma = 1, C_p = C_d = 1/3 = \theta_L = 1 - \theta_H$, and $c = 0.5, \frac{\text{erf}(-4)+1}{2}$).

plugging in $\tau = \zeta^*_{\kappa,c}(y_p)$ and $b(\tau) = \beta^*_{\kappa,c}(\tau)$ into the plaintiff's utility function in Equation (A4). For $\gamma_{b(\tau)}(\tau)$, we plug in (per Lemma A2) $\gamma_{\beta^*_{\kappa,c}(\zeta^*_{\kappa,c}(y_p))}\left(\zeta^*_{\kappa,c}(y_p)\right) = \sqrt{2}\sigma\Phi^{-1}\left(\zeta^*_{\kappa,c}(y_p)\right) - \beta^*_{\kappa,c}\left(\zeta^*_{\kappa,c}(y_p)\right) = \sqrt{2}\sigma\Phi^{-1}\left(\zeta^*_{\kappa,c}(y_p)\right) - y_p$.

Figure A2 depicts the expected utility for two different boundary values. $c = 0.5$ for the left panel and $c = \frac{\text{erf}(-4)+1}{2}$ for the right panel. The graphs show that the plaintiff's expected utility is increasing in her type. The dashed horizontal lines are drawn at $C_d + \theta_L$, the plaintiff's payoff when $\tau = 0$. Even though the left side of the graph on the right panel appears to coincide with this payoff, this portion actually lies strictly under (but very close to) $C_d + \theta_L$ and the graph will eventually go down toward $\theta_L - C_p$ (which happens to be 0 in this case) as we move further to the left. Exactly how far to the left we must move before the graph goes down toward 0 will depend on the boundary value. Figure A2 shows that the plaintiff is better off demanding $\tau = 0$ when it receives a signal below some threshold $y^0$, where $y^0$ is the unique $y_p$ value at which the plaintiff's expected utility for demanding $\zeta^*\left(y_p\right)$ equals $C_d + \theta_L$, and this value is always greater than 0 when litigation is expensive. Above this signal, the plaintiff should demand $\zeta^*(y_p)$. In the Online Appendix, we analytically establish these features of the plaintiff's expected utility under $\zeta^*_{\kappa,c}(y_p)$ consistent with these graphs. We summarize the results in Lemma A5.

Lemma A5. The Plaintiff's Expected Utility under her Interior Optimal Settlement Demand. *When the plaintiff's settlement demand is characterized*

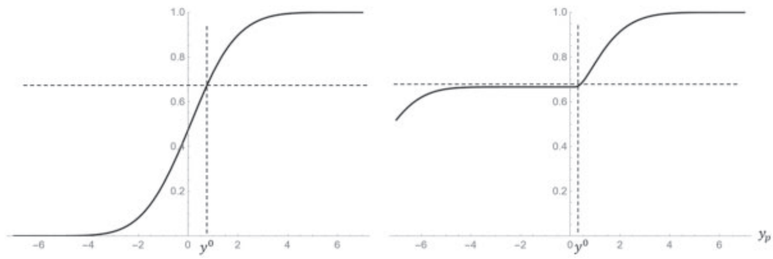


**Figure A2.** The Plaintiff's Expected Utility under $\zeta^*\left(y_p\right)$ ($\sigma = 1, C_p = C_d = 1/3 = \theta_L = 1 - \theta_H$, and $c = 0.5, \frac{\mathrm{erf}(-4)+1}{2}$).

*by Equation (A7), we have the following results. First, whether litigation is expensive or inexpensive, each plaintiff's expected payoff (according to the interior optimal settlement demand for a specified boundary condition) is strictly increasing in the plaintiff type. Second, the plaintiff's expected payoff will approach $C_d + \theta_H$ as the plaintiff type increases, and will approach $\theta_L - C_p$ as the plaintiff type decreases. Third, each plaintiff's expected payoff is strictly decreasing in the boundary value. In other words, the more aggressively each plaintiff type demands, the smaller will be the payoff. Fourth, the fraction of plaintiffs whose expected payoffs are smaller than the corner-solution payoff, $C_d + \theta_L$, will also be increasing in $c$. Fifth, when litigation is expensive, for the plaintiff types less than or equal to 0, their expected payoffs are smaller than the corner-solution payoff, $C_d + \theta_L$, and their payoffs will all increase and approach $C_d + \theta_L$ from below as the boundary value approaches 0.*

According to Lemma A5, as the plaintiff's type goes to positive infinity, her expected utility monotonically approaches $C_d + \theta_H$, which is equivalent to extracting all possible rents from the defendant (that is, $\tau = 1$). As the plaintiff's type gets low, her expected utility approaches $\theta_L - C_p$, which is equal to the expected value of going to trial for type negative infinity. Note that this value is strictly lower than $C_d + \theta_L$, which is the payoff the plaintiff can extract by simply demanding $\tau = 0$. For this reason, there cannot be any fully-separating equilibrium satisfying Equation (A7), as stated below in Corollary A1.

Corollary A1. *There is no fully separating equilibrium (that is everywhere differentiable).*

Lemma A5 also indicates that when litigation is expensive, the threshold plaintiff type will always be greater than 0. In other words, when litigation is expensive, a plaintiff who believes there is a 50–50 chance that his case is of high merit will prefer to demand $\tau = 0$. Finally, Lemma A5 says that as the boundary value increases, the expected payoff of every plaintiff type will decrease and the threshold plaintiff type—who is indifferent between demanding $\zeta^*_{\kappa,c}\left(y_p\right)$ and 0—will decrease as well.

*Perfect Bayesian Equilibria under the Plaintiff-Offer Model.* We are now ready to characterize a class of semi-pooling perfect Bayesian equilibria. But before we do that, we show that complete pooling equilibria do not exist.

Lemma A6. *Under the assumption that the support for any off-the-equilibrium-path belief is either a singleton or a continuous subset of* $\boldsymbol{R}$, *there does not exist any complete pooling equilibrium.*

*Proof of Lemma A6.* We first establish that if any complete pooling equilibrium existed, it must be at $\tau^* = 0$ only and for the case when litigation costs are expensive. Suppose there is a complete pooling equilibrium for some $\tau^* > 0$. This means that every plaintiff type demands $\tau^*$, and the demand is not informative for the defendant. Then each defendant will assume that the distribution of the plaintiff type is normal around his own signal. But then however low $\tau^*$ may be, as long as it is strictly positive, at some point, there will be a defendant whose type $y_d$ is so low (i.e., a very strong defendant) that he would still prefer to go to trial rather than settle at $\tau^*$. This is because with a complete pooling equilibrium, such a defendant will infer that the plaintiff types are distributed around his low signal. Thus, there exists $\widetilde{y_d}(\tau^*)$ such that all defendants whose types are lower than $\widetilde{y_d}(\tau^*)$ will want to go to trial when faced with the demand $\tau^*$. This means there exists a plaintiff whose type is sufficiently lower than $\widetilde{y_d}(\tau^*)$ that she believes most of the likely defendant types she will face (who she believes are distributed normally around her low $y_p$) will want to go to trial. Given her weak signal, she then has an incentive to deviate to $\tau = 0$. Therefore, if any complete pooling equilibrium existed, it must be at $\tau^* = 0$ only. This pooling, however, cannot be sustained if litigation costs are cheap since a very high type plaintiff will want to deviate, and her minimum expected payoff (i.e., if every defendant were to choose to go to trial with her) can still be chosen to be arbitrarily close to $\theta_H - C_p$, which will be greater than $C_d + \theta_L$ when litigation costs are cheap.

Suppose now that there exists a complete pooling equilibrium at $\tau^* = 0$ and litigation costs are expensive. Consider a possible point of deviation $\tau' \in (0, 1)$ and let $\Sigma$ be the support of $b(\tau')$. As mentioned in the main text, $\Sigma$ will not depend on defendant type. Rather, defendant type will affect $b(\tau')$, the specific distribution of plaintiff types over $\Sigma$. Specifically, for a defendant of type $y_d$, his belief $b(\tau')$ will either be a point belief (in which case, the belief is the same across all defendant types), or a truncated normal distribution, which is constructed by taking a normal distribution around $y_d$, truncating all areas lying outside $\Sigma$, and re-normalizing by dividing by the measure of $\Sigma$. We show that regardless of how $\Sigma$ is specified, as long as it is a continuous subset of $\boldsymbol{R}$, there will always be a threshold $y_d$ type, $\widetilde{y_d}(\tau')$, such that all defendants observing greater than $\widetilde{y_d}(\tau')$ prefer to settle with the demand $\tau'$ (given $b(\tau')$). This is straightforward if $\Sigma$ is bounded below. For example, if $\Sigma$ is bounded below by $y_{p,0} \in \boldsymbol{R}$, then the best outcome a defendant of type $y_d$ can do in going to trial against $\tau'$ is to assume that the plaintiff type is $y_{p,0}$. But even this best trial outcome will be dominated by settling for $\tau'$ when $y_d$ becomes sufficiently high. Suppose now $\Sigma$ is not bounded below. Then we have two cases: $\Sigma = \boldsymbol{R}$ or $\Sigma$ is bounded above. If $\Sigma = \boldsymbol{R}$, then the defendant's expected payout of going to trial will monotonically approach $\theta_H + C_d$ as $y_d$ goes to infinity, and thus, $\widetilde{y_d}(\tau')$ must exist. If $\Sigma$ is bounded above by, say, $y_{p,1} \in \boldsymbol{R}$, we need to show that the defendant's subjective probability that the case is high merit, $\Pr(Y > 0 | Y_p < y_{p,1}, Y_d = y_d)$, approaches 1 as $y_d$ goes to positive infinity. Specifically, we need to show that

$$\lim_{y_d \to \infty} \int_{-\infty}^{y_{p,1}} \Pr(Y > 0 | Y_p = y_p, Y_d = y_d) \left( \frac{\varphi\left(\frac{y_p - y_d}{\sqrt{2}\sigma}\right)}{\sqrt{2}\sigma\, \Phi\left(\frac{y_{p,1} - y_d}{\sqrt{2}\sigma}\right)} \right) \mathrm{d}y_p = 1.$$

To show this, let $u = y_p/\sigma$, $v = y_d/\sigma$, and $u_1 = y_{p,1}/\sigma$. Then,

$$\int_{-\infty}^{y_{p,1}} \Pr(Y > 0|Y_p = y_p, Y_d = y_d) \left( \frac{\varphi\left(\frac{y_p - y_d}{\sqrt{2}\sigma}\right)}{\sqrt{2}\sigma\,\Phi\left(\frac{y_{p,1}-y_d}{\sqrt{2}\sigma}\right)} \right) \mathrm{d}y_p$$

$$= \int_{-\infty}^{y_{p,1}} \Phi\left(\frac{y_p + y_d}{\sqrt{2}\sigma}\right) \left( \frac{\varphi\left(\frac{y_p - y_d}{\sqrt{2}\sigma}\right)}{\sqrt{2}\sigma\,\Phi\left(\frac{y_{p,1}-y_d}{\sqrt{2}\sigma}\right)} \right) \mathrm{d}y_p$$

$$= \frac{\int_{-\infty}^{u_1} \Phi\left(\frac{u+v}{\sqrt{2}}\right) \varphi\left(\frac{u-v}{\sqrt{2}}\right) \mathrm{d}u}{\sqrt{2}\Phi\left(\frac{u_1-v}{\sqrt{2}}\right)}$$

$$= \frac{\int_{-\infty}^{\frac{u_1-v}{\sqrt{2}}} \Phi\left(w + \sqrt{2}v\right) \varphi(w)\, \mathrm{d}w}{\Phi\left(\frac{u_1-v}{\sqrt{2}}\right)},$$

where $w = \frac{u-v}{\sqrt{2}}$. Then the desired limit is equal to $\lim_{v\to\infty}\left(\frac{\int_{-\infty}^{\frac{u_1-v}{\sqrt{2}}} \Phi\left(w+\sqrt{2}v\right)\varphi(w)\mathrm{d}w}{\Phi\left(\frac{u_1-v}{\sqrt{2}}\right)}\right)$.

Both the numerator and the denominator go to zero. Using L'Hôpital's rule we have

$$\lim_{v\to\infty}\left(\frac{\int_{-\infty}^{\frac{u_1-v}{\sqrt{2}}} \Phi\left(w + \sqrt{2}v\right) \varphi(w)\, \mathrm{d}w}{\Phi\left(\frac{u_1-v}{\sqrt{2}}\right)}\right) = \frac{\lim_{v\to\infty} \frac{\mathrm{d}\left(\int_{-\infty}^{\frac{u_1-v}{\sqrt{2}}} \Phi\left(w+\sqrt{2}v\right)\varphi(w)\mathrm{d}w\right)}{\mathrm{d}v}}{\lim_{v\to\infty} -\frac{1}{\sqrt{2}}\varphi\left(\frac{u_1-v}{\sqrt{2}}\right)}$$

$$= \frac{\lim_{v\to\infty}\left(\int_{-\infty}^{\frac{u_1-v}{\sqrt{2}}} \sqrt{2}\varphi\left(w + \sqrt{2}v\right) \varphi(w)\, \mathrm{d}w - \frac{1}{\sqrt{2}}\Phi\left(\frac{u_1+v}{\sqrt{2}}\right)\varphi\left(\frac{u_1-v}{\sqrt{2}}\right)\right)}{\lim_{v\to\infty} -\frac{1}{\sqrt{2}}\varphi\left(\frac{u_1-v}{\sqrt{2}}\right)}$$

$$= \lim_{v\to\infty} \Phi\left(\frac{u_1+v}{\sqrt{2}}\right) - \sqrt{2}\lim_{v\to\infty}\left(\frac{\int_{-\infty}^{\frac{u_1-v}{\sqrt{2}}} \sqrt{2}\varphi\left(w + \sqrt{2}v\right) \varphi(w)\, \mathrm{d}w}{\varphi\left(\frac{u_1-v}{\sqrt{2}}\right)}\right).$$

The first term clearly goes to 1. So it suffices to prove that the second term vanishes. Note that the numerator of the fraction simplifies as follows:

$$\int_{-\infty}^{\frac{u_1-v}{\sqrt{2}}} \sqrt{2}\varphi\left(w+\sqrt{2}v\right)\varphi\left(w\right)\mathrm{d}w$$

$$=\int_{-\infty}^{\frac{u_1-v}{\sqrt{2}}} \sqrt{2}\varphi\left(v+\sqrt{2}w\right)\varphi\left(v\right)\mathrm{d}w=\varphi\left(v\right)\int_{-\infty}^{u_1}\varphi\left(z\right)\mathrm{d}z$$

$$=\varphi\left(v\right)\Phi\left(u_1\right),$$

where $z = v + \sqrt{2}w$. Thus, the fraction inside the parentheses becomes $\frac{\varphi(v)\Phi(u_1)}{\varphi\left(\frac{u_1-v}{\sqrt{2}}\right)} = \Phi\left(u_1\right)e^{\frac{u_1^2}{4}-\frac{u_1 v}{2}-\frac{v^2}{4}}$, which clearly vanishes as $v$ approaches infinity. □

Proposition 1. Perfect Bayesian Equilibria under the $P$-Model. *In the* P-*model, for each* $\left(C_p, C_d, \theta_L, \theta_H\right) \in (0,1)^2 \times [0,1) \times (0,1]$ *such that* $0 \leqslant \theta_L < \theta_H \leqslant 1$, *there is a one-parameter class of semi-pooling perfect Bayesian equilibria, each of which satisfies the following. First, the plaintiff demands* $C_d + \theta_L$ *for all* $y_p \leqslant y^0$ *(where* $y^0$ *is the unique* $y_p$ *value at which the expected utility of the plaintiff observing* $y_p$ *and demanding according to the interior solution would equal* $C_d + \theta_L$*).*[31] *Second, the plaintiff's demand is characterized by a jump discontinuity at* $y_p = y^0$ *and increases continuously for* $y_p > y^0$*. The defendant holds the following beliefs: for* $S \in (C_d+\theta_L, C_d+\theta_H)$*, the defendant has a point belief that is consistent with the interior optimal demand strategy; for* $S = C_d+\theta_L$ *the defendant's belief is a truncated normal probability distribution, which is zero for* $y_p > y^0$ *but takes on* $\frac{\varphi_1\left(\frac{y_p-y_d}{\sqrt{2}\sigma}\right)}{\sqrt{2}\sigma\Phi\left(\frac{y^0-y_d}{\sqrt{2}\sigma}\right)}$ *for* $y_p \leqslant y^0$*, where* $\Phi(\cdot)$ *is the standard normal cumulative distribution function; for* $S \in (-\infty, C_d+\theta_L) \cup [C_d+\theta_H, \infty)$*, virtually any belief is possible because this strategy is strictly dominated for all plaintiff types.*

*Proof of Proposition 1.* Specifically, we show that for every $(\kappa, c) \in \Gamma_1 \cup \Gamma_2$, there exists a semi-pooling perfect Bayesian equilibrium that

31. We assume that if the plaintiff is indifferent between demanding $\theta_L + C_d$ or some amount greater than this value, then she will demand $\theta_L + C_d$.

satisfies the following. First, $s^*_{\kappa,c}\left(y_p\right) = 0$ for all $y_p \leqslant y^0_{\kappa,c}$ and $\zeta^*_{\kappa,c}\left(y_p\right)$ for all $y_p > y^0_{\kappa,c}$. Given the plaintiff's equilibrium settlement demand, the defendant's belief function follows immediately. To show the plaintiff's equilibrium settlement demand, note that a plaintiff whose type $y_p$ is greater than $y^0$ will (i) not want to deviate to $\tau \leqslant 0$ since her expected utility is higher under $\zeta^*\left(y_p\right)$ by construction, (ii) not mimic other plaintiff types on the equilibrium by choosing some other $\tau \in (0, 1)$ because the first-order condition was satisfied at $\tau = \zeta^*\left(y_p\right)$, and (iii) not deviate to $\tau \geqslant 1$ because that will lead to trial with certainty and thus to lower expected utility. A plaintiff whose type $y_p$ is less than or equal $y^0$ will necessarily choose $\tau = 0$ (which will be accepted with certainty). That strategy dominates any $\tau \in (-\infty, 0) \cup [1, \infty)$ because the plaintiff will end up with a higher settlement amount. It will also dominate $\tau = \zeta^*\left(y_p\right)$ by construction (see Figure A2), which in turn dominates any other $\tau \in (0, 1)$ because the first-order condition was satisfied at $\tau = \zeta^*\left(y_p\right)$. The defendant's belief function is consistent on the equilibrium path and he is minimizing his payout. □

*Equilibrium Refinement.* In this section, we prove the following lemma.

LEMMA A7 (Equilibrium Refinement).

(a) *Each semi-pooling equilibrium corresponding to* $(\kappa, c) \in \Gamma_1 \cup \Gamma_2$ *survives the "intuitive" criterion under Cho and Kreps (1987)*;
(b) *none of them survive D1 under Banks and Sobel (1987);*
(c) *only the equilibrium corresponding to* $(\kappa, c_\kappa)$, *where* $\kappa \in (0, 1)$ *and* $c_\kappa$ *is the lowest feasible boundary value for such* $\kappa$, *is "undefeated" under Mailath et al. (1993).*

*Proof of Lemma A7.* Given a plaintiff's demand of $\tau \in (0, \zeta^*\left(y^0\right))$, we first consider the possibility of all defendant types simply accepting, which would be the best possible outcome for any plaintiff who chooses to demand $\tau$. Note that even in such a scenario, there will still be some very strong plaintiff types (types *much* higher than $y^0$) who would prefer to stay at their current equilibrium payoff and risk some trials. This is because according to Lemma A5, as $y_p$ increases, the plaintiff can extract an amount very close to $C_d + \theta_H$, which is equivalent to settling at $\tau$ very close to 1. Therefore, settling at any $\tau \in \left(0, \zeta^* \left(y^0\right)\right)$ (even with certainty) will be dominated by the equilibrium payoff for some very strong plaintiff types. Our off-the-equilibrium-path belief specification places zero weight on all such high plaintiff types. This is clear since the off-the-equilibrium-path belief places weights only on plaintiff types below $y^0$.

Now we ask whether there is some plaintiff type whose equilibrium payoff is lower than the minimum possible outcome from choosing $\tau \in (0, \zeta^* \left(y^0\right))$, which would be if all defendant types simply rejected (i.e., always ending in a trial against every defendant type). The answer here is no. No plaintiff type will ever prefer trial-against-every-defendant-type to what she is getting under the current equilibrium payoff, for the same reason that no plaintiff wishes to demand $\tau = 1$ from Lemma A1. Hence, the "intuitive" criterion is satisfied and (a) is proved.

To show (b), note that D1 would require the defendant to believe that any plaintiff making any off-the-equilibrium demand between $\tau = 0$ and $\tau = \zeta^*_{\kappa,c} \left(y^0\right)$will necessarily be the threshold plaintiff, $y_p = y^0$. The belief specification would therefore have to be modified. The question then is whether this threshold plaintiff, who at equilibrium is indifferent between demanding $\tau = 0$ or $\tau = \zeta^*_{\kappa,c} \left(y^0\right)$, will now have an incentive to deviate and demand $\tau \in \left(0, \zeta^*_{\kappa,c}\left(y^0\right)\right)$ *if* he is guaranteed that his type $y^0$ will be truthfully revealed to the defendant even when he deviates to an off-the-equilibrium demand? The answer turns out to be yes: such specification of the defendant's belief will in fact incentivize the threshold plaintiff to want to deviate to $\tau \in \left(0, \zeta^*_{\kappa,c}\left(y^0\right)\right)$. To see this, recall that the expected utility function for the threshold plaintiff is given by Equation (A5). But we now have a revised belief function, which is that $b(\tau) = y^0$ for all $\tau \in (0, \zeta^*_{\kappa,c} \left(y^0\right)]$. Thus, for this range of settlement demands, the expected utility function of plaintiff type $y^0$ can be rewritten as

$$U_p\left(y^0, \tau; y^0\right) = (\theta_H - \theta_L) \times \left[ \left( \int_{-\infty}^{\gamma_{y^0}(\tau)} \Phi\left(\frac{y^0 + y_d}{\sqrt{2}\sigma}\right) f_{Y_d|Y_p=y^0}(y_d) \mathrm{d}y_d \right) + \left(1 - F_{Y_d|Y_p=y^0}\left(\gamma_{y^0}(\tau)\right)\right)(\tau + \kappa) \right] + \left(\theta_L - C_p\right).$$

When we left-differentiate[32] this equation with respect to $\tau$, we get

$$\left(\frac{1}{\theta_H-\theta_L}\right)\frac{\mathrm{d}_- U_p\left(y^0,\tau;y^0\right)}{\mathrm{d}\tau}$$
$$=\left(1-F_{Y_d|Y_p=y^0}\left(\gamma_{y^0}(\tau)\right)\right)-\kappa f_{Y_d|Y_p=y^0}\left(\gamma_{y^0}(\tau)\right)\left(\frac{\mathrm{d}\gamma_{y^0}(\tau)}{\mathrm{d}\tau}\right).$$

Since $\Phi\left(\frac{y_p+\gamma_{y_p}(\tau)}{\sqrt{2}\sigma}\right)=\tau$, $\gamma_{y_p}(\tau)=\sqrt{2}\sigma\Phi^{-1}(\tau)-y_p$ and $\frac{\mathrm{d}\gamma_{y_p}(\tau)}{\mathrm{d}\tau}=\frac{\sqrt{2}\sigma}{\varphi\left(\Phi^{-1}(\tau)\right)}=\frac{\mathrm{d}\gamma_{b(\tau)}(\tau)}{\mathrm{d}\tau}+b'(\tau)$. Therefore, the above expression evaluated at $\tau=\zeta^*_{\kappa,c}\left(y^0\right)$ becomes

$$\left(\frac{1}{\theta_H-\theta_L}\right)\frac{\mathrm{d}_- U_p\left(y^0,\tau;y^0\right)}{\mathrm{d}\tau}$$
$$=\left(1-F_{Y_d|Y_p=y^0}\left(\gamma_{y^0}\left(\zeta^*_{\kappa,c}\left(y^0\right)\right)\right)\right)$$
$$-\kappa f_{Y_d|Y_p=y^0}\left(\gamma_{y_p}\left(\zeta^*_{\kappa,c}\left(y^0\right)\right)\right)\left(\frac{\mathrm{d}\gamma_{y^0}(\tau)}{\mathrm{d}\tau}\right)|_{\tau=\zeta^*_{\kappa,c}(y^0)}$$
$$=\left(1-F_{Y_d|Y_p=y^0}\left(\gamma_{y^0}\left(\zeta^*_{\kappa,c}\left(y^0\right)\right)\right)\right)$$
$$-\kappa f_{Y_d|Y_p=y^0}\left(\gamma_{y^0}\left(\zeta^*_{\kappa,c}\left(y^0\right)\right)\right)\left(\frac{\mathrm{d}\gamma_{\beta^*(\tau)}(\tau)}{\mathrm{d}\tau}+\frac{\mathrm{d}\beta^*(\tau)}{\mathrm{d}\tau}\right)|_{\tau=\zeta^*_{\kappa,c}(y^0)}$$
$$=A-\kappa f_{Y_d|Y_p=y^0}\left(\gamma_{y^0}\left(\zeta^*_{\kappa,c}\left(y^0\right)\right)\right)\frac{\mathrm{d}\beta^*(\tau)}{\mathrm{d}\tau}|_{\tau=\zeta^*_{\kappa,c}(y^0)},$$

where $\beta^*(\tau)$ is the equilibrium belief function from the interior solution and

$$A=\left(1-F_{Y_d|Y_p=y^0}\left(\gamma_{y^0}\left(\zeta^*_{\kappa,c}\left(y^0\right)\right)\right)\right)$$
$$-\kappa f_{Y_d|Y_p=y^0}\left(\gamma_{y^0}\left(\zeta^*_{\kappa,c}\left(y^0\right)\right)\right)\left(\frac{\mathrm{d}\gamma_{\beta^*(\tau)}(\tau)}{\mathrm{d}\tau}\right)|_{\tau=\zeta^*_{\kappa,c}(y^0)}.$$

32. We cannot right-differentiate this expression since it is only valid from the left-limit.

But by Equation (A6), $A = 0$. Therefore, the first-order condition becomes

$$\left(\frac{1}{\theta_H - \theta_L}\right) \frac{\mathrm{d}_- U_p\left(y^0, \tau; y^0\right)}{\mathrm{d}\tau}$$
$$= -\kappa f_{Y_d|Y_p=y^0}\left(\gamma_{y^0}\left(\zeta^*_{\kappa,c}\left(y^0\right)\right)\right) \frac{\mathrm{d}\beta^*(\tau)}{\mathrm{d}\tau}|_{\tau=\zeta^*_{\kappa,c}(y^0)},$$

which is negative since $\frac{\mathrm{d}\beta^*(\tau)}{\mathrm{d}\tau} > 0$. In other words, the left-derivative of the plaintiff's expected utility is negative, and therefore, the expected utility is decreasing in $\tau$ as we approach from the left. Therefore, the plaintiff who was originally indifferent between $\tau = 0$ and $\tau = \zeta^*_{\kappa,c}\left(y^0\right)$ will, under the revised belief specification, have an incentive to deviate to a demand value slightly smaller than $\zeta^*_{\kappa,c}\left(y^0\right)$. Therefore, we do not have an equilibrium.

To show (c), recall that Mailath et al. (1993) defines the undefeated equilibrium as follows (p. 254).

Definition A1. $\sigma^* = (\mu^*, \rho^*, \beta^*)$ *is a pure strategy sequential equilibrium* if:

(D1.1) $\forall t \in T: \mu^*(t) \in \operatorname{argmax}_{m\in M} u(m, \rho^*(m), t)$;
(D1.2) $\forall m \in M: \rho^*(m) = BR(m, \beta^*(m))$;
(D1.3) $\forall t \in T$ and $m \in M: \beta^*(t|m) = p(t)\mu^*(m|t)/\sum_{t'\in T} p\left(t'\right)\mu^*(m|t')$ if the denominator is positive, where $\mu^*(m|t) = 1$ if $\mu^*(t) = m$ and 0 otherwise.

Here, the notations are standard. $\mu : T \to M$ is the sender's strategy, $\rho : M \to R$ is the receiver's response strategy, and $\beta : M \to \Delta_T$ is the receiver's belief function (where $\Delta_T$ is the set of all probability distributions on set $T$). $t \in T$ is sender's type, and $u(\sigma, t)$ is the sender type $t$'s payoff associated with $\sigma$ (with an abuse of notation), and $p(t)$ is the common knowledge prior probability of $t \in T$. Now denote the set of pure strategy sequential equilibria for the game $G$ by PSE$(G)$.

Definition A2. $\sigma \equiv (\mu, \rho, \beta) \in$ PSE$(G)$ *defeats* $\sigma' \equiv (\mu', \rho', \beta') \in$ PSE$(G)$ *if* $\exists m \in M$ *such that:*

(D2.1) $\forall t \in T: \mu'(t) \neq m$, *and* $K \equiv \{t \in T|\mu(t) = m\} \neq \emptyset$;

(D2.2) $\forall t \in K : u(\sigma, t) \geqslant u(\sigma', t)$, *and* $\exists t \in K : u(\sigma, t) > u(\sigma', t)$; and

(D2.3) $\forall t \in K : \beta'(t|m) \neq p(t)\pi(t)/\sum_{t' \in T} p(t')\pi(t')$ *for any* $\pi : T \rightarrow [0, 1]$ *satisfying* $t' \in K$ *and* $u(\sigma, t') > u(\sigma', t') \Rightarrow \pi(t') = 1$, *and* $t' \notin K \Rightarrow \pi(t') = 0$.

Under this set-up, a perfect sequential equilibrium $\sigma \in \mathrm{PSE}(G)$ is *undefeated* if there does not exist $\sigma' \in \mathrm{PSE}(G)$ that defeats $\sigma$.

In order to apply this refinement criterion to our game, we have to modify (D1.3) and (D2.3) slightly because once the respective signals are observed, the parties no longer have a common prior regarding the plaintiff type distribution. Therefore, we replace $p(t)$ with $f_{Y_p|Y_d=y_d}(t)$, which is the defendant's conditional probability density that the plaintiff is type $t$ after having observed signal $y_d$.

We now show that, given two semi-pooling equilibria with $(\kappa, c)$ and $(\kappa, c')$ where $c < c'$, the $(\kappa, c)$-equilibrium defeats the $(\kappa, c')$-equilibrium. In our semi-pooling equilibria, given a nontrivial settlement demand amount (i.e., $\tau > 0$), there is either a unique plaintiff type who would demand it or no plaintiff would demand it. Therefore, $\beta^*(t|m)$ will always be either 0 or 1.

To show (D2.1), note first that since $c < c'$, we have $y^0_{\kappa,c} < y^0_{\kappa,c'}$ (see proof of Lemma A5 in the Online Appendix). Now pick any $z \in (y^0_{\kappa,c}, y^0_{\kappa,c'})$. Then we have $\mu'(z) \equiv s^*_{\kappa,c'}(z) = 0$ but $\mu(z) \equiv s^*_{\kappa,c}(z) = \zeta^*_{\kappa,c}(z) > 0$. Meanwhile, $m = \zeta^*_{\kappa,c}(z) < \zeta^*_{\kappa,c}(y^0_{\kappa,c'}) < \zeta^*_{\kappa,c'}(y^0_{\kappa,c'})$, which implies that $m = \zeta^*_{\kappa,c}(z)$ is not a message that is observed in the $(\kappa, c')$-equilibrium since $\zeta^*_{\kappa,c'}(y^0_{\kappa,c'})$ is the smallest positive demand that is made under that equilibrium. Now let $K = \{z\}$, and (D2.1) is established. To show (D2.2), the first part is established by Lemma A5, and the second part is established since $u(\sigma, t) \equiv U_p\left(z, \zeta^*_{\kappa,c}(z); \beta^*_{\kappa,c}(\tau)\right) > U_p\left(y^0_{\kappa,c}, \zeta^*_{\kappa,c}\left(y^0_{\kappa,c}\right); \beta^*_{\kappa,c}(\tau)\right) = C_d + \theta_L = U_p\left(z, 0; \beta^*_{\kappa,c'}(\tau)\right) = U_p\left(z, s^*_{\kappa,c'}(z); \beta^*_{\kappa,c'}(\tau)\right) \equiv u\left(\sigma', t\right)$. Finally, to show (D2.3), note that since $K = \{z\}$, we will have $\pi(y_p) = 1$ if $y_p = z$ and $\pi(y_p) = 0$ for all other values of $y_p$. At this point, note that we also have

$$\beta^*(t|m) \equiv \beta^*\left(z|\zeta^*_{\kappa,c}(z)\right) = 1 = \frac{f_{Y_p|Y_d=y_d}(z)\,\mu^*\left(\zeta^*_{\kappa,c}(z)\,|\,z\right)}{\sum_{t' \in T} f_{Y_p|Y_d=y_d}(t')\,\mu^*\left(\zeta^*_{\kappa,c}(z)\,|\,t'\right)},$$

since $\zeta^*_{\kappa,c}(z)$ lies on the separating portion, and therefore $\mu^*\left(\zeta^*_{\kappa,c}(z) \mid t'\right) = 0$ for all $t'$ except for $t' = z$.

But $\beta'^*(t|m) \equiv \beta'^*\left(z|\zeta^*_{\kappa,c}(z)\right) = 0$ under our belief specification since ${\zeta^*_{\kappa,c'}}^{-1}\left(\zeta^*_{\kappa,c}(z)\right) \neq z$. And therefore,

$$\beta'^*(t|m) \equiv \beta'^*\left(z|\zeta^*_{\kappa,c}(z)\right) = 0 \neq 1 = \frac{f_{Y_p|Y_d=y_d}(z)\,\pi(z)}{\sum_{t'\in T}\beta^*\left(t'|\zeta^*_{\kappa,c}(z)\right)\pi(t')}.$$

So we have (D2.3). □

*The Probability of Rejection for Each Demand.* Let $\rho(\tau; y_p)$ be the probability that a plaintiff of type $y_p$ expects the defendant to reject a settlement demand of $\tau$. We can compute $\rho(\tau; y_p)$ as follows. Recall that a demand of $\tau = 0$ will be accepted by all defendant types, and a demand of $\tau = 1$ will be rejected by all defendant types. For $\tau \in (0, 1)$, the defendant will reject if and only if $y_d < \gamma_{b(\tau)}(\tau)$. Using Lemma A2, we can write this probability, conditional on the plaintiff's observing $y_p$, as follows:

$$\begin{aligned}\Pr\left(Y_d < \gamma_{b(\tau)}(\tau) \,|\, Y_p = y_p\right) &= F_{Y_d|Y_p=y_p}\left(\sqrt{2}\sigma\,\Phi^{-1}(\tau) - b^*_{\kappa,c}(\tau)\right)\\ &= \Phi\left(\frac{\sqrt{2}\sigma\,\Phi^{-1}(\tau) - {\zeta^*_{\kappa,c}}^{-1}(\tau) - y_p}{\sqrt{2}\sigma}\right).\end{aligned}$$

Therefore, we have

$$\rho\left(\tau; y_p\right) = \begin{cases} 0 \text{ if } \tau = 0, \\ \Phi\left(\frac{\sqrt{2}\sigma\,\Phi^{-1}(\tau) - {\zeta^*_{\kappa,c}}^{-1}(\tau) - y_p}{\sqrt{2}\sigma}\right) \text{ if } \tau \in (0,1), \text{ and} \\ 1 \text{ if } \tau = 1. \end{cases}$$

From the functional form, it is immediate that the higher the plaintiff type the lower the probability of rejection for demanding the same settlement amount $\tau$. It turns out that $\rho\left(\tau; y_p\right)$ is strictly increasing in $\tau$ but is characterized by jump discontinuities at both ends (at $\tau = 0$ and $\tau = 1$). Specifically, we have the following.

LEMMA A8. *$\rho\left(\tau; y_p\right)$ is strictly increasing but is characterized by jump discontinuities at both ends.*

*Proof of Lemma A8.* For $\tau \in (0,1)$, $\rho\left(\tau; y_p\right) = \Phi\left(\frac{\sqrt{2}\sigma\Phi^{-1}(\tau) - \zeta_{\kappa,c}^{*}{}^{-1}(\tau) - y_p}{\sqrt{2}\sigma}\right)$. If we substitute $\tau = \zeta_{\kappa,c}^{*}(y_p')$, it suffices to analyze the behavior of $\rho\left(\zeta_{\kappa,c}^{*}(y_p'); y_p\right)$ because $\tau$ is an increasing function in $y_p'$ (Lemma A4), and $\zeta_{\kappa,c}^{*}(y_p')$ spans all of $(0,1)$ as $y_p'$ goes from negative infinity to positive infinity (Lemma A4). Rewriting $y_p' = \sigma u'$, we have

$$\begin{aligned}
\rho\left(\tau; y_p\right) &= \rho\left(\zeta_{\kappa,c}^{*}(y_p'); y_p\right) \\
&= \rho\left(\zeta_{\kappa,c}^{*}(\sigma u'); \sigma u\right) \\
&= \Phi\left(\frac{\sqrt{2}\sigma\Phi^{-1}\left(\zeta_{\kappa,c}^{*}(\sigma u')\right) - \zeta_{\kappa,c}^{*}{}^{-1}\left(\zeta_{\kappa,c}^{*}(\sigma u')\right) - \sigma u}{\sqrt{2}\sigma}\right) \\
&= \Phi\left(\frac{\left(2n(u') - u'\right) - u}{\sqrt{2}}\right).
\end{aligned}$$

By Lemma A9, $2n(u') - u'$ is a strictly increasing function in $u'$ which is bounded above and below by two horizontal asymptotes. Therefore, $\rho\left(\tau; y_p\right)$ is strictly increasing in $\tau$, but for any fixed $u$, the argument, $\frac{(2n(u') - u') - u}{\sqrt{2}}$, will not approach either negative infinity or positive infinity in either direction, and hence $\rho\left(\tau; y_p\right)$ is discontinuous at both end points. □

*Selection Implications.* To analyze the selection implications, we first consider whether a particular pair of $(y_p, y_d)$ will go to trial, and then consider the probability that a dispute of merit $y$ will signal as $y_p$ to the plaintiff and $y_d$ to the defendant.

Note that since every defendant type will accept the settlement demand of $\tau = 0$, no plaintiff whose type is less than or equal to $y_{\kappa,c}^{0}$ will ever go to trial. In addition, according to Proposition 1, any equilibrium settlement demand greater than 0 must lie on the separating portion of the offer curve. Thus, if $\tau > 0$, then $\tau \in (\zeta_{\kappa,c}^{*}(y_{\kappa,c}^{0}), 1)$. On this portion of settlement demand, a dispute will go to trial if and only if $y_d < \gamma_{b(\tau)}(\tau) = \gamma_{b_{\kappa,c}^{*}(s_{\kappa,c}^{*}(y_p))}\left(s_{\kappa,c}^{*}\left(y_p\right)\right) = \sqrt{2}\sigma\Phi^{-1}\left(\zeta_{\kappa,c}^{*}\left(y_p\right)\right) - y_p$. Therefore, the set of $\left(y_p, y_d\right)$ that will result in a trial is as follows:

$$R_\sigma\left(y_p, y_d\right) = \left\{\left(y_p, y_d\right) | y_d < \sqrt{2}\sigma\Phi^{-1}\left(\zeta_{\kappa,c}^{*}\left(y_p\right)\right) - y_p \text{ and } y_p \geqslant y_{\kappa,c}^{0}\right\}.$$

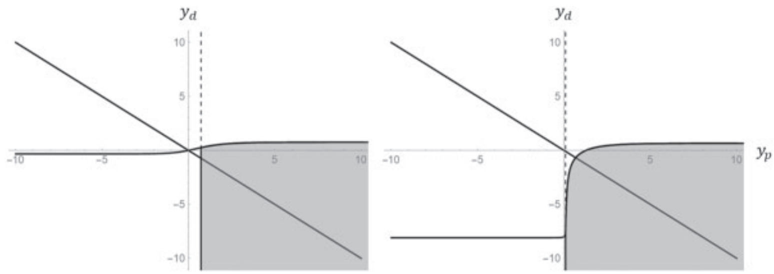


**Figure A3.** The Litigation Condition Set: $y_d \leqslant \sqrt{2}\sigma\,\Phi^{-1}\left(\zeta^*_{\kappa,c}\left(y_p\right)\right) - y_p$ and $y_p \geqslant y^0$ ($\sigma = 1, C_p = C_d = 1/3 = \theta_L = 1 - \theta_H$, and $c = 0.5$, $\frac{\text{erf}(-4)+1}{2}$).

We will call $R_\sigma\left(y_p, y_d\right)$ the *litigation condition set*.

Figure A3 depicts $R_\sigma\left(y_p, y_d\right)$ (the gray area) in the general $y_p y_d$-coordinate. $c = 0.5$ for the left panel and $c = \frac{\text{erf}(-4)+1}{2}$ for the right panel. The roughly horizontal curve in the left panel graph is defined by $y_d = \gamma_{y_p}\left(\zeta^*_{\kappa,c}\left(y_p\right)\right)$, and depicts the threshold-defendant type that would reject each plaintiff type's settlement demand. The same graph is depicted in the right panel; however, the graph dips below for the left side of the graph. In general, the right side of this graph will remain largely similar, but the left side, while remaining nearly flat, will dip below depending on the initial condition. As $c$ approaches zero, the lower asymptote defining the boundary of the left side of the graph will approach negative infinity. The dotted vertical line is drawn at $y_p = y^0_{\kappa,c}$. As is expected, litigation is more likely if $y_p$ is high (the plaintiff is confident) and $y_d$ is low (the defendant is confident), because in that case the mismatch between the plaintiff's settlement demand and the defendant's willingness to pay will be great.

The shape of the threshold-defendant curve merits some discussion. Consider first the following lemma. The proof is included in the Online Appendix.

Lemma A9. *In the separating portion of a perfect Bayesian equilibrium,* $2n(u) - u$ *is a strictly increasing function in* $u$, *and* $2n(u) - u$ *has a finite range for each initial condition* $n(0) \in \boldsymbol{R}$. *In addition,* $n(u) - u$ *approaches negative infinity as* $u$ *approaches positive infinity, and* $n(u) - u$ *approaches positive infinity as* $u$ *approaches negative infinity.*

Note that the condition $y_d < \sqrt{2}\sigma\Phi^{-1}\left(\zeta^*_{\kappa,c}\left(y_p\right)\right) - y_p$ is equivalent to $v < 2n(u) - u$. Thus, according to Lemma A9, the threshold-defendant curve is strictly increasing in $y_p$ but is bounded both above and below by horizontal asymptotes. The locus of the horizontal asymptotes will vary with $c$. As $c$ decreases, the left half of the graph will become more and more negative (while remaining nearly flat), but the right half of the graph remains largely unchanged (and remain mostly positive). In fact, as $c$ approaches 0, the left half of the graph can get arbitrarily negative, and thus, in the limit, the graph behaves as if it is defined only for $y_p > 0$. Meanwhile, regardless of $c$, $y^0_{\kappa,c}$ will always remain positive when litigation is expensive.

In other words, there is a threshold-defendant type, $y^1_d$, such that in equilibrium every defendant weaker than type $y^1_d$ will accept every settlement demand $0 \leq \tau < 1$, and a lower threshold defendant type, $y^0_d < y^1_d$, such that in equilibrium every defendant stronger than type $y^0_d$ will reject every nontrivial settlement demand. Part of the reason why we observe this shape is that the plaintiff's interior settlement demand, $\zeta^*\left(y_p\right)$, is itself characterized by two horizontal asymptotes.[33] In equilibrium, all plaintiffs must make settlement demands against the restriction that all demands $\tau \geqslant 1$ will be rejected with certainty and the demand of $\tau = 0$ will be accepted with certainty. Therefore, even as $y_p$ approaches infinity, high-type plaintiffs must make demands conservatively so as to maintain $\tau < 1$ and avoid certain rejections. In equilibrium, the plaintiff's conservative demand in turn motivates sufficiently weak defendants to accept all observed demands. By contrast, we see that sufficiently confident defendant types will reject all nontrivial settlement demands in equilibrium.

33. The fact that $\zeta^*\left(y_p\right)$ is characterized by two horizontal asymptotes indicates that $\frac{\mathrm{d}\zeta^*(y_p)}{\mathrm{d}y_p}$ will approach zero in either direction. Note that $\frac{\mathrm{d}\gamma_{y_p}\left(\zeta^*_{\kappa,c}(y_p)\right)}{\mathrm{d}y_p} = \left(\frac{\mathrm{d}\gamma_{\beta^*(\tau)}(\tau)}{\mathrm{d}\tau}\Big|_{\tau=\zeta^*(y_p)}\right)\left(\frac{\mathrm{d}\zeta^*(y_p)}{\mathrm{d}y_p}\right) = \left(\frac{1}{\kappa h_{Y_d|Y_p=y_p}\left(\gamma_{\beta^*(\zeta^*(y_p))}(\zeta^*(y_p))\right)}\right)\left(\frac{\mathrm{d}\zeta^*(y_p)}{\mathrm{d}y_p}\right)$, where $h_{Y_d|Y_p=y_p}(\cdot)$ is the hazard rate for $f_{Y_d|Y_p=y_p}(\cdot)$. Thus, as long as $\left(\frac{1}{\kappa h_{Y_d|Y_p=y_p}\left(\gamma_{\beta^*(\zeta^*(y_p))}(\zeta^*(y_p))\right)}\right)$ does not outpace $\frac{\mathrm{d}\zeta^*(y_p)}{\mathrm{d}y_p}$, $\frac{\mathrm{d}\gamma_{y_p}\left(\zeta^*_{\kappa,c}(y_p)\right)}{\mathrm{d}y_p}$ will likewise approach zero in either direction.

Figure A3 shows that the plaintiff trial win rate will be greater than 50%. This is because $R_\sigma(y_p, y_d)$ is *not* symmetric across the line $y_d = -y_p$ but is instead tilted to the first quadrant. In that region, both the plaintiff and the defendant are observing signals that are favorable to the plaintiff's *ex ante* probability of prevailing at trial. Because the signals are unbiased, cases above the line $y_d = -y_p$ are likely to result in plaintiff trial victories, while cases below the line are likely to result in defendant wins. Note that because $Y_p$ and $Y_d$ are unbiased, the density of any point on the graph is symmetric across the line $y_d = -y_p$. As a result, the fact that the portion of the shaded area above the line $y_d = -y_p$ is greater than the portion below indicates that the plaintiff will win more than 50% at trial. In addition, Figure A3 illustrates that extreme cases on both ends are likely to settle and close cases are more likely to go to trial. Given a particular case whose merit is $y$, the probability that it will go to trial is simply the probability that it will produce signals $y_p$ and $y_d$ which belong to $R_\sigma(y_p, y_d)$. The shape of $R_\sigma(y_p, y_d)$ shows that a *necessary* condition for a dispute to go to trial is that $y_p > y^0$ *and* $y_p < y_{d,1}$. But as $y$ becomes high, it is unlikely to produce $y_p < y_{d,1}$; likewise, as $y$ becomes low, it is unlikely to produce $y_p > y^0$.

Meanwhile, Figure A3 also explains that extremely weak and extremely strong cases are likely to settle for different reasons. Extreme cases in which $y$ is small will tend to settle because the litigation condition set is bounded by $y_p = y^0$. Thus, the corner solution plays an important role here. On the other hand, extreme cases to the right of 0 are likely to settle because, as we discussed, defendants who observe sufficiently weak signals will accept all equilibrium demands.

The probability that a dispute of merit $y$ will signal $(y_p, y_d)$ that belongs to $R_\sigma(y_p, y_d)$ can be calculated by placing a bivariate normal distribution above the planes in Figure A3 (centering it at $(y, y)$) and calculating the volume under the surface but above the shaded area only. See Lee and Klerman (2016). If we let $\Pi_\sigma(y)$ denote the probability that a dispute of merit $y$ will go to trial, we have

$$\Pi_\sigma(y) = \iint\limits_{R_\sigma(y_p, y_d)} \varphi_\sigma(y - y_p)\varphi_\sigma(y - y_d)\mathrm{d}y_p\mathrm{d}y_d.$$

Figure 3 in the main text plots $\Pi_\sigma(y)$. Observe that the graph of $\Pi_\sigma(y)$ is greater toward the right of 0, consistent with the observed asymmetry from Figure A3. Under Priest and Klein's model, the litigation probability function is symmetric around 0 (when the plaintiff and the defendant have the same amount at stake, as here), and symmetry is necessary to lead to the plaintiff trial win rate of 50% in the limit.[34] See Lee and Klerman (2016). Under our set up, although the plaintiff and the defendant face the same amount at stake, the litigation probability function is greater toward the right. Therefore, the plaintiff trial win rate will be higher than 50%. The precise amount will vary depending on the specific parameters. When $g_Y(x) = 1$, the plaintiff trial win rate can be calculated as $(\theta_H - \theta_L)\left(\frac{\int_0^\infty \Pi_\sigma(y)\mathrm{d}y}{\int_{-\infty}^\infty \Pi_\sigma(y)\mathrm{d}y}\right) + \theta_L$.

## The Defendant-Offer Model

LEMMA A10. *The interior solution $\zeta^*(y_d)$ to the defendant-offer model satisfies the following differential equation*:

$$\frac{\mathrm{d}\zeta^*(y_d)}{\mathrm{d}y_d} = \left(\frac{\varphi\left(\Phi^{-1}\left(\zeta^*\left(y_d\right)\right)\right)}{\sqrt{2}\sigma}\right)\left(1 - \frac{\varphi\left(\Phi^{-1}\left(\zeta^*\left(y_d\right)\right)\right)}{\kappa h\left(\frac{\sqrt{2}y_d}{\sigma} - \Phi^{-1}\left(\zeta^*\left(y_d\right)\right)\right)}\right)^{-1}. \tag{A12}$$

*Proof of Lemma A10.* Let $v = \frac{y_d}{\sigma}$ and $\zeta^*(y_d) = \Phi(\sqrt{2}m(v))$. We show that $m(v)$ is defined by the following $\sigma$-independent differential equation:

$$\frac{\mathrm{d}m(v)}{\mathrm{d}v} = \frac{1}{2}\left(1 - \frac{\varphi\left(\sqrt{2}m(v)\right)}{\kappa h\left(-\sqrt{2}\left(m(v) - v\right)\right)}\right)^{-1}. \tag{A13}$$

which is equivalent to Equation (A12).

We will use analogous notations without redefining them. Thus, $b(\tau)$ denotes the plaintiff's belief as to the defendant's type upon receiving a settlement offer of $\tau$. A rational plaintiff accepts the defendant's settlement offer if and only if the defendant's offer is greater than or equal to the plaintiff's expected judgment less cost of going to trial. That is, the plaintiff will accept if and only if

$$S \geqslant \theta_H \Phi\left(\frac{y_p + b(\tau)}{\sqrt{2}\sigma}\right) + \theta_L\left(1 - \Phi\left(\frac{y_p + b(\tau)}{\sqrt{2}\sigma}\right)\right) - C_p.$$

34. Note that although Priest and Klein assume $\theta_L = 1 - \theta_H = 0$, the 50% result will obtain as long as $\theta_L = 1 - \theta_H$.

This becomes: $\tau \geqslant \Phi\left(\frac{y_p+\mathrm{b}(\tau)}{\sqrt{2}\sigma}\right)$. Similarly as before, we let $\gamma_{b(\tau)}(\tau)$ be the threshold plaintiff type below which each plaintiff will accept an offer of $\tau$. The interim expected utility to the defendant who makes a settlement offer $\tau$ is given by the following equation:

$$U_d\left(y_d,\tau;\mathrm{b}(\tau)\right) = -\left(\theta_H-\theta_L\right)\times\left(\int_{\gamma_{b(\tau)}(\tau)}^{\infty}\Phi\left(\frac{y_p+y_d}{\sqrt{2}\sigma}\right) f_{Y_p|Y_d=y_d}\left(y_p\right)\mathrm{d}y_p \right.$$
$$\left. + F_{Y_p|Y_d=y_d}\left(\gamma_{b(\tau)}(\tau)\right)(\tau-\kappa)\right) - \left(\theta_L + C_d\right). \quad \text{(A14)}$$

Differentiating Equation (A14) with respect to $\tau$ using the Chain Rule and substituting $\Pr(Y>0 \mid Y_d = b(\tau),\ Y_p = \gamma_{b(\tau)}(\tau)) = \tau$, we obtain:

$$\frac{\mathrm{d}U_d\left(y_d,\tau;b(\tau)\right)}{\mathrm{d}\tau} = -\left(\theta_H-\theta_L\right)\times\left(f_{Y_p|Y_d=y_d}\left(\gamma_{b(\tau)}(\tau)\right)\left(\frac{\mathrm{d}\gamma_{b(\tau)}(\tau)}{\mathrm{d}\tau}\right)(-\kappa)\right.$$
$$\left. + F_{Y_p|Y_d=y_d}\left(\gamma_{b(\tau)}(\tau)\right)\right). \quad \text{(A15)}$$

From Equation (A15), we have

$$\left(\kappa\left(\frac{\mathrm{d}\gamma_{b(\tau)}(\tau)}{\mathrm{d}\tau}\right)\right)^{-1} = \frac{f_{Y_p|Y_d=y_d}\left(\gamma_{b(\tau)}(\tau)\right)}{F_{Y_p|Y_d=y_d}\left(\gamma_{b(\tau)}(\tau)\right)} = \frac{1}{\sqrt{2}\sigma}\frac{\varphi\left(\frac{\gamma_{b(\tau)}(\tau)-y_d}{\sqrt{2}\sigma}\right)}{\Phi\left(\frac{\gamma_{b(\tau)}(\tau)-y_d}{\sqrt{2}\sigma}\right)}$$
$$= \frac{1}{\sqrt{2}\sigma}h\left(\frac{y_d-\gamma_{b(\tau)}(\tau)}{\sqrt{2}\sigma}\right).$$

As before, $\left(\frac{\mathrm{d}\gamma_{b(\tau)}(\tau)}{\mathrm{d}\tau}|_{\tau=\zeta^*(y_d)}\right) = \frac{\sqrt{2}\sigma}{\varphi\left(\sqrt{2}m(v)\right)}\left(1-\frac{1}{2m'(v)}\right)$ and $\frac{y_d-\gamma_{y_d}(\zeta^*(y_d))}{\sqrt{2}\sigma} = \frac{y_d-(\sqrt{2}\sigma\Phi^{-1}(\zeta^*(y_d))-b(\zeta^*(y_d)))}{\sqrt{2}\sigma} = \frac{2y_d-\sqrt{2}\sigma\Phi^{-1}(\zeta^*(y_d))}{\sqrt{2}\sigma} = \sqrt{2}(v-m(v))$. So we have

$$\left(\frac{\sqrt{2}\sigma\kappa}{\varphi\left(\sqrt{2}m(v)\right)}\left(1-\frac{1}{2m'(v)}\right)\right)^{-1} = \frac{1}{\sqrt{2}\sigma}h\left(-\sqrt{2}(m(v)-v)\right),$$

which simplifies to Equation (A13). □

Proposition 2. The Symmetry Between the $P$-Model and the $D$-Model. *When the plaintiff and the defendant face identical litigation costs, the* *litigation probability function of the* D*-model will be the reflection of the litigation probability function from the* P*-model around* $y = 0$*. In other words, for each* $y \in \boldsymbol{R}$*, the probability that a case of merit* $y$ *will go to trial in the* P*-model is the same as the probability that a case of merit* $-y$ *will go to trial in the* D*-model. Furthermore, if, in addition,* $\theta_H = 1 - \theta_L$*, then the plaintiff trial win rate in the* D*-model is one minus the plaintiff trial win rate from the* P*-model.*

*Proof of Proposition 2.* Let $C_d = C_p = C$. It suffices to show that the litigation condition set (the region of integration) for one model is a reflection of the other across the line $v = -u$ in the $uv$-coordinate. Note first that given $n(x)$ which satisfies Equation (A8), $m(x) = -n(-x)$ will satisfy Equation (A13). This is clear since $\frac{dm(x)}{dx} = -\frac{dn(-x)}{dx} = \frac{1}{2}\left(1 - \frac{\varphi(\sqrt{2}n(-x))}{\kappa h(\sqrt{2}(n(-x)+x))}\right)^{-1} = \frac{1}{2}\left(1 - \frac{\varphi(-\sqrt{2}m(x))}{\kappa h(-\sqrt{2}(m(x)-x))}\right)^{-1} = \frac{1}{2}\left(1 - \frac{\varphi(\sqrt{2}m(x))}{\kappa h(-\sqrt{2}(m(x)-x))}\right)^{-1}$, and the last equation is Equation (A13). Now consider the region of integration for the plaintiff-offer model, $R_p(u,v)$ and the region for the defendant-offer mode, $R_d(u,v)$. Then $R_p(u,v) = \left\{(u,v) \in \boldsymbol{R}^2 \mid v < 2n(u) - u \text{ and } u \geqslant u^0\right\}$ and $R_d(u,v) = \left\{(u,v) \in \boldsymbol{R}^2 \mid u > 2m(v) - v \text{ and } v \leqslant v^0\right\}$. $u^0$ is defined implicitly by

$$
\begin{aligned}
(\theta_H - \theta_L) \times \Bigg[ & \frac{1}{\sqrt{2}} \int_{-\infty}^{2n(u^0)-u^0} \Phi\left(\frac{v + u^0}{\sqrt{2}}\right) \varphi\left(\frac{v - u^0}{\sqrt{2}}\right) \mathrm{d}v \\
& + \left(1 - \Phi\left(\sqrt{2}\left(n\left(u^0\right) - u^0\right)\right)\right)\left(\Phi\left(\sqrt{2}n\left(u^0\right)\right) + \kappa\right)\Bigg] \\
& + (\theta_L - C) = C + \theta_L
\end{aligned}
$$

and $v^0$ is defined implicitly by

$$
\begin{aligned}
-(\theta_H - \theta_L) \times \Bigg[ & \frac{1}{\sqrt{2}} \int_{2m(v^0)-v^0}^{\infty} \Phi\left(\frac{v^0 + u}{\sqrt{2}}\right) \varphi\left(\frac{v^0 - u}{\sqrt{2}}\right) \mathrm{d}u \\
& + \Phi\left(\sqrt{2}\left(m\left(v^0\right) - v^0\right)\right)\left(\Phi\left(\sqrt{2}m\left(v^0\right)\right) - \kappa\right)\Bigg] \\
& - (\theta_L + C) = C - \theta_{H.}
\end{aligned}
$$

We can rewrite these conditions as follows:

$$k\left(u^0\right) \equiv \frac{1}{\sqrt{2}} \int_{-\infty}^{2n\left(u^0\right)-u^0} \Phi\left(\frac{v+u^0}{\sqrt{2}}\right) \varphi\left(\frac{v-u^0}{\sqrt{2}}\right) \mathrm{d}v \\ + \left(1-\Phi\left(\sqrt{2}\left(n\left(u^0\right)-u^0\right)\right)\right)\left(\Phi\left(\sqrt{2}n\left(u^0\right)\right)+\kappa\right) = \kappa$$

and

$$\begin{aligned} l\left(v^0\right) \equiv & \frac{1}{\sqrt{2}} \int_{2m\left(v^0\right)-v^0}^{\infty} \Phi\left(\frac{v^0+u}{\sqrt{2}}\right) \varphi\left(\frac{v^0-u}{\sqrt{2}}\right) \mathrm{d}u \\ & + \Phi\left(\sqrt{2}\left(m\left(v^0\right)-v^0\right)\right)\left[\Phi\left(\sqrt{2}m\left(v^0\right)\right)-\kappa\right] \\ = & 1-\kappa . \end{aligned}$$

We show that if $(u, v) \in R_p(u, v)$, then $(-v, -u) \in R_d(u, v)$. If $(u, v) \in R_p(u, v)$, then $v < 2n(u) - u$ and $u \geqslant u^0$. $v < 2n(u) - u = -2m(-u) - u$. Therefore, $-v > 2m(-u) - (-u)$. This last inequality shows that $(-v, -u)$ satisfies the first of the two constraints to belong to $R_d(u, v)$. To show that it also satisfies the second constraint, we must show that $-u \leqslant v^0$. Since we already know that $u \geqslant u^0$, it suffices to show that $u^0 = -v^0$. It suffices to show that $k\left(-v^0\right) = \kappa$ if and only if $l\left(v^0\right) = 1 - \kappa$. This is true because

$$\begin{aligned} \kappa = k\left(-v^0\right) = & \frac{1}{\sqrt{2}} \int_{-\infty}^{2n\left(-v^0\right)+v^0} \Phi\left(\frac{v-v^0}{\sqrt{2}}\right) \varphi\left(\frac{v+v^0}{\sqrt{2}}\right) \mathrm{d}v \\ & + \left(1-\Phi\left(\sqrt{2}\left(n\left(-v^0\right)+v^0\right)\right)\right)\left(\Phi\left(\sqrt{2}n\left(-v^0\right)\right)+\kappa\right) \\ = & \frac{1}{\sqrt{2}} \int_{-\infty}^{-2m\left(v^0\right)+v^0} \Phi\left(\frac{v-v^0}{\sqrt{2}}\right) \varphi\left(\frac{v+v^0}{\sqrt{2}}\right) \mathrm{d}v \\ & + \left(1-\Phi\left(\sqrt{2}\left(-m\left(v^0\right)+v^0\right)\right)\right)\left(\Phi\left(-\sqrt{2}m\left(v^0\right)\right)+\kappa\right) \\ = & \frac{1}{\sqrt{2}} \int_{2m\left(v^0\right)-v^0}^{\infty} \Phi\left(\frac{-u-v^0}{\sqrt{2}}\right) \varphi\left(\frac{-u+v^0}{\sqrt{2}}\right) \mathrm{d}u \\ & + \Phi\left(\sqrt{2}\left(m\left(v^0\right)-v^0\right)\right)\left(1-\Phi\left(\sqrt{2}m\left(v^0\right)\right)+\kappa\right) \\ = & \frac{1}{\sqrt{2}} \int_{2m\left(v^0\right)-v^0}^{\infty} \left(1-\Phi\left(\frac{u+v^0}{\sqrt{2}}\right)\right) \varphi\left(\frac{-u+v^0}{\sqrt{2}}\right) \mathrm{d}u \end{aligned}$$

$$+\Phi\left(\sqrt{2}\left(m\left(v^0\right)-v^0\right)\right)\left(1-\Phi\left(\sqrt{2}m\left(v^0\right)\right)+\kappa\right)$$
$$=1-l\left(v^0\right).$$

Note that the fourth equality makes use of a change of dummy variable, $v=-u$. The proof is complete. □

PROPOSITION 3. The Irrelevance of the Dispute Distribution for the Limit Results in the Take-It-or-Leave-It Offer Models. *Under both the* P-*model and the* D-*model, given a distribution of disputes that is strictly positive, bounded above, and continuous, if the litigants make naïve inferences, the plaintiff trial win rate in the limit as* $\sigma$ *approaches zero will be equal to the plaintiff trial win rate obtained when* $\sigma=1$ *and the distribution of disputes was assumed to be improper uniform.*

*Proof of Proposition 3.* The logic behind this is similar to the one discussed in Lee and Klerman (2016). Let $u=\frac{y_p}{\sigma}$, $v=\frac{y_d}{\sigma}$, $z=\frac{y}{\sigma}$, and $u^0=\frac{y^0}{\sigma}$. We prove the proposition for the $P$-model, but the same reasoning applies to the $D$-model as well. Recall that the litigation probability function can be calculated according to the following double integral:

$$\Pi_\sigma(y)=\iint\limits_{R_\sigma(y_p,y_d)}\varphi_\sigma(y-y_p)\varphi_\sigma(y-y_d)\mathrm{d}y_p\mathrm{d}y_d,$$

where $R_\sigma(y_p,y_d)=\{(y_p,y_d)\in\boldsymbol{R}^2\,|\,y_d<(2\sigma)\mathrm{erf}^{-1}(2\zeta^*(y_p)-1)-y_p$ and $y_p\geqslant y^0\}$.

Rewriting this integral in terms of $u$ and $v$, we have

$$\Pi_\sigma(y)=\Pi_\sigma(\sigma z)=\iint\limits_{R_\sigma(u,v)}\varphi(u-z)\,\varphi(v-z)\,\mathrm{d}u\mathrm{d}v,$$

where $R_\sigma(u,v)=\{(u,v)\in\boldsymbol{R}^2\,|\,\sigma v<\gamma_{\sigma u}(\zeta^*(\sigma u))=(2\sigma)\,\mathrm{erf}^{-1}(2\zeta^*(\sigma u)-1)-(\sigma u)$ and $\sigma u\geqslant\sigma u^0\}$. It is easy to show that $R_\sigma(u,v)=R(u,v)\equiv\left\{(u,v)\in\boldsymbol{R}^2\,|\,v<2n(u)-u \text{ and } u\geqslant u^0\right\}$, which is a $\sigma$-independent set. Thus, neither the integrand nor the region of integration depends on $\sigma$. Furthermore, by Lemma A9, $2n(u)-u$ is bounded above and $R(u,v)$ is bounded on the left by $u=u^0$. Then by the argument used to prove Proposition 3 of Lee and Klerman (2016), the limit value of the plaintiff trial win rate will be that which was obtained under the assumption that $g_Y(x)$ is improper uniform and $\sigma = 1$. □

## B. Model with the Chatterjee–Samuelson Mechanism

LEMMA B1. *Any best response in the litigation game can be represented by a non-decreasing function.*

*Proof of Lemma B1.* This proof is modeled after the one used in Friedman and Wittman (2007). Let $d(v)$ be an arbitrary defendant strategy. Suppose that plaintiff strategy $p(u)$ does not have the desired monotonicity property. We will show that there is a better response, $q(u)$, that is closer to monotonic. If $p(u)$ is not nondecreasing, then there are points $u_1 < u_2$ such that $p(u_1) > p(u_2)$. Define $q(u)$ as follows: $q\,(u_1) = p(u_2)$, $q\,(u_2) = p(u_1)$, and otherwise $q\,(u) = p(u)$. Then the payoff sum at points $u_1$ and $u_2$ of $q(u)$ is $\Pi^p(q\,(u_1)\,, u_1, d\,(v)) + \Pi^p(q\,(u_2)\,, u_2, d\,(v)) = A + B_1 + B_2$, where

$$A = \int_{\{v\,|\,p(u_2)\leqslant d(v)\}} \left(\frac{d(v)+p(u_2)}{2}\right) f_{V|U=u}\,(v)\,\mathrm{d}v + \int_{\{v\,|\,p(u_1)\leqslant d(v)\}} \left(\frac{d(v)+p(u_1)}{2}\right) f_{V|U=u}\,(v)\,\mathrm{d}v,$$

$$B_1 = \int_{\{v\,|\,p(u_2)>d(v)\}} \left((\theta_H-\theta_L)\,\Phi\left(\frac{u_1+v}{\sqrt{2}}\right)+(\theta_L-C)\right) f_{V|U=u}\,(v)\,\mathrm{d}v, \text{ and}$$

$$B_2 = \int_{\{v\,|\,p(u_1)>d(v)\}} \left((\theta_H-\theta_L)\,\Phi\left(\frac{u_2+v}{\sqrt{2}}\right)+(\theta_L-C)\right) f_{V|U=u}\,(v)\,\mathrm{d}v.$$

Likewise, the payoff sum for $p(u)$ is $\Pi^p\,(p(u_1), u_1, d(v)) + \Pi^p(\mathrm{p}(u_2), u_2, d(v)) = A + B_1' + B_2'$, where

$$B_1' = \int_{\{v\,|\,p(u_1)>d(v)\}} \left((\theta_H-\theta_L)\,\Phi\left(\frac{u_1+v}{\sqrt{2}}\right)+(\theta_L-C)\right) f_{V|U=u}\,(v)\,\mathrm{d}v, \text{ and}$$

$$B_2' = \int_{\{v\,|\,p(u_2)>d(v)\}} \left((\theta_H-\theta_L)\,\Phi\left(\frac{u_2+v}{\sqrt{2}}\right)+(\theta_L-C)\right) f_{V|U=u}\,(v)\,\mathrm{d}v.$$

Therefore,

$$
\begin{aligned}
&[\Pi^p(q(u_1), u_1, d(v)) + \Pi^p(q(u_2), u_2, d(v))] \\
&\quad - [\Pi^p(p(u_1), u_1, d(v)) + \Pi^p(p(u_2), u_2, d(v))] \\
&= (B_1 + B_2) - \left(B_1' + B_2'\right) = \int_{\{v \mid p(u_2) \leqslant d(v) < p(u_1)\}} \varphi(u_1, u_2, v) f_{V|U=u}(v)\, \mathrm{d}v,
\end{aligned}
$$

where $\varphi(u_1, u_2, v) = (\theta_H - \theta_L)\left[\Phi\left(\frac{u_2+v}{\sqrt{2}}\right) - \Phi\left(\frac{u_1+v}{\sqrt{2}}\right)\right]$ is a strictly positive function. Thus, the difference is strictly positive if $\{v \mid p(u_2) \leqslant d(v) < p(u_1)\}$ is non-empty. If the set is empty, then there is no difference between the expected payoff from $q(u)$ and that from $p(u)$ at any $u$, and the plaintiff may as well be assumed to choose $q(u)$ over $p(u)$. The argument for the defendant is similar. □

Proposition 4. Symmetric Nash Equilibria under the Chatterjee–Samuelson Bargaining Model. *Under the Chatterjee–Samuelson bargaining model with* $g_Y(x) = 1$, *the following is true.*

(a) *There exists a continuous family of symmetric Nash equilibria*;
(b) *the plaintiff trial win rate is* $\frac{\theta_H+\theta_L}{2}$ *for any symmetric* NE; *and*
(c) *a sufficient condition that ensures that extreme cases on both ends will be more likely to settle is that both the plaintiff's strategy and the defendant's strategy eventually coincide at a fixed value in each direction (as* $y_p$ *and* $y_d$ *approach positive infinity and negative infinity).*

*Proof of Proposition 4.* Point (a) was shown in the main text. To establish (b), we begin by proving the existence of $\gamma(\sigma) \in \boldsymbol{R}$ such that the step function strategies discussed in the main text indeed constitute a NE. Suppose such $\gamma(\sigma)$ existed. Then the plaintiff must be indifferent between demanding $\theta_L$ or demanding $\theta_H$ when $y_p = -\gamma(\sigma)$ and the defendant must be indifferent between offering $\theta_L$ and offering $\theta_H$ when $y_d = \gamma(\sigma)$. Define $\pi(x) \equiv \Pr\left(y_p < -x | y_d = x\right) = \Phi\left(\frac{-\sqrt{2}x}{\sigma}\right) = \Pr\left(y_d > x | y_p = -x\right)$. Similarly, let $\psi(x) \equiv \Pr\left(Y \geqslant 0 \mid y_d = x, y_p \geqslant -x\right) = 1 - \Pr\left(Y \geqslant 0 \mid y_p = -x, y_d < x\right)$.[35] Then, for a given $\sigma > 0$, the defendant

35. The probability that $y_d = \gamma(\sigma)$ or $y_p = -\gamma(\sigma)$ has measure zero.

is indifferent at $y_d = \gamma_d(\sigma)$ if and only if his expected payout from either offer is the same. Thus,

$$\pi(\gamma(\sigma))\left(\frac{\theta_H+\theta_L}{2}\right)+(1-\pi(\gamma(\sigma)))(\theta_H)$$
$$=\pi(\gamma(\sigma))(\theta_L)+(1-\pi(\gamma(\sigma)))((\theta_H-\theta_L)\psi(\gamma(\sigma))+\theta_L+C),$$

which is equivalent to $\pi(\gamma(\sigma)) = \frac{(\theta_H-\theta_L)(1-\psi(\gamma(\sigma)))-C}{(\theta_H-\theta_L)\left(\frac{1}{2}-\psi(\gamma(\sigma))\right)-C}$. Likewise, the plaintiff indifference condition can be written as

$$\pi(\gamma(\sigma))\left(\frac{\theta_H+\theta_L}{2}\right)+(1-\pi(\gamma(\sigma)))(\theta_L)=\pi(\gamma(\sigma))(\theta_H)$$
$$+(1-\pi(\gamma(\sigma)))((\theta_H-\theta_L)(1-\psi(\gamma(\sigma)))+\theta_L-C),$$

which is also equivalent to $\pi(\gamma(\sigma)) = \frac{(\theta_H-\theta_L)(1-\psi(\gamma(\sigma)))-C}{(\theta_H-\theta_L)\left(\frac{1}{2}-\psi(\gamma(\sigma))\right)-C}$. Thus, under the set-up, the plaintiff indifference condition and the defendant indifference condition coincide. In this case, we simply need to prove there exists $x \in \boldsymbol{R}$ such that $\pi(x) = \frac{(\theta_H-\theta_L)(1-\psi(x))-C}{(\theta_H-\theta_L)\left(\frac{1}{2}-\psi(x)\right)-C}$. Note that as $x$ goes from negative infinity to positive infinity, $\pi(x) = \Pr\left(y_p < -x | y_d = x\right)$ decreases from 1 to 0. Note also that as $x$ goes from negative infinity $\psi(x) = \Pr\left(Y \geqslant 0 \,|\, y_d = x, y_p \geqslant -x\right)$ will go from $\lim\limits_{x\to-\infty}\psi(x)$ to $\lim\limits_{x\to\infty}\psi(x) = 1$. It is easy to show that $\Pr\left(Y \geqslant 0 \,|\, y_d = x, y_p \geqslant -x\right)$ is increasing in $x$. In addition, since $\Pr\left(Y \geqslant 0 \,|\, y_d = x, y_p \geqslant -x\right) > \Pr\left(Y \geqslant 0 \,|\, y_d = x, y_p = -x\right) = \frac{1}{2}$, this value is always greater than $\frac{1}{2}$. In turn, $\frac{(\theta_H-\theta_L)(1-\psi(x))-C}{(\theta_H-\theta_L)\left(\frac{1}{2}-\psi(x)\right)-C} = 1 - \frac{\frac{1}{2}(\theta_H-\theta_L)}{C+(\theta_H-\theta_L)\left(\psi(x)-\frac{1}{2}\right)}$ increases from $1 - \frac{\frac{1}{2}(\theta_H-\theta_L)}{C+(\theta_H-\theta_L)\left(\psi(-\infty)-\frac{1}{2}\right)}$ to $\frac{C}{\frac{1}{2}(\theta_H-\theta_L)+C}$. Thus, by the Intermediate Value Theorem guarantees us that there will be at least one $x \in \boldsymbol{R}$ such that $\pi(x) = \frac{(\theta_H-\theta_L)(1-\psi(x))-C}{(\theta_H-\theta_L)\left(\frac{1}{2}-\psi(x)\right)-C}$.

Next, we show that this is indeed a NE. We must also show that for all values of $y_p$ below some threshold, the plaintiff cannot do better than demanding $\theta_L$, and for all values of $y_p$ above the threshold the plaintiff cannot do better than demanding $\theta_H$, and similarly for the defendant. By symmetry, we need only show one party's case. Notice first that, given the plaintiff's strategy, it is never optimal at any $y_d$ for the defendant to make an offer strictly below $\theta_L$ because this strategy is strictly dominated by the offer of $\theta_L$. For all cases that would have litigated had the defendant offered $\theta_L$, the outcome is the same; but for all cases that would have settled had the defendant offered $\theta_L$, the defendant will instead incur a minimum expected payout of $\theta_L + C$. Second, it is also never optimal at any $y_d$ for the defendant to make a settlement offer that is strictly between $\theta_L$ and $\theta_H$. This is because the plaintiff is playing by the discontinuous two-step strategy of playing either $\theta_L$ or $\theta_H$ himself. If the defendant were to make a settlement offer strictly between $\theta_L$ and $\theta_H$, he will end up (i) litigating all the cases he would have litigated had he offered $\theta_L$ instead (that is, those cases in which the plaintiff observes $y_p$ above the threshold) but (ii) will also be settling all other cases (that is, those cases in which the plaintiff observes $y_p$ below the threshold) at a higher settlement value than had he simply offered $\theta_L$. Therefore, he is better off offering $\theta_L$ than any intermediate value. Third, it is never optimal for the defendant to make a settlement offer strictly greater than $\theta_H$, since that is dominated by an offer of $\theta_H$. Thus, given the plaintiff's strategy, the defendant's best response must be one that offers either $\theta_L$ or $\theta_H$ for each $y_d$. Furthermore, the indifference condition at $y_d = \gamma(\sigma)$ shows that for $y_d > \gamma(\sigma)$ the defendant will be better off offering $\theta_H$, and for $y_d < \gamma(\sigma)$, the defendant will be better off offering $\theta_L$. Therefore, the defendant's specified strategy is the best response to the plaintiff's strategy, and likewise, the plaintiff's specified strategy is the best response to the defendant's strategy. Finally, note that the same argument goes through when we work with $\theta_L - \epsilon$ and $\theta_H + \epsilon$ where $\epsilon$ is sufficiently small.

To show (c), note that the litigation condition set is defined as $R_\sigma(y_p, y_d) = \{(y_p, y_d) | p(y_p; \sigma) > d(y_d; \sigma)\}$. This means that once $p(y_p; \sigma)$ and $d(y_d; \sigma)$ achieve their maximum value (or minimum value), the inequality will never be satisfied beyond that point. This means there exists $y_{d,1}$ such that beyond this value, no defendant will ever go to trial (since he is offering the highest settlement amount any plaintiff is demanding). Similarly, there exists $y_{p,0}$ such that below this value, no plaintiff will ever go to trial (since she is demanding the lowest settlement amount any defendant is offering). For this reason, the litigation condition set is bounded both above and to the left, and extreme cases are thus likely to settle. □

Proposition 5. Asymmetric Nash Equilibria under the Chatterjee–Samuelson Bargaining Model. *Under the Chatterjee–Samuelson bargaining model with* $g_Y(x) = 1$, *the following is true.*

(a) *There exists a continuous family of asymmetric Nash equilibria*;
(b) *the plaintiff trial win rate will be* $\theta_L$ *under the obstinate plaintiff equilibrium; and*
(c) *the plaintiff trial win rate will be* $\theta_H$ *under the obstinate defendant equilibrium.*

*Proof of Proposition 5.* Point (a) was proved by our construction of obstinate Nash equilibria. To show (b), note that under an obstinate plaintiff equilibrium, there will exist a $y_d^* \in \boldsymbol{R}$ such that disputes will go to trial if and only if $y_d < y_d^*$. In this case, the plaintiff trial win rate (when $g_Y(x) = 1$) is $(\theta_H - \theta_L)\left(\frac{\int_0^\infty \Pi_\sigma(y)dy}{\int_{-\infty}^\infty \Pi_\sigma(y)dy}\right) + \theta_L$, where $\Pi_\sigma(y) = \iint_{R_\sigma(y_p,y_d)} \varphi_\sigma(y - y_p)\varphi_\sigma(y - y_d)dy_p dy_d$ and $R_\sigma(y_p, y_d) = \{(y_p, y_d) \,|y_d < y_d^*\}$. Since $R_\sigma(y_p, y_d)$ is bounded above by $y_d^*$ but extends indefinitely for negative $y_d$ and $y_p$values, $\Pi_\sigma(y)$ approaches 1 as $y$ approaches negative infinity and approaches 0 as $y$ approaches positive infinity by Chebyshev's inequality. By the argument used in Proposition 3 of Lee and Klerman (2016), $\int_0^\infty \Pi_\sigma(y)\,dy$ converges to a finite value. But in this case, $\int_{-\infty}^\infty \Pi_\sigma(y)\,dy$ will approach infinity. Thus, the plaintiff trial win rate will be $\theta_L$. To show (c), note that under an obstinate defendant equilibrium, there will exist $y_p^* \in \boldsymbol{R}$ such that disputes will go to trial if and only if $y_p > y_p^*$. Rewriting the plaintiff trial win rate as $\theta_H - (\theta_H - \theta_L)\left(\frac{\int_{-\infty}^0 \Pi_\sigma(y)\mathrm{d}y}{\int_{-\infty}^\infty \Pi_\sigma(y)\mathrm{d}y}\right)$, we can apply the analogous reasoning to show that the trial win rate will be $\theta_H$.□

Proposition 6. Symmetric and Asymmetric Limit Equilibria under the Chatterjee–Samuelson Bargaining Model. *Suppose the distribution of disputes is strictly positive, bounded above, and continuous. Then under the Chatterjee–Samuelson bargaining mechanism, whether the litigants make naïve inferences or sophisticated inferences, the following is true.*

(a) *There exists at least one class of symmetric limit equilibria, and for all symmetric limit equilibria, the plaintiff trial win rate is* $\frac{\theta_H + \theta_L}{2}$ *as* $\sigma$ *approaches zero; and*

(b) *there exist classes of obstinate limit equilibria (which are asymmetric in the limit), and the plaintiff trial win rate is* $\theta_L$ *for obstinate plaintiff limit equilibria and* $\theta_H$ *for obstinate defendant limit equilibria as* $\sigma$ *approaches zero.*

*Proof of Proposition 6.* The plaintiff trial win rate in (a) follows immediately from looking at the litigation condition set. We need only show that there exists a family of Nash equilibria which in the limit approaches the one we constructed in Proposition 4. To do this, we make the following change of variables: $u = y_p/\sigma$, $v = y_d/\sigma$. To see the first part of the result, notice that the litigation condition set under the $uv$-coordinate is defined as follows: $R(u,v) = \{(u,v) \mid \lim_{\sigma\to 0^+} p(\sigma u;\sigma) > \lim_{\sigma\to 0^+} d(\sigma v;\sigma)\}$. This region will be symmetric around the line $v = -u$ if and only if we can show that whenever $(u,v) \in R(u,v)$, we must also have $(-v,-u) \in R(u,v)$. The rest of the analysis follows from the proof of Proposition 4 since we must have $\lim_{\sigma\to 0^+} p(\sigma u;\sigma) > \lim_{\sigma\to 0^+} d(\sigma v;\sigma)$ if and only if we have $\lim_{\sigma\to 0^+} p(-\sigma v;\sigma) > \lim_{\sigma\to 0^+} d(-\sigma u;\sigma)$. Therefore, the region of integration in the limit is symmetric. Importantly, under the Chatterjee–Samuelson bargaining model, the litigation condition set in the limit is *not* determined by the parties' inferences, but only by the parties' strategies, which are assumed to be symmetric in the limit. Therefore, regardless of whether the litigants' inferences are naïve or sophisticated, we will have a symmetric region of integration in the limit. Given this symmetry, the 50% result follows from the arguments used in Lee and Klerman (2016), Proposition 3.

It now remains to show that there will always be at least one continuous family of Nash equilibria for $\sigma \in (0,\bar{\sigma})$ for some $\bar{\sigma} > 0$ such that in the limit we obtain a symmetric limit equilibrium. Our strategy is to show that there exists $\bar{\sigma} > 0$ such that for all $\sigma \in (0,\bar{\sigma})$, there will generally[36] exist a pair of continuous functions $\left(\gamma_p(\cdot),\gamma_d(\cdot)\right) : \boldsymbol{R}^+ \to \boldsymbol{R}^2$ such

36. We say "generally" because the proof makes use of the Implicit Function Theorem and thus will depend on a particular Jacobian not taking on the value of zero at the particular equilibrium value. Because the particular Jacobian is not identically zero, this will generally be the case, although it may be possible to *construct* an example in which the Jacobian can take on the value of zero at the particular equilibrium point. Calculation using *Mathematica* confirmed that the Jacobian is indeed nonzero for normal distributions.

that $\lim_{\sigma\to 0^+}\gamma_p(\sigma) = \gamma = \lim_{\sigma\to 0^+}\gamma_d(\sigma)$ and for each $\sigma \in (0,\bar{\sigma})$, the following $(\mathrm{p}(u;\sigma), \mathrm{d}(v;\sigma))$ is a NE in the $\sigma$-game and its limit is a limit equilibrium:

$$p(u;\sigma) = \begin{cases} \theta_H + \theta_L \text{ for } u \geqslant -\gamma_p(\sigma) \\ 0 \text{ for } u < -\gamma_p(\sigma) \end{cases} \quad \text{and}$$

$$d(v;\sigma) = \begin{cases} \theta_H + \theta_L \text{ for } v \geqslant \gamma_d(\sigma) \\ 0 \text{ for } v < \gamma_d(\sigma) \end{cases}.$$

Proceeding as before, we can write the two indifference conditions as follows:

$$\Pr\left(u < -\gamma_p(\sigma)\,|v = \gamma_d(\sigma)\right) = \frac{(\theta_H - \theta_L)\left(1 - \Pr\left(Y \geqslant 0\,|\,v = \gamma_d(\sigma), u \geqslant -\gamma_p(\sigma)\right)\right) - C}{(\theta_H - \theta_L)\left(\frac{1}{2} - \Pr\left(Y \geqslant 0\,|\,v = \gamma_d(\sigma), u \geqslant -\gamma_p(\sigma)\right)\right) - C}$$

and

$$\Pr\left(v \geqslant \gamma_d(\sigma)\,|u = -\gamma_p(\sigma)\right) = \frac{(\theta_H - \theta_L)\left(1 - \Pr\left(Y < 0\,|\,u = -\gamma_p(\sigma), v < \gamma_d(\sigma)\right)\right) - C}{(\theta_H - \theta_L)\left(\frac{1}{2} - \Pr\left(Y < 0\,|\,u = -\gamma_p(\sigma), v < \gamma_d(\sigma)\right)\right) - C}.$$

We will call these two conditions Condition $X_1$ and Condition $X_2$, respectively. At this point, our strategy to constructing this continuous family of Nash equilibria is as follows. We first find out what the limit equilibrium must be if such a continuous family exists. Then we show that the limit is indeed a NE of the $\sigma$-game in the limit. Then we apply the Implicit Function Theorem to conclude that there must indeed be continuous families in a small neighborhood around $\sigma = 0$ that satisfy the two indifference conditions. Therefore, assume such a continuous family exists and take the limits of Condition $X_1$ and Condition $X_2$ as $\sigma$ goes to zero. Notice

$$\begin{aligned}
&\lim_{\sigma\to 0^+} \Pr\left(u < -\gamma_p(\sigma)\,|v = \gamma_d(\sigma)\right) \\
&= \lim_{\sigma\to 0^+}\left(\frac{\int_{-\infty}^{\infty} g_Y(\sigma z)\,\varphi(z - \gamma_d(\sigma))\,\Phi\left(-\gamma_p(\sigma) - z\right)dz}{\int_{-\infty}^{\infty} g_Y(\sigma z)\,\varphi(z - \gamma_d(\sigma))\,dz}\right) \\
&= \int_{-\infty}^{\infty} \varphi(z - \gamma)\,\Phi(-\gamma - z)dz = \Pr(u < -\gamma|v = \gamma).
\end{aligned}$$

Note that the integral specification here assumes that the parties are making sophisticated inferences by taking the distribution of disputes into account. To prove the result for naïve inferences, one can simply replace $g_Y(\sigma z)$ with 1 and follow the rest of the proof. The second equality in the above equation comes from Lebesgue's Dominated Convergence Theorem. If we let $g_u \in \boldsymbol{R}$ be the upper bound for $g_Y(x)$, then since $|g_Y(\sigma z)\varphi(z-\gamma_d(\sigma))\Phi(-\gamma_d(\sigma)-z)| \leqslant g_u\varphi(z-\gamma_d(\sigma))$ for all $z$, and the latter function will integrate to a finite value (namely, $g_u$) over all $z$. Thus, we can take the limit under the integral and factor out $g_Y(0)$, which is nonzero. Likewise, we can show that $\lim_{\sigma\to 0^+} \Pr(v \geqslant \gamma_d(\sigma)|u = -\gamma_p(\sigma)) = \Pr(v \geqslant \gamma|u = -\gamma)$. Thus, $\lim_{\sigma\to 0^+} \Pr(u < -\gamma_p(\sigma)|v = \gamma_d(\sigma)) = \lim_{\sigma\to 0^+} \Pr(v \geqslant \gamma_d(\sigma)|u = -\gamma_p(\sigma))$. Similarly, in the limit we will also have $\lim_{\sigma\to 0^+} \Pr(Y \geqslant 0|v = \gamma_d(\sigma), u \geqslant -\gamma_p(\sigma)) = \lim_{\sigma\to 0^+} \Pr(Y < 0\,|\,u = -\gamma_p(\sigma), v < \gamma_d(\sigma))$. This means that in the limit, Condition $X_1$ and Condition $X_2$ will coincide. As before, the Intermediate Value Theorem guarantees us that there is a suitable $\gamma > 0$ such that the two conditions are both satisfied in the limit. We further know that this is a NE in the limit by Proposition 4. Now we show that there exist a pair of continuous functions $(\gamma_p(\cdot), \gamma_d(\cdot)) : \boldsymbol{R}^+ \to \boldsymbol{R}^2$ such that $\lim_{\sigma\to 0^+} \gamma_p(\sigma) = \gamma = \lim_{\sigma\to 0^+} \gamma_d(\sigma)$ and $(p(y_p;\sigma), d(y_d;\sigma))$ is a NE for each $\sigma$. We rewrite Conditions $X_1$ and $X_2$ as follows:

$$\begin{aligned} X_1(\sigma, x_1, x_2) = {} & \left(\frac{\int_{-\infty}^{\infty} g(\sigma z)\,\varphi(z-x_2)\,\Phi(-x_1-z)\mathrm{d}z}{\int_{-\infty}^{\infty} g(\sigma z)\varphi(z-x_2)\mathrm{d}z}\right)\left(\frac{\theta_H+\theta_L}{2}\right) \\ & + \left(1-\left(\frac{\int_{-\infty}^{\infty} g(\sigma z)\,\varphi(z-x_2)\,\Phi(-x_1-z)\mathrm{d}z}{\int_{-\infty}^{\infty} g(\sigma z)\varphi(z-x_2)\mathrm{d}z}\right)\right) \\ & \times \Bigg((\theta_H+\theta_L)-(\theta_H-\theta_L) \\ & \quad \times \frac{\int_0^{\infty} g(\sigma z)\,\varphi(z-x_2)\,\Phi(x_1+z)\mathrm{d}z}{\int_{-\infty}^{\infty} g(\sigma z)\,\varphi(z-x_2)\,\Phi(x_1+z)\mathrm{d}z} + \theta_L + C\Bigg) = 0 \end{aligned}$$

and

$$\begin{aligned} X_2(\sigma, x_1, x_2) = {} & \left(\frac{\int_{-\infty}^{\infty} g(\sigma z)\,\varphi(z+x_1)\,\Phi(x_2-z)\mathrm{d}z}{\int_{-\infty}^{\infty} g(\sigma z)\varphi(z+x_1)\mathrm{d}z}\right)\left(\frac{\theta_H+\theta_L}{2}\right) \\ & + \left(1-\left(\frac{\int_{-\infty}^{\infty} g(\sigma z)\,\varphi(z+x_1)\,\Phi(x_2-z)\mathrm{d}z}{\int_{-\infty}^{\infty} g(\sigma z)\varphi(z+x_1)\mathrm{d}z}\right)\right) \end{aligned}$$

$$\times\left((\theta_H-\theta_L)\left(1-\frac{\int_0^{\infty} g(\sigma z)\,\varphi(z+x_1)\,\Phi(z-x_2)\mathrm{d}z}{\int_{-\infty}^{\infty} g(\sigma z)\,\varphi(z+x_1)\,\Phi(z-x_2)\mathrm{d}z}\right)\right.$$

$$\left.+\theta_L-C\right)=0.$$

Then $K(\sigma,x_1,x_2)=(X_1(\sigma,x_1,x_2),X_2(\sigma,x_1,x_2))$ is a continuously differentiable function from $\boldsymbol{R}^3$ to $\boldsymbol{R}^2$ such that $K(0,\gamma,\gamma)=(0,0)$. Then by the Implicit Function Theorem,[37] as long as the Jacobian matrix is invertible at $\sigma=0$, there is a small neighborhood around $\sigma=0$[38] for which we can find a unique $(\gamma_p(\sigma),\gamma_d(\sigma))$ for each $\sigma$ such that $K(\sigma,\gamma_p(\sigma),\gamma_d(\sigma))=(0,0)$ and $\lim_{\sigma\to 0^+}\gamma_p(\sigma)=\gamma=\lim_{\sigma\to 0^+}\gamma_d(\sigma)$. Thus, we need only check that the Jacobian matrix is invertible at $\sigma=0$. Looking at the equation, we see that the determinant cannot be identically zero since $\left(\frac{\partial X_1(0,x_1,x_2)}{\partial x_1}\right)\left(\frac{\partial X_2(0,x_1,x_2)}{\partial x_2}\right)\neq\left(\frac{\partial X_1(0,x_1,x_2)}{\partial x_2}\right)\left(\frac{\partial X_2(0,x_1,x_2)}{\partial x_1}\right)$. (The left-hand side has terms involving mostly $\varphi(x)$s while the right-hand side has terms involving $\Phi(x)$s and $\varphi'(x)$s.) Calculation using *Mathematica* confirmed that the Jacobian was indeed not zero when working with normal distributions.

To establish (b), we first show the existence of obstinate plaintiff/defendant limit equilibria. Note that $P(Y\geqslant 0\mid Y_d=x)+C$ is continuous in $\sigma$. We therefore have a continuous family of Nash equilibria with parameter $\sigma$. Furthermore, we can rewrite the condition as follows:

$$P(Z\geqslant 0\mid V=v)=\frac{\int_0^{\infty} g(\sigma z)\varphi(z-v)dz}{\int_{-\infty}^{\infty} g(\sigma z)\varphi(z-v)dz}.$$

Then as $\sigma$ goes to zero, $P(Z\geqslant 0\mid V=v)$ approaches $\int_0^{\infty}\varphi(z-v)dz$, which can take on any value between 0 and 1 depending on $v$. Therefore, the corresponding $v$ that satisfies $P(Z\geqslant 0\mid V=v)+C=s$ will be uniquely determined, and this pair of strategies will be an equilibrium for the $\sigma$-game in the limit. Since $\frac{\int_0^{\infty} g(\sigma z)\varphi(z-v)dz}{\int_{-\infty}^{\infty} g(\sigma z)\varphi(z-v)dz}$ is continuous in $\sigma$ and for each $\sigma$, there will be a unique $v^*(s;\sigma)$, this equilibrium will be a limit of a continuous family of asymmetric Nash equilibria.

37. We thank Ken Alexander for suggesting the use of the Implicit Function Theorem to complete this argument.

38. Although in this model, $\sigma$, as standard deviation, is necessarily positive, both $X_1(\sigma,x_1,x_2)$ and $X_2(\sigma,x_1,x_2)$, simply as mathematical functions in three variables $\sigma$, $x_1$, and $x_2$, are continuous in $\sigma$ around $\sigma=0$ and well-defined for $\sigma<0$ as well.

Finally, we show that the plaintiff trial win rate in the limit is $\theta_L$ for the obstinate plaintiff limit equilibria and $\theta_H$ for the obstinate defendant limit equilibria. Under the normalized coordinates, the plaintiff trial win rate can be written as

$$(\theta_H - \theta_L)\left(\frac{\int_0^\infty \Pi_\sigma(z)\, g_Y(\sigma z)dz}{\int_{-\infty}^\infty \Pi_\sigma(z)\, g_Y(\sigma z)dz}\right) + \theta_L,$$

where $\Pi_\sigma(z) = \iint_{R_\sigma(u,v)} \varphi_1(z-u)\,\varphi_1(z-v)\,\mathrm{d}u\mathrm{d}v$. For obstinate plaintiff limit equilibria, $R_\sigma(u,v) = \{(u,v)\,|v < v^*(s;\sigma)$. It suffices to show that $\frac{\int_0^\infty \Pi_\sigma(z) g_Y(\sigma z)dz}{\int_{-\infty}^\infty \Pi_\sigma(z) g_Y(\sigma z)dz}$ approaches zero as $\sigma$ goes to zero. We can rewrite this as follows:

$$\frac{\int_0^\infty \Pi_\sigma(z)\, g_Y(\sigma z)dz}{\int_{-\infty}^\infty \Pi_\sigma(z)\, g_Y(\sigma z)dz} = \frac{\int_0^\infty \Pi_\sigma(z)\, g_Y(\sigma z)dz}{\int_{-\infty}^0 \Pi_\sigma(z)\, g_Y(\sigma z)dz + \int_0^\infty \Pi_\sigma(z)\, g_Y(\sigma z)dz}$$

$$= \frac{\int_0^\infty \Pi_\sigma(z)\, g_Y(\sigma z)dz}{\int_{-\infty}^0 g_Y(\sigma z)dz - \int_{-\infty}^0 (1-\Pi_\sigma(z))\, g_Y(\sigma z)dz + \int_0^\infty \Pi_\sigma(z)\, g_Y(\sigma z)dz}$$

By the argument used in Propositions 3 and 4 of Lee and Klerman (2016), it is easy to show that $\int_0^\infty \Pi_\sigma(z)\, g_Y(\sigma z)dz$ approaches $\int_0^\infty \Pi_0(z)\, g_Y(0)dz$, as $\sigma$ approaches zero, where $\Pi_0(z) = \iint_{R_0(u,v)} \varphi_1(z-u)\varphi_1(z-v)\mathrm{d}u\mathrm{d}v$ and $R_0(u,v) = \{(u,v)\,|v < v^*(s;0)\}$. $\int_0^\infty \Pi_0(z)\, g_Y(0)dz$ is a finite value since $\Pi_0(z)$ approaches 0 at the speed of $z^{-2}$. (See Proposition 3, Lee and Klerman (2016)).

Meanwhile, $\int_{-\infty}^0 g_Y(\sigma z)dz = \frac{1}{\sigma}\int_{-\infty}^0 g_Y(y)dy$ approaches an infinite value, as $\sigma$ approaches zero. Lastly, $\int_{-\infty}^0 (1-\Pi_\sigma(z))\, g_Y(\sigma z)dz$ approaches $\int_{-\infty}^0 (1-\Pi_0(z))\, g_Y(0)dz$, which also is a finite value. Thus, the plaintiff trial win rate approaches $\theta_L$ in the limit. Likewise, for obstinate defendant limit equilibria, the plaintiff trial win rate will approach $\theta_H$ in the limit. $\square$

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