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Information and the Optimal Ownership Structure of Firms

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Abstract

I develop a property rights theory of the firm in which managers bargain over the sharing of quasi-rents in the presence of private information. I analyze the interdependence between the ownership structure of firms and the bargaining inefficiency that is due to the presence of private information and derive the optimal ownership structure that minimizes the bargaining inefficiency. I first assume that managers can only contract over the ownership structure and show that they optimally choose one that minimises (maximises) their aggregate disagreement payoff if the minimum expected quasi-rents are large (small). I then extend my analysis and allow the managers to contract over the ownership structure and the bargaining game that is played ex post. I show that the main results continue to hold if and only if ownership structures are deterministic and cannot be made contingent on information that is revealed ex post.

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1 Introduction

Managers who bargain with each other over the sharing of quasi-rents often do so in the presence of private information. For instance, suppliers often know more about the costs of producing specialized inputs while downstream firms often know more about how much they value such inputs. It is well known that, under fairly general conditions, voluntary bilateral bargaining in the presence of private information must be inefficient, in the sense that not all gains from trade are realized instantaneously (see Myerson and Satterthwaite (1983)). In this paper I argue that the size of the bargaining inefficiency that arises when two managers bargain over the sharing of quasi-rents in the presence of private information depends crucially on the ownership structure of their firms. I formally model this interdependence and derive the optimal ownership structure that minimizes the bargaining inefficiency.

Economists have believed for a long time that informational imperfections are an important factor in determining the boundaries of firms. This belief can be traced back to Coase (1937) who argues that the resources used to discover and haggle over the terms of trade constitute a major cost of market transactions and that these costs can be reduced if the transactions are brought into the firm[1]. Similarly, the modern transaction cost theory of the firm, as developed by Williamson (1975, 1985), argues that bargaining between firms over the sharing of quasi-rents can be costly due to the presence of private information and that these haggling costs can be reduced if the firms integrate[2]. This literature, however, does not describe explicitly the mechanism through which integration leads to a reduction in the bargaining inefficiency. It essentially assumes that managers have less scope to act less opportunistically if a transaction takes place within a firm rather than across two independent firms and that, due to this behavioral change, integration can lead to a reduction in the bargaining inefficiency[3].

[1] Coase (1937), p.390-391.

[2] See, for instance, Williamson (1975), p.31-37.

[3] Williamson (1975) stresses four reasons why integration can reduce inefficiencies that are due to the presence of private information: first, integrated firms can create better incentives to induce managers to act less opportunistically than non-integrated firms. Second, integrated firms have stronger auditing powers than non-integrated firms. Third, integrated firms can settle disputes more efficiently than non-integrated firms. Fourth, integration facilitates communication (see Williamson (1975), p.31 -

The conceptual framework of the property rights theory of the firm that I adopt in this paper allows me to be more precise about the mechanism through which integration, or any other ownership change, affects the bargaining inefficiency that arises when managers have private information. In the property rights theory, as developed by Grossman and Hart (1986) and Hart and Moore (1990), changes in the ownership structure of firms only affect the behavior of managers by changing the disagreement payoffs they realize if they do not agree to trade with each other. I show that by altering the disagreement payoffs, ownership changes can have significant effects on how efficiently managers bargain with each other in the presence of private information and that the ownership structures that minimize the bargaining inefficiency are frequently observed in the real world.

To illustrate my main arguments, consider the following basic set up. There are two managers, a buyer and a seller, and two machines, a_1 and a_2. The seller can use a_2 to produce either a 'widget' that is specialized to the buyer's needs or a standard widget that can be sold on a spot market. The buyer can use a_1 to turn either the special or a standard widget into a final good. The managers' 'trade payoffs,' that is the seller's cost of producing the special widget and the buyer's valuation of it, are independent and privately known only to the respective manager. Also, these payoffs are independent of the ownership structure since, if trade takes place, both managers have access to both machines regardless of who owns them. For simplicity, suppose that the managers' 'disagreement payoffs,' that is the payoff each manager realizes if the special widget is not traded, are common knowledge. In contrast to the trade payoffs, the disagreement payoffs do depend on the ownership structure since, in the case of disagreement, only the owner of a machine can use it. Finally, suppose that sometimes there are positive quasi-rents, that is the gains from trading the special widget are larger than the sum of both managers' disagreement payoffs, and that sometimes the quasi-rents are negative.

Ex post the managers bargain over the price of the special widget. If they agree on a price, they simply transfer the special widget and if they do not agree, they just realize

37). All these arguments essentlially assume a restriction on the contracts that can be written between independent firms compared to those that can be written between internal divisions.

their respective disagreement payoffs. Note that because of the presence of bilateral private information the managers will sometimes not agree to trade the special widget although there are positive gains from trade. In this sense bargaining is inefficient.

Ex ante, before learning the realization of their trade payoffs and engaging in bargaining, the managers contract over the ownership structure, that is they contract over who owns what machine. For simplicity suppose that only the ownership structure is contractible and that the ex post bargaining game is given exogenously[4]. The key question then is what ownership structure the managers agree on or, equivalently, what ownership structure minimizes the bargaining inefficiency.

It turns out that the answer to this question is surprisingly simple. The managers either choose the ownership structure that minimizes their expected quasi-rents (or, equivalently, maximizes their aggregate disagreement payoff) or they choose the one that maximizes their expected quasi-rents (or, equivalently, minimizes their aggregate disagreement payoff). Which one of the two they choose depends crucially the 'minimum expected quasi-rents,' that is it depends on how high the expected quasi-rents are if the aggregate disagreement payoff is maximized. In particular, the managers choose the ownership structure that minimizes the expected quasi-rents if, for a given distribution of trade payoffs, the minimum expected quasi-rents are small and they choose the ownership structure that maximizes the expected quasi-rents otherwise.

To get an intuition for why this result holds, consider the trade-off the managers face when they decide on an ownership change. Such a change has no effect on the size of the expected bargaining inefficiency if it simply leads to a redistribution of disagreement payoffs between the managers without altering the aggregate disagreement payoff. A change in the ownership structure that leads to a reduction in the aggregate disagreement payoff, however, does affect the bargaining inefficiency. On the one hand, such a change increases the probability that efficient trading opportunities are realized which, ceteris paribus, reduces the bargaining inefficiency. On the other hand, it also increases the cost of disagreement and might even induce the managers to trade 'too often;' ceteris

[4] In the formal analysis I allow the managers to choose both their disagreement payoffs and the bargaining game they play ex post.

paribus these two effects increase the bargaining inefficiency.

If the aggregate disagreement payoff is large, the managers disagree very often and, as a result, the costs of a marginal reduction in the aggregate disagreement payoff are large and indeed dominate the benefit. Thus, if the aggregate disagreement payoff is large, reducing it marginally is inefficient. If the aggregate disagreement payoff is small, however, the managers do not disagree very often and a marginal reduction in the aggregate disagreement payoff is welfare enhancing. The result that welfare is decreasing in the aggregate disagreement payoff if the aggregate disagreement payoff is small and that it is increasing if the aggregate disagreement payoff is large is key in my analysis. It implies, not only that the managers always either want to minimize or maximize the aggregate disagreement payoff, but also that they want to do the former if the maximum aggregate disagreement payoff is 'large' and that they want to do the latter otherwise.

There are four ownership structures that can be optimal in my model: buyer-integration, seller-integration, non-integration, and joint ownership. All of them are, of course, frequently observed in the real world. The property rights literature has been criticized for not predicting often enough the optimality of joint ownership and of asset clusters, such as buyer- and seller-integration (see, for instance, Holmström (1999)). My analysis shows that joint ownership and asset clusters can be optimal in a property rights model in which ex post bargaining is inefficient due to the presence of private information.

Managers often take actions that increase the expected quasi-rents in their trading relationships but do not maximize them[5]. For instance, managers sometimes exchange 'ugly princess hostages' (see, for example, Williamson (1985) and, in the context of military conflicts, Schelling (1960)). The term refers to a practice in which managers exchange ownership of assets that are very valuable to themselves but which have little or no value to the other party. In the Japanese car industry, for instance, it can be observed that physical assets which are specific to a particular car manufacturer are

[5] To maximize their expected quasi-rents, the managers would need to agree on joint ownership. Joint ownership maximizes expected quasi-rents since it prevents either manager from using the assets without the agreement of the other.

often owned by its supplier[6]. Also, firms sometimes increase expected quasi-rents by selling off assets that can be used unilaterally during a conflict. For instance, after settling a costly dispute about their alliance, KLM and Northwest Airlines deliberately increased their interdependence by eliminating their duplicate support operations[7]. It is also well known that firms sometimes use exclusive sourcing arrangements to increase expected quasi-rents. It is often argued informally that managers take such actions to ensure that their trading relationships 'work well' and that conflicts are settled quickly. My model shows that increasing expected quasi-rents can indeed reduce the probability of ex post disagreements. However, to provide an explanation for why managers might want to increase expected quasi-rents but not maximize them, my model would need to be extended to include an additional cost of reducing the aggregate disagreement payoff. I conjecture that this could be done, for instance, by allowing for ex ante investments.

This paper is related to the literature on the property rights theory of the firm that was first introduced by Grossman and Hart (1986) and Hart and Moore (1990), and was developed further in a number of papers, including de Meza and Lockwood (1998), Rajan and Zingales (1998), Baker, Gibbons, and Murphy (1999). This literature assumes that bargaining between managers is efficient and studies the role of asset ownership in determining ex ante investment inefficiencies. In contrast to this literature, I study the role of asset ownership in determining ex post bargaining inefficiencies rather than ex ante investment inefficiencies. I do so, not because I think that investment inefficiencies are not an important factor in determining the boundaries of firms, but because I think that ex post inefficiencies also play an important role. There are only very few papers in the property rights literature that allow for ex post inefficiency and, to my knowledge, none are related to the issues addressed here[8].

Apart from the property rights literature, my paper is also related to Arrow (1975) and Riordan (1990) who study vertical integration under imperfect information. The key assumption in their models is that vertical integration reduces the amount of private

[6] Holmström and Roberts (1998), p.80-81.

[7] Holmström and Roberts (1998), p.84.

[8] Hart and Moore (1999), for instance, study the design of a firm's constitution when ex post asymmetric information prevents recontracting.

information. Thus, for instance, a downstream firm is assumed to know less about an independent upstream firm than it knows about an inhouse supplier. This assumption contrasts with recent papers by Aghion and Tirole (1997) and Dessein (1999) which show that the incentives of agents to transmit information to a principal may actually be reduced, and thus informational asymmetries be increased, when a principal gains more control over an agent's actions. In contrast to Arrow (1975) and Riordan (1990), I therefore assume that ownership changes do not affect the amount of private information per se and instead merely determine the managers' disagreement payoffs.

Technically, the general model that I present in the second half of the paper is closely related to Myerson and Satterthwaite (1983). Among other important results, they derive the optimal mechanism for a bilateral bargaining situation with two-sided asymmetric information. The key difference between their analysis and mine is that they derive the optimal mechanism for given disagreement payoffs while I solve for the optimal mechanism and the optimal disagreement payoffs. In spite of this difference I can draw extensively on their results in solving the model. I show that the efficiency of the bargaining mechanism depends only on the aggregate disagreement payoff and not on the distribution if individual disagreement payoffs. Also, I show that it is always optimal to either choose the maximum or the minimum aggregate disagreement payoff and that the former is optimal if the maximum aggregate disagreement payoff is large while the latter is optimal otherwise.

In the next section I present the basic set-up, solve for the optimal ownership structure, and discuss the implications of the analysis. In this section I simplify the analysis by assuming that contracts are incomplete, in the sense that the managers cannot contract ex ante over the bargaining game that takes place ex post, and by restricting attention to uniform distributions. The main worry with this approach, of course, is that the results may rely crucially on these simplifying assumptions[9]. To address this concern I generalize the simple model in section 3 by allowing the managers to contract over the ownership structure and the bargaining game and by allowing for more general

[9] Aghion, Dewatripont, and Rey (1994) study renegotiation design as a solution to the hold-up problem and show that contracts can solve the hold-up problem if they can assign bargaining power to one agent and specify the default option when renegotiation fails.

distributions. The analysis in this section shows that the main results of the simple model continue to hold if and only if ownership structures cannot be made contingent on information that is revealed ex post. Finally, in section 4 I briefly discuss other applications of the analysis and conclude.

2 The Simple Model with Incomplete Contracting

There are two risk neutral and liquidity unconstrained managers, a buyer (she) and a seller (he), and two assets a_1 and a_2. The assets are owned by the managers. The set of assets owned only by the buyer is denoted by $A_b \subseteq \{a_1, a_2\}$ and that owned only by the seller is denoted by $A_s \subseteq \{a_1, a_2\}$. The ownership structure is given by $A = (A_b, A_s)$.

The managers can either trade with each other or with third parties in the market. If the managers trade with each other, they both have access to both assets, regardless of the ownership structure. The seller can then use the assets to produce an input at cost $c \in \Re$ and the buyer can use them to turn the input into a final good that she values at $\pi \in \Re$. It is common knowledge that the 'trade payoffs' π and c are independently drawn from a uniform distribution with support $[0, 1]$[10]. If the managers trade with third parties, each can only use the assets over which he or she has sole ownership rights. The individual disagreement payoffs that the buyer and the seller then realize are denoted by $b(A_b)$ and $s(A_s)$ respectively. The aggregate disagreement payoff is denoted by $d(A) \equiv b(A_b) + s(A_s)$. All disagreement payoffs are common knowledge.

Ex ante the managers contract over the ownership structure A without yet knowing the realizations of the trade payoffs π and c. In this simple model they cannot contract over anything but A; in particular, they cannot contract over the bargaining game that takes place ex post. For the time being I justify this assumption informally by supposing that the complex nature of the input makes it impossible to describe it in any ex ante contract[11].

At the interim stage, the buyer learns the realization of π and the seller learns the

[10] In section 3 I relax this assumption and allow for more general distributions.

[11] In section 3 I relax this assumption and allow the managers to contract over the ownership structure and the ex post bargaining game.

realization of c. Ex post, the managers then bargain over the price of the input. The exogenously given bargaining game takes the form of a simple double auction: the buyer and the seller respectively submit sealed bids $p_b \in \Re$ and $p_s \in \Re$. If $p_b \geq p_s$, trade takes place at price $p = \frac{1}{2}(p_b + p_s)$. The buyer's payoff is then given by $\pi - p$ and the seller's by $p - c$. If $p_b < p_s$, trade does not take place and the managers realize their respective disagreement payoffs $b(A_b)$ and $s(A_s)$.

I denote the ownership structure that minimizes the aggregate disagreement payoff by $\underline{A} \equiv \arg\min_A d(A)$ and the ownership structure that maximizes the aggregate disagreement payoff by $\overline{A} \equiv \arg\max_A d(A)$. The corresponding aggregate disagreement payoffs are denoted by $\underline{d} \equiv d(\underline{A})$ and $\overline{d} \equiv d(\overline{A})$. Also, I assume that the bounds on the aggregate disagreement payoffs satisfy $-1 = \underline{d} \leq \overline{d} \leq 1$. These assumptions simply ensure that, at least sometimes, trading the special widget is (weakly) optimal and, at least sometimes, not trading it is (weakly) optimal. I make these assumptions only because they simplify the exposition. It would be trivial to analyze the implications of relaxing them.

I refer to an ownership structure in which the buyer owns both assets as 'buyer integration' and define 'seller integration' accordingly. Under 'non-integration' each manager owns one asset and under 'joint ownership' both managers own both assets. Finally, I refer to $E_{c,\pi}\left[\pi - c - d \mid \pi - c - d \geq 0\right]$ as the 'expected quasi-rents' and to $E_{c,\pi}\left[\pi - c - \overline{d} \mid \pi - c - \overline{d} \geq 0\right]$ as the 'minimum expected quasi-rents.'

Before moving on to the analysis, it is useful to make a number of conceptual observations. First, in the model I adopt all the key assumptions of the property rights literature. In particular, I follow the literature by assuming that contracts are incomplete and that asset ownership determines the disagreement payoffs but not the trade payoffs. Second, I also follow the property rights literature by assuming that the managers can contractually commit ex ante to ownership structures that might be inefficient if disagreement occurs ex post. This assumption is as strong, or as weak, in my model, in which disagreement can occur on the equilibrium path, as in the standard property rights model (see Hart (1995)), in which equilibrium only takes place off the equilibrium path[12]. Third, I assume that the distribution of the trade payoffs is independent of the ownership structure. As noted in the introduction, I make this assumption since I do not want the ownership structure to have a direct effect on the amount of private information. Finally, I assume that there is only uncertainty about the managers' trade payoffs and not about their disagreement payoffs. This is not a critical restriction. A model with uncertainty over the disagreement payoffs gives the same results as those described below as long as ownership changes are assumed to shift the distributions of the disagreement payoffs. The important point here is only that the reservation payoffs, that is the difference between the trade and the disagreement payoffs, are uncertain.

2.1 The Analysis

I first solve the bargaining game that takes place ex post and then derive the optimal ownership structure on which the managers agree ex ante.

2.1.1 The Bargaining Stage

At the ex post bargaining stage the buyer and the seller play a double auction to determine the terms of trade (taking the ownership structure as given). It is well known (see, for example, Chatterjee and Samuelson (1983)) that the double auction can have many Bayesian Nash equilibria. I focus on the 'linear Bayesian Nash equilibrium' in which the managers' strategies are linear functions of their trade payoffs. I do so primarily because Myerson and Satterthwaite (1983) have shown that if the players' reservation prices are uniformly distributed on $[0, 1]$ this linear Bayesian Nash equilibrium is the most efficient Bayesian Nash equilibrium of the double auction, and indeed of any voluntary bargaining game. Thus, at least for a part of the parameter space, the bargaining inefficiency on which I focus is not an artifact of the particular bargaining game or equilibrium I consider but is the minimum bargaining inefficiency that must arise in this economic setting.

[12] I conjecture that my main results could be derived in a model in which the managers can renegotiate the ownership structures ex post as long as this renegotiation is costly. Renegotiation may be costly, for instance, because each manager has private information about how much they value the asset outside of the trading relationship.

The following proposition describes the equilibrium strategies and differs from the standard analysis of the double auction (see, for instance, Gibbons (1992)) only by allowing explicitly for non-zero disagreement payoffs.

Proposition 1 *The strategies*

$$p_b(\pi) = \begin{cases} \widetilde{p}_b(\pi) & \textit{if } \pi \leq \frac{1}{4}(5+3d(A)) \\ \widetilde{p}_s(1) & \textit{if } \pi > \frac{1}{4}(5+3d(A)) \end{cases}$$

and

$$p_s(c) = \begin{cases} \widetilde{p}_s(c) & \textit{if } c \geq -\frac{1}{4}(1+3d(A)) \\ \widetilde{p}_b(0) & \textit{if } c < -\frac{1}{4}(1+3d(A)), \end{cases}$$

where

$$\widetilde{p}_b(\pi) = \frac{1}{12}\left(1 - 9b(A_b) + 3s(A_s)\right) + \frac{2}{3}\pi$$

and

$$\widetilde{p}_s(c) = \frac{1}{12}(3 - 3b(A_b) + 9s(A_s)) + \frac{2}{3}c,$$

form a linear Bayesian Nash equilibrium of the double auction.

Proof: see appendix.

For the remainder of the paper, when I refer to 'the equilibrium of the double auction,' I mean the linear Bayesian Nash equilibrium in which the players adopt the strategies $p_b(\pi)$ and $p_s(c)$. It is important to observe that for any $d(A) \in [-1, 1)$ this equilibrium is ex post inefficient. To see this, note that in equilibrium trade takes place if and only if

$$\pi - c \geq d(A) + \frac{1}{4}(1 - d(A)).$$

Thus, ex post efficient trading opportunities are not realized for any $\pi - c \in (d(A), d(A) + \frac{1}{4}(1 - d(A)))$. The inefficiency of the double auction, which was first analyzed in Chatterjee and Samuelson (1983), is also illustrated in Figure 1: for any $d(A) \in [-1, 1)$ ex post efficiency requires trade to take place in the 'trade' are above the bold line $(g(A, c))$

and no trade to take place in the 'no trade' area below the bold line ($g(A,c)$). The double auction does not achieve ex post efficiency since it does not realize efficient trading opportunities in the 'inefficiency' area between the two diagonal lines $g(A,c)$ and $h(A,c)$.

2.1.2 The Contracting Stage

Ex ante the managers contract over the ownership structure A. Since they are risk neutral and not wealth constrained they agree on the ownership structure that maximizes social welfare, independent of its ex post distribution. Formally, the managers choose the optimal ownership structure that solves

$$\max_A W(d(A)), \tag{1}$$

where $W\left(d\left(A\right)\right)$ denotes social welfare and is given by

$$W(d\left(A\right)) \equiv d\left(A\right) + E_{c,\pi}\left[(\pi - c - d\left(A\right))q(c,\pi,d(A))\right], \tag{2}$$

for

$$q\left(c,\pi,d(A)\right) \equiv \begin{cases} 1 & \text{if } \pi - c \geq d\left(A\right) + \frac{1}{4}(1 - d\left(A\right)) \\ 0 & \text{otherwise.} \end{cases}$$

Note that social welfare depends on the aggregate disagreement payoff $d(A)$ and not on the distribution of the individual disagreement payoffs $b(A_b)$ and $s(A_s)$. This is the case since, first, social welfare puts equal weight on each manager's individual payoff and, second, in the equilibrium of the double auction the probability of trade depends on the sum of the disagreement payoffs and not on their distribution. Therefore a change in the ownership structure only affects efficiency if it alters the aggregate disagreement payoff and not if it simply leads to a redistribution of disagreement payoffs from one manager to the other. This establishes the following lemma.

Lemma 2 *The efficiency of any ownership structure A depends on the aggregate disagreement payoff $d(A)$ and not on the distribution of the individual disagreement payoffs $b(A_b)$ and $s(A_s)$.*

The key to solving the contracting problem (1) is to understand how a change in the aggregate disagreement payoff affects social welfare. Suppose the managers consider a change in the ownership structure from $\overline{A}$ to A' and that this leads to a reduction in the aggregate disagreement payoff from $\overline{d}$ to $d' \equiv d(A') < \overline{d}$. The effect on social welfare of such a change is given by

$$\begin{aligned} W(d') - W(\overline{d}) \;\; = \;\; & -(\overline{d} - d')\,(1 - q(c, \pi, d(A'))) + \\ & E_{c,\pi}\left[(\pi - c - \overline{d})(q(c, \pi, d(A')) - q(c, \pi, d(\overline{A})))\right]. \end{aligned} \tag{3}$$

On the one hand, the managers realize a lower aggregate payoff if they disagree even after the ownership change. This effect is always negative and is captured by the first term on the RHS of (3). On the other hand, however, a reduction in the aggregate disagreement payoff also commits at least one of the managers to a more cautious bargaining strategy. This increases the probability that trade takes place ex post. In particular, for any realizations of π and c that satisfy $\pi - c \in \left(\frac{3}{4}d' + \frac{1}{4}, \frac{3}{4}\overline{d} + \frac{1}{4}\right]$ trade will take place only after the ownership change. The second term on the RHS of (3) captures the welfare implication of the increase in the probability of trade. Its sign is ambiguous and depends crucially on the size of the reduction in the aggregate disagreement payoff. If the reduction is small, in the sense that $\frac{3}{4}d' + \frac{1}{4} \geq \overline{d}$, the gains $\pi - c$ from each of the additional trades are larger than the aggregate disagreement payoff $\overline{d}$. Thus, in this case the increase in the probability of trade is unambiguously welfare improving. For larger reductions in the aggregate disagreement payoff, however, some of the additional trades the managers realize are 'ex ante inefficient,' in the sense that they satisfy $\pi - c < \overline{d}$. While these trades are ex post efficient given d', the managers would be better off realizing the maximum aggregate disagreement payoff than engaging in these trades. Thus, for large reductions in the aggregate disagreement payoff, that is for $\frac{3}{4}d' + \frac{1}{4} < \overline{d}$, the increased probability of trade has an ambiguous effect on social welfare.

These effects are also illustrated in Figures 2 and 3. For $A = \overline{A}$ the equilibrium of the double auction is inefficient since trade does not take place for realizations of π and c between $g(\overline{A}, c)$ and $h(\overline{A}, c)$. As shown in Figure 2, an ownership change that leads to a small reduction in the aggregate disagreement payoff shifts down $h(\cdot, c)$ to a position somewhere above $g(\overline{A}, c)$. Thus the managers realize more efficient trades (the area between $h(\overline{A}, c)$ and $h(A', c)$) but also receive a lower payoff in the case of disagreement (below $h(A', c)$). Figure 3 shows that an ownership change that leads to a large reduction in the aggregate disagreement payoff shifts $h(\cdot, c)$ to a position below $g(\overline{A}, c)$. Thus, such a change not only increases the probability of efficient trades (the area between $h(\overline{A}, c)$ and $g(\overline{A}, c)$) and the cost of disagreement (the area below $h(A', c)$) but also increases the probability that the managers realize ex ante inefficient trades (the area between $g(\overline{A}, c)$ and $h(A', c)$).

Faced with this trade-off, what ownership structure should the managers choose? It turns out the answer to this question is surprisingly straightforward:

Proposition 3 *At the ex ante stage it is weakly optimal for the parties to agree on the ownership structure*

$$A^* = \begin{cases} \underline{A} & \text{if } \overline{d} \leq d_{crit} \\ \overline{A} & \text{otherwise,} \end{cases}$$

where $d_{crit} \equiv \frac{1}{3}\left(4 - \sqrt{5}\right)$.

Proof: Let $d \in [-1, 1]$ and consider the social welfare function $W(d)$. From (2) it follows that

$$W(d) = \begin{cases} \frac{1}{64}(1+3d)(9+10d-3d^2) & \text{if } -\frac{1}{3} \leq d \leq 1 \\ \\ \frac{1}{192}(1+3d)(5+3d)^2 & \text{if } -1 \leq d \leq -\frac{1}{3} \end{cases}$$

and

$$W'(d) = \begin{cases} -\frac{9}{64}(1-d)^2 + 1 - \frac{9}{32}(1-d)^2 & \text{if } -\frac{1}{3} \leq d \leq 1 \\ \\ -\frac{3}{64}(5+3d)(1-d) + \frac{1}{32}(5+3d)^2 & \text{if } -1 \leq d \leq -\frac{1}{3}. \end{cases}$$

The only stationary point is therefore given by $d = -\frac{7}{9}$ and, since $W''\left(-\frac{7}{9}\right) = \frac{3}{8}$, this stationary point is a local minimum. Also, $W(-1) = W\left(\frac{1}{3}\left(4-\sqrt{5}\right)\right)$. Thus, the solution to (1) is given by $\underline{A}$ if $\overline{d} \leq \frac{1}{3}\left(4-\sqrt{5}\right)$ and by $\overline{A}$ otherwise. ■

Thus the managers optimally choose the ownership structure that minimizes their expected quasi-rents if, for a given distribution of trade payoffs, the minimum expected quasi-rents are small (i.e. when $\overline{d}$ is large) and they choose the ownership structure that maximizes their expected quasi-rents otherwise.

To see the intuition for this proposition consider first Figure 4 that plots the social welfare function $W(d)$ for $d \in [-1, 1]$. The key feature of $W(d)$ is that it only has one interior stationary point and that this stationary point is a minimum. This not only implies that only $\underline{A}$ and $\overline{A}$ can be optimal but also that the former is optimal if $\overline{d}$ is small and that the latter is optimal if $\overline{d}$ is large. To understand why the social welfare function has this shape, consider the welfare effect of a marginal increase in d. It follows from the analysis above that the welfare benefit of a marginal increase in d is given by the higher payoff the managers realize in the case of disagreement while the welfare cost is given by the reduction in the probability that efficient trades take place[13]. If d is small, disagreement does not occur very often. Thus, the welfare benefit of an increase in d is quite small and is indeed dominated by the welfare cost. The larger d, however, the more likely it is that disagreement occurs and thus the larger the welfare benefit of a further in increase in d. For d large enough, the welfare benefit of a further increase in d eventually dominates the welfare cost. The property that a marginal increase in d is welfare reducing for small d and welfare enhancing for large d is the key reason behind the main results of the simple model as summarized in Proposition 3.

2.2 Discussion

The ownership structures that the model predicts are widely observed in the real world. To see this, suppose, without loss of generality, that a_1 is more useful to the buyer than

[13] Above I showed that another benefit associated with an increase in d is the reduction in the probability that managers realize ex ante inefficient trades. Note that this second benefit effect does not opperate on the margin, i.e. it is only realized for sufficiently large discrete changes in d.

a_2 and that a_2 is more useful to the seller than a_1. Thus,

$$b(a_1) + s(a_2) \geq b(a_2) + s(a_1).$$

Furthermore, consider the following definitions:

Definition 4 *The assets a_1 and a_2 are 'non-synergistic' if and only if*

$$b(a_1) + s(a_2) \geq \max\left[b\left(a_1, a_2\right) + s(\emptyset), b(\emptyset) + s\left(a_1, a_2\right)\right].$$

Definition 5 *The assets a_1 and a_2 are 'buyer-synergistic' if and only if*

$$b\left(a_1, a_2\right) + s(\emptyset) \geq \max\left[b(a_1) + s(a_2), b(\emptyset) + s\left(a_1, a_2\right)\right].$$

Definition 6 *The assets a_1 and a_2 are 'seller-synergistic' if and only if*

$$b(\emptyset) + s\left(a_1, a_2\right) \geq \max\left[b(a_1) + s(a_2), b\left(a_1, a_2\right) + s(\emptyset)\right].$$

Hence, assets are buyer-synergistic if, in the case of disagreement, the aggregate payoff is higher under buyer-integration than under seller- or non-integration. Similarly, assets are seller-synergistic if, in the case of disagreement, the aggregate payoff is higher under seller-integration than under buyer- or non-integration. The assets are non-synergistic if, in the case of disagreement, the aggregate payoff is higher under non-integration than under either buyer- or seller-integration. Finally, note that joint ownership of assets minimizes the aggregate disagreement payoff since, under this arrangement, neither manager can use the assets in the case of disagreement. It then immediately follows from Proposition 3 that the optimal ownership structure is given by

$$A^* = \begin{cases} \text{joint ownership} & \text{if } \overline{d} \leq d_{crit} \\ \text{buyer-integration} & \text{if } \overline{d} > d_{crit} \text{ and assets are buyer-synergistic} \\ \text{seller-integration} & \text{if } \overline{d} > d_{crit} \text{ and assets are seller-synergistic} \\ \text{non-integration} & \text{if } \overline{d} > d_{crit} \text{ and assets are non-synergistic} \end{cases}$$

Thus, joint ownership is optimal when, for a given distribution of trade payoffs, the minimum expected quasi-rents are large, i.e. when $\overline{d}$ is small. This is true independent of whether the assets are synergistic or non-synergistic. Intuitively, when the minimum expected quasi-rents are large, the managers do not disagree very often. Thus the welfare costs of further increasing the quasi-rents by moving to joint ownership are quite small and are indeed dominated by the welfare gain.

In the basic property rights model with ex ante investments joint ownership cannot be optimal (see Hart (1995)). Since joint ownership arrangements are, however, frequently observed in the real world it is important to develop theoretic arguments that might explain their existence. A number of recent papers have extended the basic property rights model and shown that under certain conditions joint ownership can provide optimal investment incentives[14]. This paper suggests an additional reason for why joint ownership may be optimal, namely that it minimizes ex post bargaining inefficiencies.

Joint ownership is not optimal when, for a given distribution of trade payoffs, the minimum expected quasi-rents are small, i.e. when $\overline{d}$ is large. Instead, in this case integration is optimal if the assets are synergistic and non-integration is optimal if the assets are non-synergistic. When the minimum expected quasi-rents are small, the managers disagree very often. The welfare cost of a reduction in the aggregate disagreement payoff is therefore quite large and dominates the welfare benefit.

The property rights literature has been criticized for not predicting asset clusters often enough (see Holmström (1999)). Asset clusters are, of course, observed very often since most firms own large numbers of assets while their workers typically have no ownership rights over the assets they use in the production process. In a recent paper Holmström (1999, p.88) asks: "So why do firms own essentially all the nonhuman assets it uses in production? Why do workers - or for that matter any other stakeholder - rarely own any such assets? This strikes me as one of the most basic regularities that a theory of the firm needs to explain." In my model asset clusters arise when assets are synergistic and, for a given distribution of trade payoffs, the minimum expected quasi

[14] Rajan and Zingales (1998) show that joint ownership can be optimal in a property rights model if the investments reduce the players' outside options. Halonen (1995) shows that in an infinitely repeated game with ex ante investments joint ownership can be optimal for reputational reasons. Also, de Meza and Lockwood (1998) and Chiu (1998) show that joint ownership can be optimal when ex post bargaining takes the form of a Rubinstein alternating offers game with outside options.

rents are small. In this case the managers know that they will disagree very often and simply want to ensure that they realize as high a payoff as possible whenever they do disagree. When assets are synergistic this is achieved by clustering the ownership rights of all assets and giving them to the party that has the highest outside value for the assets.

It is reassuring that all four potentially optimal ownership structures can be commonly observed. However, it of course remains an open empirical question whether they actually arise under the conditions predicted by the model. Note, though, that since the predictions in my model only depend on the level of the expected quasi-rents and the nature of the productive assets, and not on marginal investment incentives, they should, in principle, be easier to test empirically than the predictions of the existing property rights models (for a discussion of the empirical evidence of the property rights theory see Whinston (2000)).

3 The General Model with Complete Contracting

I now extend the simple model by allowing the managers to contract over the ownership structure *and* the bargaining game that determines the price of the widget ex post. Also, I now allow for more general distributions of the trade payoffs. I first show that the main results of the simple model continue to hold in this more general set up. In section 3.4 I then argue that the results do not hold once I also allow for contingent ownership structures, that is once I allow for ownership structures to depend on information that is revealed after the initial contracting stage.

The general model differs from that introduced in section 2 in four ways. First, at the ex ante stage the managers can now contract, not only over the ownership structure of the assets, but also over the ex post bargaining game that determines the price of the input. The managers can choose any bargaining game that satisfies the balanced budget constraint. Thus, I only require the payments to the seller to always equal the payments from the buyer. Second, the cumulative density functions of the trade payoffs $\pi \in [\underline{\pi}, \overline{\pi}]$ and $c \in [\underline{c}, \overline{c}]$ are now given by $F_b(\pi)$ and $F_s(c)$ respectively. I assume that the density functions $f_b(\pi)$ and $f_s(c)$ are continuous and strictly positive and that the distributions satisfy the monotone hazard rate conditions

$$\frac{d}{d\pi}(\frac{f_b(\pi)}{1-F_b(\pi)}) \geq 0, \ \forall \pi \in [\underline{\pi}, \overline{\pi}], \quad \text{and} \quad \frac{d}{dc}(\frac{f_s(c)}{F_s(c)}) \leq 0, \forall c \in [\underline{c}, \overline{c}].$$

Third, the bounds on the disagreement payoffs must now satisfy $\overline{d} < \overline{\pi} - \underline{c} \equiv \overline{\Delta}$ and $\underline{d} > \underline{\pi} - \overline{c} \equiv \underline{\Delta}$. The first restriction ensures that, at least sometimes, there are gains from trade while the second restriction ensures that, at least sometimes, there are no gains from trade. These restrictions simplify the exposition and it is trivial to analyze the implications of relaxing them.

Finally, I now assume that the managers can decide whether or not to participate in the ex post bargaining game after learning the realizations of the trade payoffs at the interim stage. If either the buyer or the seller decides not to participate in the bargaining game each manager realizes his or her respective disagreement payoff $b(A)$ and $s(A)$[15].

Before proceeding with the analysis, it is useful to note again how this general set up relates to the model analyzed in Myerson and Satterthwaite (1983). The key difference between their analysis and mine is that they solve for the optimal mechanism for given disagreement payoffs while I solve for the optimal mechanism and the optimal disagreement payoffs. My solution draws on their analysis, nevertheless.

Another paper that is related to the general model that I study here is Cramton, Gibbons, and Klemperer (1987). They extend Myerson and Satterthwaite (1983) by allowing for more general ownership arrangements over the good that is to be traded (and also by allowing for more than two players). In contrast, I retain the assumption in Myerson and Satterthwaite (1983) that the seller initially owns the good that is to be traded but allow for general ownership arrangements over the assets that are required in producing it.

The next section introduces the concept of a direct bargaining mechanism that I use in section 3.2 to solve the model. Section 3.3 gives a simple example of the general model with uniform distributions of trade payoffs and section 3.4 discusses the implications of allowing for contingent ownership structures.

[15] The results in the simple model would be unchanged if I altered it by allowing the managers to opt out of the bargaining game at the interim stage.

3.1 The Direct Bargaining Mechanism

In the analysis I make use of the well-known Revelation Principle which states that, for any Bayesian Nash equilibrium of any bargaining game, there exists a Bayesian incentive compatible direct mechanism that leads to the same outcome (see, for example, Myerson (1979, 1981)). Thus, instead of studying the very large set of all possible indirect bargaining games to which the parties can commit, I can restrict myself, without loss of generality, to Bayesian incentive compatible direct mechanisms.

In a direct bargaining mechanism the players make reports of their trade payoffs after learning the realizations at the interim stage. A mediator then decides on the probability of trade and on the expected price. Denoting the buyer's and the seller's reports by $\widehat{\pi}$ and $\widehat{c}$ respectively, a direct mechanism is then characterized by two functions: the probability of trade $q(\widehat{c}, \widehat{\pi})$ and the expected price of the good $t(\widehat{c}, \widehat{\pi})$. For a given mechanism $(q(\cdot), t(\cdot))$, it is useful to define

$$\overline{q}_s(\widehat{c}) \equiv E_\pi[q(\widehat{c}, \pi)], \qquad \overline{q}_b(\widehat{\pi}) \equiv E_c[q(c, \widehat{\pi})],$$

$$\overline{t}_s(\widehat{c}) \equiv E_\pi[t(\widehat{c}, \pi)], \text{ and } \qquad \overline{t}_b(\widehat{\pi}) \equiv E_c[t(c, \widehat{\pi})].$$

Thus, $\overline{q}_s(\widehat{c})$ and $\overline{t}_s(\widehat{c})$ respectively denote the seller's expected probability of trade and the transfer he expects to receive, given that he announces $\widehat{c}$ and that the buyer truthfully reveals her type. Similarly, $\overline{q}_b(\widehat{\pi})$ and $\overline{t}_b(\widehat{\pi})$ respectively denote the buyer's expected probability of trade and the transfer she expects to make if she announces $\widehat{\pi}$ and the seller truthfully reveals his type. Furthermore, let

$$U_b(\pi) \equiv -\overline{t}_b(\pi) + [1 - \overline{q}_b(\pi)]\, b(A_b) + \pi \overline{q}_b(\pi)$$

and

$$U_s(c) \equiv \overline{t}_s(c) + [1 - \overline{q}_s(c)]\, s(A_s) - c\overline{q}_s(c)$$

be the buyer's and the seller's expected utility from truthfully reporting their own type when the other party is also expected to be truthful.

A direct mechanism is 'Bayesian incentive compatible' if it is a Bayesian Nash equilibrium for the players to report their true trade payoffs. This will be the case if and only if the Bayesian incentive compatibility constraints

$$U_b(\pi) \geq b(A_b) + (\pi - b(A_b))\overline{q}_b(\pi') - \overline{t}_b(\pi') \qquad \forall \pi, \pi' \in [\underline{\pi}, \overline{\pi}] \tag{IC}$$

and

$$U_s(c) \geq s(A_s) + \overline{t}_s(c') - (c + s(A_s))\overline{q}_s(c') \qquad \forall c, c' \in [\underline{c}, \overline{c}].$$

are satisfied.

A Bayesian incentive compatible mechanism is 'interim individually rational' if, after learning their own type at the interim stage, each party prefers participating in the mechanism to realizing its disagreement payoff. For this to be the case the interim individual rationality constraints

$$U_b(\pi) \geq b(A_b) \qquad \forall \pi \in [\underline{\pi}, \overline{\pi}] \tag{IR}$$

and

$$U_s(c) \geq s(A_s) \qquad \forall c \in [\underline{c}, \overline{c}]$$

need to be satisfied.

3.2 The Analysis

At the ex ante stage, the risk neutral and liquidity unconstrained managers agree on the incentive compatible and individually rational mechanism $(q(\cdot), t(\cdot))$ and on the ownership structure A that maximize expected social welfare $W(d(A),\ q(\cdot))$, defined as

$$W(d(A),\ q(\cdot)) \equiv d(A) + E_{\pi,c}\left[(\pi - c - d(A))q(c, \pi)\right]. \tag{4}$$

Formally, the contracting problem can be stated as

$$\max_{A, q(\cdot), t(\cdot)} W(d(A),\ q(\cdot)) \quad \text{subject to IC and IR.} \tag{5}$$

To derive the solution to this contracting problem, consider the following two well-known lemmas.

Lemma 7 *A mechanism* $(q(\cdot), t(\cdot))$ *is Bayesian incentive compatible if and only if* $\overline{q}_s(c)$ *is non-increasing,* $\overline{q}_b(\pi)$ *is non-decreasing,*

$$U_s(c) = U_s(\overline{c}) + \int_c^{\overline{c}} \overline{q}_s(t)dt,$$

and

$$U_b(\pi) = U_b(\underline{\pi}) + \int_{\underline{\pi}}^{\pi} \overline{q}_b(t)dt.$$

Proof: see appendix.

I can therefore replace the (IC) constraint in the contracting problem (5) by the (four) conditions specified in the lemma. The last two of these conditions can be used further to prove the next lemma.

Lemma 8 *A Bayesian incentive compatible and interim individually rational mechanism* $(q(\cdot), t(\cdot))$ *satisfies*

$$U_s(\overline{c}) + U_b(\underline{\pi}) - d(A) = E_{c,\pi}[(\pi - c - d(A) - \frac{1 - F_b(\pi)}{f_b(\pi)} - \frac{F_s(c)}{f_s(c)})q(c,\pi)] \geq 0.$$

Proof: see appendix.

The optimal trading rule and ownership structure therefore solve

$$\max_{q(\cdot),A} W(d(A), q(\cdot)) \tag{6}$$

s.t.

$$E_{c,\pi}[(\pi - c - d(A) - \frac{1 - F_b(\pi)}{f_b(\pi)} - \frac{F_s(c)}{f_s(c)})q(c,\pi)] \geq 0, \tag{IR*}$$

$$\overline{q}_s(c) \text{ is non-increasing, and } \overline{q}_b(\pi) \text{ is non-decreasing.} \tag{IC*}$$

Given the solution to this problem, the optimal transfer rule can then be derived using Lemmas 7 and 8.

It turns out, and I verify this claim below, that the (IC*) constraints in the contracting problem (6) are not binding. Two observations can then be made. First, the individual disagreement payoffs enter the contracting problem symmetrically and additively. Thus, just as in the simple model, the aggregate disagreement payoff, and not the distribution of the individual disagreement payoffs, matters in the contracting problem. Second, a marginal change in the aggregate disagreement payoff affects the contracting problem in two ways. On the one hand, for a given trading rule, an increase in the aggregate disagreement payoff increases the payoff the managers realize in the case of disagreement. This effect is clearly welfare improving. On the other hand, however, a marginal increase in the aggregate disagreement payoff also makes the participation constraint more binding and thus reduces the set of feasible trading rules. Other things equal this effect is welfare reducing.

In this general model the managers therefore face a very similar trade-off to the trade-off in the simple model. The following proposition shows that the solutions to the contracting problems are also very similar in the two models.

Proposition 9 *The optimal ownership structure and trading rule are given by*

$$A^* = \begin{cases} \underline{A} & \text{if } d(\overline{A}) \leq d_{crit} \\ \overline{A} & \text{otherwise.} \end{cases}$$

$$q^*(c,\pi) = \begin{cases} 1 & \text{if } \pi - c - d(A^*) \geq \frac{\lambda(d(A^*))}{1+\lambda(d(A^*))}[\frac{1-F_b(\pi)}{f_b(\pi)} + \frac{F_s(c)}{f_s(c)}] \\ 0 & \text{otherwise.} \end{cases}$$

where $d_{crit} \in [\underline{\Delta}, \overline{\Delta}]$ *and* $\lambda(d(A^*)) \in (0, \infty)$*.*

Proof:

In the contracting problem (6), replace the discrete variable $d(A)$ with the continuously defined $d \in (\underline{\Delta}, \overline{\Delta})$. The augmented contracting problem that needs to be considered is therefore given by

$$\max_{q(\cdot)\in[0,1], d\in[\underline{d},\overline{d}]} W(d, q(\cdot)) \tag{7}$$

s.t.

$$E_{c,\pi}[(\pi - c - d - \frac{1-F_b(\pi)}{f_b(\pi)} - \frac{F_s(c)}{f_s(c)})q(c,\pi)] \geq 0, \tag{IR**}$$

$$\overline{q}_s(c) \text{ is non-increasing, and } \overline{q}_b(\pi) \text{ is non-decreasing.} \tag{IC**}$$

I first solve for the optimal disagreement payoff d^{**} and trading rule $q^{**}(\cdot)$ that solve (7) subject to (IR^{**}) and then confirm that $q^{**}(\cdot)$ also satisfies the (IC^{**}) constraint. The Lagrangian that I need to consider is then given by

$$L(q(\cdot), d, \lambda) \equiv d + E_{c,\pi}\left[[\pi - c - d + \lambda(\pi - c - d - \frac{1-F_b(\pi)}{f_b(\pi)} - \frac{F_s(c)}{f_s(c)})]q(c,\pi)\right],$$

where $\lambda \in [0, \infty)$ is the Lagrangian multiplier.

In Lemma 10 I show that the (IR^{**}) constraint must be binding at the optimum[16]. The necessary first order conditions are therefore given by

$$\frac{\partial L(q(\cdot), d, \lambda)}{\partial q(\cdot)} = \begin{cases} \geq 0 & \text{if } q(\cdot) = 1 \\ = 0 & \text{if } 0 < q(\cdot) < 1 \\ \leq 0 & \text{if } q(\cdot) = 0, \end{cases} \qquad \frac{\partial L(q(\cdot), d, \lambda)}{\partial d} = \begin{cases} \geq 0 & \text{if } d = \overline{d} \\ = 0 & \text{if } \underline{d} < d < \overline{d} \\ \leq 0 & \text{if } d = \underline{d}, \end{cases}$$

and

$$\frac{\partial L(q(\cdot), d, \lambda)}{\partial \lambda} = 0.$$

For a given d, the optimal trading rule then is

$$q(c, \pi, d, \lambda(d)) = \begin{cases} 1 & \text{if } \pi - c - d \geq \frac{\lambda(d)}{1+\lambda(d)}[\frac{1-F_b(\pi)}{f_b(\pi)} + \frac{F_s(c)}{f_s(c)}] \\ 0 & \text{otherwise,} \end{cases}$$

where $\lambda(d)$ solves

$$\frac{\partial L(q(c, \pi, d, \lambda), d, \lambda)}{\partial \lambda} = 0. \tag{8}$$

In Lemma 11 I show that for any $d \in (\underline{\Delta}, \overline{\Delta})$ there exists a $\lambda(d) > 0$ that solves (8)[17].

To find the optimal disagreement payoff d^{**}, consider the bordered Hessian

$$H(q(\cdot), d, \lambda) \equiv \begin{pmatrix} \frac{\partial^2 L(q(\cdot),d,\lambda)}{\partial \lambda^2} & \frac{\partial^2 L(q(\cdot),d,\lambda)}{\partial \lambda \partial d} & \frac{\partial^2 L(q(\cdot),d,\lambda)}{\partial \lambda \partial q(\cdot)} \\ \frac{\partial^2 L(q(\cdot),d,\lambda)}{\partial \lambda \partial d} & \frac{\partial^2 L(q(\cdot),d,\lambda)}{\partial d^2} & \frac{\partial^2 L(q(\cdot),d,\lambda)}{\partial d \partial q(\cdot)} \\ \frac{\partial^2 L(q(\cdot),d,\lambda)}{\partial \lambda \partial q(\cdot)} & \frac{\partial^2 L(q(\cdot),d,\lambda)}{\partial d \partial q(\cdot)} & \frac{\partial^2 L(q(\cdot),d,\lambda)}{\partial q(\cdot)^2} \end{pmatrix}.$$

Evaluated for $q(c, \pi, d, \lambda(d))$ and $\lambda(d)$ its determinant is given by

[16] I relegate this lemma to the appendix since it follows immediately from Myerson and Satterthwaite (1983).

[17] I also relegate this lemma to the appendix since it follows immediately from Myerson and Satterthwaite (1983).

$$H(q(c,\pi,d,\lambda(d)),d,\lambda(d)) = 2(1+\lambda(d))E_{c,\pi}\left[q(c,\pi,d,\lambda(d))\right] \times E_{c,\pi}\left[\pi - c - d - \frac{1-F_b(\pi)}{f_b(\pi)} - \frac{F_s(c)}{f_s(c)}\right].$$

Since

$$E_{c,\pi}\left[\pi - c - d - \frac{1-F_b(\pi)}{f_b(\pi)} - \frac{F_s(c)}{f_s(c)}\right] = \underline{\pi} - \overline{c} - d < 0 \qquad \forall d \in (\underline{\Delta}, \overline{\Delta})$$

and

$$E_{c,\pi}\left[q(c,\pi,d,\lambda(d))\right] > 0 \qquad \forall d \in (\underline{\Delta}, \overline{\Delta})$$

it follows that

$$|H(q(c,\pi,d,\lambda(d)),d,\lambda(d))| < 0 \;\; \forall d \in (\underline{\Delta}, \overline{\Delta}).$$

Thus, any interior $d \in (\underline{d}, \overline{d})$ that is a local extremum satisfies the sufficient conditions for a minimum. This implies that the optimal disagreement payoff is always given by a corner solution, i.e. $d^{**} \in \{\underline{d}, \overline{d}\}$. The solution to the Lagrangian maximization is therefore given by $d^{**} = \underline{d}$ and $q^{**}(\cdot) = q(c,\pi,\underline{d},\lambda(\underline{d}))$ if $W(\underline{d}, q(c,\pi,\underline{d},\lambda(\underline{d}))) \geq W(\overline{d}, q(c,\pi,\overline{d},\lambda(\overline{d})))$ and by $d^{**} = \overline{d}$ and $q^{**}(\cdot) = q(c,\pi,\overline{d},\lambda(\overline{d}))$ otherwise.

Since $W(d, q(c,\pi,d,\lambda(d)))$ does not have an interior local maximum for any $d \in (\underline{\Delta}, \overline{\Delta})$, there exists a $d_{crit} \in [\underline{\Delta}, \overline{\Delta}]$ such that $W(\underline{d}, q(c,\pi,\underline{d},\lambda(\underline{d}))) \geq W(\overline{d}, q(c,\pi,\overline{d},\lambda(\overline{d})))$ if and only if $\overline{d} \leq d_{crit}$. Thus I can restate the solution of the Lagrangian as been given by $d^{**} = \underline{d}$ and $q^{**}(\cdot) = q(c,\pi,\underline{d},\lambda(\underline{d}))$ if $\overline{d} \leq d_{crit}$ and $d^{**} = \overline{d}$ and $q^{**}(\cdot) = q(c,\pi,\overline{d},\lambda(\overline{d}))$ otherwise.

Finally, note that $q^{**}(c,\pi,d,\lambda(d))$ satisfies (IC^{**}) so that $q^{**}(\cdot)$ and d^{**} solve the overall contracting problem (7). ■

The optimal trading rule, of course, corresponds exactly to the one derived in Myerson and Satterthwaite (1983): trade either takes place with certainty or not at all and ex post efficient trades are not realized for any $\pi - c \in \left[d, d + \frac{\lambda(d(A^*))}{1+\lambda(d(A^*))}\left[\frac{1-F_b(\pi)}{f_b(\pi)} + \frac{F_s(c)}{f_s(c)}\right]\right)$. More importantly for us, Proposition 9 shows that the managers choose $\overline{A}$ if, for a given distribution of trade payoffs, the minimum expected quasi-rents are small (i.e. $\overline{d}$ is large), and they choose $\underline{A}$ otherwise. The solution to the contracting problem is therefore remarkably similar to that presented in the simple model. In contrast to the simple model, however, I cannot derive a closed form solution of the critical level d_{crit}. This is not a substantive problem though if it can be shown that, at least sometimes, $d_{crit} \in (\underline{\Delta}, \overline{\Delta})$, so that neither $\underline{A}$ nor $\overline{A}$ is always optimal. In the next section I prove this by solving the simple example in which the managers' trade payoffs are uniformly distributed on $[0,1]$.

3.3 An Example with Uniform Distributions

Suppose that π and c are uniformly distributed on $[0,1]$. It follows from the analysis above that, for any $d \in (-1,1)$, the optimal trading rule is given by

$$q(c, \pi, d, \lambda(d)) = \begin{cases} 1 & \text{if } \pi - c - d \geq \frac{\lambda(d)}{1+2\lambda(d)}(1-d) \\ 0 & \text{otherwise,} \end{cases}$$

where $\lambda(d) \in (0, \infty)$ solves

$$E_{c,\pi}\left[(2(\pi - c) - 1 - d)\, q(c, \pi, d, \lambda)\right] = 0. \tag{9}$$

Figure 5 plots $W(d, q(c, \pi, d, \lambda(d)))$ for $d \in (-1,1)$[18]. The most important features of $W(d, q(c, \pi, d, \lambda(d)))$ are, first, that it has only one interior extremum and, second, that this extremum is a local minimum. This, of course, establishes that it can indeed be the case that $d_{crit} \in (\underline{\Delta}, \overline{\Delta})$, i.e. neither $\underline{A}$ nor $\overline{A}$ is optimal for all parameter configurations. Note also the similarity of Figure 4 which plots the social welfare function for the double auction and Figure 5 that plots it for the 'best' bargaining game.

3.4 Contingent Ownership Structures

So far I have restricted attention to non-contingent ownership structures. I believe that often transaction costs prevent agents from making ownership structures contingent

[18] To generate this figure I solved $W(d, q(c, \pi, d, \lambda(d)))$ numerically by using Maple software.

on information that is revealed after the initial contracting stage and that the above analysis can be applied to such situations. Nevertheless, it is interesting to discuss the theoretical implications of allowing for contingent ownership structures and I do so in this section. In this discussion I rely on existing results and do not derive any formal arguments.

Reconsider the complete contracting set up introduced above but now suppose that the ownership structures can be made contingent on the announcements the managers make after learning their own trade payoffs. It is useful to distinguish between 'interim disagreement payoffs,' that are the disagreement payoffs the parties realize if, at the interim stage, either manager decides not to participate in the bargaining game, and 'ex post disagreement payoffs,' by which I mean the disagreement payoffs the managers realize if both participate in the bargaining game but then do not reach an agreement. I denote the buyer's and the seller's respective interim disagreement payoffs by $\beta(A)$ and $\sigma(A)$ and their respective ex post disagreement payoffs by $b(A)$ and $s(A)$. Note that assuming deterministic ownership structures, as I did above, imposes the restrictions that $\beta(A) = b(A)$ and $\sigma(A) = s(A)$ and that neither can be made contingent on ex post announcements.

There are two specifications of the model with message-contingent ownership structures that are worth considering. In the first one, the ex post disagreement payoffs are a function of the ownership structure but the interim disagreement payoffs are exogenously given and normalized to zero. Thus, in this specification, the parties can contractually specify a trading rule $q(\widehat{c}, \widehat{\pi})$, a transfer rule $t(\widehat{c}, \widehat{\pi})$, and an ownership structure $A(\widehat{c}, \widehat{\pi})$ but they take as given the interim disagreement payoffs $\beta = \sigma = 0$. It can be shown that in such a model it is always optimal for the managers to choose $A(\widehat{c}, \widehat{\pi}) = \overline{A}$ for all $\widehat{c}$ and $\widehat{\pi}$. The intuition for this result is straightforward. Recall that in the case of deterministic ownership structures a reduction in the aggregate disagreement payoff can increase expected welfare only because it relaxes the interim participation constraints and thereby increases the set of feasible trading rules. In a model in which ex post disagreement payoffs are contractible but interim disagreement payoffs are exogenously given it can never be optimal to reduce the ex post disagreement payoffs since this would merely increase the cost of disagreement but not relax the interim participation constraint[19].

The second specification of the model with contingent ownership structures allows for both the interim and the ex post disagreement payoffs to be a function of the ownership structure. Thus, in this specification, the parties can contractually specify a trading rule $q(\widehat{c}, \widehat{\pi})$, a transfer rule $t(\widehat{c}, \widehat{\pi})$ and an ownership structure $A(\widehat{c}, \widehat{\pi})$. The ownership structure then determines the ex post disagreement payoffs $b(A(\widehat{c}, \widehat{\pi}))$ and $s(A(\widehat{c}, \widehat{\pi}))$, and the interim disagreement payoffs $\beta(A(\widehat{c}, \widehat{\pi}))$ and $\sigma(A(\widehat{c}, \widehat{\pi}))$. In such a model the managers can always achieve first best. To do so they simply specify ownership structures which ensure that interim disagreement payoffs that are low enough to relax the interim participation constraints and then play a d'Aspremont-Gérard-Varet mechanism (see d'Aspremont-Gérard-Varet (1979)) to achieve first best. The main trade-off that I have focused on in this paper, namely that a reduction in the aggregate disagreement payoff increases the cost of disagreement but also reduces the probability of disagreement, is therefore absent in such a model. This is the case since the probability of disagreement can be reduced (by reducing the interim disagreement payoffs) without having to increase the cost of disagreement (by reducing the ex post disagreement payoffs).

In summary, once I allow for complete contracting, the main results of the simple model continue to hold if and only if ownership structures cannot be made contingent on ex post announcements. The key reason for this is that the interim and the ex post disagreement payoffs are only 'linked,' in the sense that the former cannot be changed without also changing the latter, when ownership structures are deterministic. Without such a link the parties' choice of the optimal ownership structure is not determined by the trade-off between a higher cost and a lower probability of disagreement that is central in the simple model.

[19]This result is analytically equivalent to showing that in the model studied in Myerson and Satterthwaite (1983) it is never optimal for the parties to specify transfer rules in which the expected payment by the buyer is larger than the expected payment to the seller (see section 5 in Myerson and Satterthwaite (1983)).

4 Applications and Conclusions

There are many situations in which players bargain over the sharing of quasi-rents in the presence of private information. In this paper I have shown that, in such a situation, the players may have an incentive to take actions prior to the bargaining stage to reduce their aggregate ex post disagreement payoff. Such a reduction increases the probability that the players reach efficient agreements but also increases the costs of disagreements and might even induce them to agree 'too often.' I have shown that it is optimal for players to minimize the aggregate disagreement payoff if, for a given distribution of trade payoffs, the minimum expected quasi-rents are large and to maximize their aggregate disagreement payoff otherwise.

I believe that this analysis cannot only be applied to the ownership structure of firms, as I have done above, but might also be applicable to other institutions and contractual arrangements. An obvious example is the marriage contract which reduces the aggregate disagreement payoff of a couple by giving veto rights over certain actions to both parties and which is typically signed by two people who anticipate to be locked-in in the future and who might reasonably expect future bargaining inefficiencies due to presence of private information[20]. In this interpretation the marriage contract is a means of facilitating domestic decision making, albeit one that comes at the cost of lower payoffs in the case of potential disagreements.

Another potential application, and one that is more closely related to the theory of the firm, is the optimal design of bankruptcy procedures (for an introduction see, for instance, Hart (1995)). Bankruptcy procedures which put a hold on the claims of creditors and allow the incumbent management a period of time to reorganize their enterprise (such as Chapter 11 in the US) have been criticized for being cumbersome and time consuming. The arguments in this paper suggest that it might be possible to accelerate such bankruptcy procedures by reducing the aggregate payoff the parties realize during

[20] In Matouschek (2000) I develop a similar model as the one studied here but allow for a dynamic bilateral bargaining game. In this framework I study the effects of changes in the *inside options* on the duration of temporary disagreements. Instead of increasing the probability of trade, a reduction in the aggregate inside option is shown to accelerate agreement (while making temporary disagreements more costly). The results are very similar to those presented above.

the negotiations, for example by limiting the business transactions the management is allowed to perform, and that such a change can reduce ex post inefficiencies that are due to private information.

The analysis also seems applicable to the optimal design of strike legislation (for theoretical and empirical applications of asymmetric information bargaining games to strikes see, for example, Cramton and Tracy (1992) and Kennan and Wilson (1990, 1993)). In this context one could ask, for instance, if a firm should commit itself contractually not to use temporary replacement workers or not to run down inventories during strikes.

Finally, to the extent that the model can be extended to multilateral bargaining situations, arguments similar to those presented above might also be used to explain the institution used by the Roman Catholic Church to elect a new pope. A new pope is elected by an assembly of cardinals who are locked up in a part of Vatican Palace until they reach an agreement. This institution, called a 'conclave', originated in the 13th century when the cardinals failed to elect a new pope for two years. A local magistrate then decided to improve the cardinals' incentives by locking them up in the episcopal palace, removing its roof, and allowing them nothing but bread and water until they elected the next pope[21]. The observation that this institution has not been abandoned, and only somewhat adapted, suggests it might be efficient for the church as a whole, including the decision making cardinals, to accelerate the decision making process by lowering the payoff the cardinals realize during their negotiations.

While I believe that the basic model can be applied to a number of institutions, it is also evident that the formal analysis very stylized. Generalizing the model, for example by either allowing for private information about the players' disagreement payoffs or for multilateral bargaining, could make for interesting future work.

[21] For more details see www.britannica.com.

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5 Appendix

Proof of Proposition 1:

I first verify that $p_b(\pi)$ is a best response to $p_s(c)$ and then the reverse. It is useful to define $x \equiv \max\left[\widetilde{p}_s(0), \frac{1}{12}(1 - 9b(A_b) + 3s(A_s))\right]$. If the seller plays $p_s(c)$, the buyer's best response must solve

$$\max_{p_b} B(p_b, \pi), \tag{10}$$

where

$$B(p_b, \pi) \equiv b(A_b) + \left[\pi - b(A_b) - \frac{1}{2}\left(p_b + E\left(p_s(c) \mid p_s(c) \leq p_b\right)\right)\right] \operatorname{prob}\left(p_s(c) \leq p_b\right),$$

$$\operatorname{prob}(p_s(c) \leq p_b) = \begin{cases} 0 & \text{if } p_b < x \\ \frac{2}{3}\left(p_b - \widetilde{p}_s(0)\right) & \text{if } x \leq p_b \leq p_s(1) \\ 1 & \text{if } p_b > p_s(1), \end{cases}$$

and

$$E\left(p_s(c) \mid p_s(c) \leq p_b\right) = \begin{cases} \frac{1}{p_b - \widetilde{p}_s(0)}\left[\frac{1}{2}x^2 + \frac{1}{2}p_b^2 - \widetilde{p}_s(0)x\right] & \text{if } x \leq p_b \leq p_s(1) \\ \frac{3}{2}\left[\frac{1}{2}x^2 + \frac{1}{2}p_s(1)^2 - \widetilde{p}_s(0)x\right] & \text{if } p_b > p_s(1). \end{cases}$$

Note that it can never be optimal to set $p_b > p_s(1)$ since $B(p_b, \pi) < B\left(p_s(1), \pi\right)$ for any $p_b > p_s(1)$. Also, it can never be strictly optimal to set $p_b < x$ since $B(p_b, \pi) \leq B\left(x, \pi\right)$ for any $p_b < x$. Finally, note that $B(p_b, \pi)$ is strictly concave in p_b for any $x \leq p_b \leq p_s(1)$. The first order conditions for the buyer's maximization problem are therefore given by

$$\begin{aligned} \frac{\partial B(p_b, \pi)}{\partial p_b} &\leq 0 \text{ if } p_b = x, \ \frac{\partial B(p_b, \pi)}{\partial p_b} = 0 \text{ if } x < p_b < p_s(1), \text{ and} \\ \frac{\partial B(p_b, \pi)}{\partial p_b} &\geq 0 \text{ if } p_b = p_s(1). \end{aligned}$$

This implies that the buyer's best response is given by $p_b(\pi) = \frac{1}{12}\left(1 - 9b(A_b) + 3s(A_s)\right) + \frac{2}{3}\pi$ for any $\pi \leq \frac{1}{4}(5 + 3d(A))$ and $p_b(\pi) = \widetilde{p}_s(1)$ for any $\pi > \frac{1}{4}(5 + 3d(A))$.

Next I verify that $p_s(c)$ is a best response to $p_b(\pi)$. It is useful to define $y \equiv \min\left[\frac{1}{12}(11 - 3b(A_b) + 9s(A_s)), \widetilde{p}_b(1)\right]$. If the buyer plays $p_b(\pi)$, the seller's best response must solve

$$\max_{p_s} S(p_s, c), \tag{11}$$

where

$$S(p_s, c) \equiv s(A_s) + \left[\frac{1}{2}\left(p_s + E\left(p_b(\pi) \mid p_b(\pi) \geq p_s\right)\right) - c - s(A_s)\right] \text{prob}\left(p_b(\pi) \geq p_s\right),$$

$$\text{prob}\left(p_b(\pi) \geq p_s\right) = \begin{cases} 0 & \text{if } p_s > y \\ \frac{\widetilde{p}_b(1) - p_s}{\widetilde{p}_b(1) - \widetilde{p}_b(0)} & \text{if } p_b(0) \leq p_s \leq y \\ 1 & \text{if } p_s < p_b(0), \end{cases}$$

and

$$E\left(p_b(\pi) \mid p_b(\pi) \geq p_s\right) = \begin{cases} \frac{1}{\widetilde{p}_b(1) - p_s}\left[y\widetilde{p}_b(1) - \frac{1}{2}y^2 - \frac{1}{2}p_s^2\right] & \text{if } p_b(0) \leq p_s \leq y \\ \frac{3}{2}\left[y\widetilde{p}_b(1) - \frac{1}{2}y^2 - \frac{1}{2}p_b(0)^2\right] & \text{if } p_s < p_b(0). \end{cases} .$$

Note that it can never be strictly optimal to set $p_s > y$ since $S(p_s, c) \leq S(y, c)$ for any $p_s > y$. Note also that it can never be optimal to set $p_s < p_b(0)$ since $S(p_s, c) < S(p_b(0), c)$ for any $p_s < p_b(0)$. Finally, note that $S(p_s, c)$ is strictly concave in p_s for any $p_b(0) \leq p_s \leq y$. The first order conditions for the seller's maximization problem are therefore given by

$$\begin{aligned} \frac{\partial S(p_s, \pi)}{\partial p_s} &\leq 0 \text{ if } p_s = p_b(0), \ \frac{\partial S(p_s, \pi)}{\partial p_s} = 0 \text{ if } p_b(0) < p_s < y, \text{ and} \\ \frac{\partial S(p_s, \pi)}{\partial p_s} &\geq 0 \text{ if } p_s = y. \end{aligned}$$

This implies that a best response is given by $p_s(c) = \frac{1}{12}\left(3 + 9s(A_s) - 3b(A_b)\right) + \frac{2}{3}c$ for any $c \geq -\frac{1}{4}(1 + 3d(A))$ and $p_s(c) = \widetilde{p}_b(0)$ for any $c < -\frac{1}{4}(1 + 3d(A))$. ■

Proof of Lemma 7:

Since Lemma 7 and its proof are standard I only provide the proof for the seller. The proof for the buyer is very similar.

Only if: for any $c, c' \in [\underline{c}, \overline{c}]$ (IC) implies that

$$\begin{aligned} U_s(c) &\geq U_s(c') + \overline{q}_s(c')(c' - c) \\ U_s(c') &\geq U_s(c) + \overline{q}_s(c)(c - c'). \end{aligned}$$

Thus

$$\overline{q}_s(c)(c'-c) \geq U_s(c) - U_s(c') \geq \overline{q}_s(c')(c'-c)$$

which implies that $\overline{q}_s(c)$ is non-increasing. Dividing by $c'-c$ and taking $c' \longrightarrow c$ gives $U_s'(c) = -\overline{q}_s(c)$ almost everywhere. Integrating then gives $U_s(c) = U_s(\overline{c}) + \int_c^{\overline{c}} \overline{q}_s(t)dt$.

If: I need to show that

$$(c+s)(\overline{q}_s(c) - \overline{q}_s(c')) + \overline{t}_s(c') - \overline{t}_s(c) \leq 0 \quad \forall c, c' \in [\underline{c}, \overline{c}]. \tag{12}$$

Rearranging $U_s(c) = U_s(\overline{c}) + \int_c^{\overline{c}} \overline{q}_s(t)dt$ gives

$$\overline{t}_s(c) = U_s(\overline{c}) + \int_c^{\overline{c}} \overline{q}_s(t)dt + \overline{q}_s(c)(c+s) - s.$$

Substituting for $\overline{t}_s(c)$ and $\overline{t}_s(c')$ into 12 then gives

$$\overline{q}_s(c')(c'-c) + \int_{c'}^{\overline{c}} \overline{q}_s(t)dt - \int_c^{\overline{c}} \overline{q}_s(t)dt = \overline{q}_s(c')(c'-c) - \int_c^{c'} \overline{q}_s(t)dt \leq 0.$$

Finally, note that

$$\overline{q}_s(c')(c'-c) - \int_c^{c'} \overline{q}_s(t)dt = \int_{c'}^{c} \overline{q}_s(t) - \overline{q}_s(c')dt \leq 0$$

since $\overline{q}_s(\cdot)$ is non-increasing. ■

Proof of Lemma 8:

It follows from Lemma 7 that

$$\overline{t}_s(c) = U_s(\overline{c}) + \int_c^{\overline{c}} \overline{q}_s(t)dt + \overline{q}_s(c)(c+s) - s.$$

Taking expectations with respect to c then gives

$$\begin{aligned}\int_{\underline{\pi}}^{\overline{\pi}} \int_{\underline{c}}^{\overline{c}} t(c,\pi) f_s(c) f_b(\pi) dc d\pi &= U_s(\overline{c}) - s + \int_{\underline{\pi}}^{\overline{\pi}} \int_{\underline{c}}^{\overline{c}} (c+s) q(c,\pi) f_s(c) f_b(\pi) dc d\pi \\ &\quad + \int_{\underline{c}}^{\overline{c}} \int_c^{\overline{c}} \overline{q}_s(t) dt f_s(c) dc.\end{aligned}$$

Integrating by parts then gives

$$\begin{aligned}&\int_{\underline{\pi}}^{\overline{\pi}} \int_{\underline{c}}^{\overline{c}} t(c,\pi) f_s(c) f_b(\pi) dc d\pi \qquad (13)\\ = \quad & U_s(\overline{c}) - s + \int_{\underline{\pi}}^{\overline{\pi}} \int_{\underline{c}}^{\overline{c}} q(c,\pi) \left[c + s + \frac{F_s(c)}{f_s(c)}\right] f_s(c) f_b(\pi) dc d\pi\end{aligned}$$

Similarly, it follows from Lemma 7 that

$$\overline{t}_b(\pi) = b - U_s(\underline{\pi}) + \overline{q}_b(\pi)(\pi - b) - \int_{\underline{\pi}}^{\pi} \overline{q}_b(t)dt.$$

Taking expectations with respect to π then gives

$$\begin{aligned} & \int_{\underline{\pi}}^{\overline{\pi}} \int_{\underline{c}}^{\overline{c}} t(c,\pi) f_s(c) f_b(\pi) dc d\pi \\ = \quad & b - U_s(\underline{\pi}) + \int_{\underline{\pi}}^{\overline{\pi}} \int_{\underline{c}}^{\overline{c}} (\pi - b) q(c,\pi) f_s(c) f_b(\pi) dc d\pi - \int_{\underline{\pi}}^{\overline{\pi}} \int_{\underline{\pi}}^{\pi} \overline{q}_b(t) dt f_b(\pi) d\pi. \end{aligned}$$

Integrating by parts then gives

$$\begin{aligned} & \int_{\underline{\pi}}^{\overline{\pi}} \int_{\underline{c}}^{\overline{c}} t(c,\pi) f_s(c) f_b(\pi) dc d\pi \qquad (14) \\ = \quad & b - U_s(\underline{\pi}) + \int_{\underline{\pi}}^{\overline{\pi}} \int_{\underline{c}}^{\overline{c}} q(c,\pi) \left[\pi - b - \frac{1 - F_b(\pi)}{f_b(\pi)} \right] f_s(c) f_b(\pi) dc d\pi. \end{aligned}$$

Equating (13) and (14) then gives

$$U_s(\overline{c}) + U_s(\underline{\pi}) - s - b = \int_{\underline{\pi}}^{\overline{\pi}} \int_{\underline{c}}^{\overline{c}} q(c,\pi) \left[\pi - c - s - b - \frac{1 - F_b(\pi)}{f_b(\pi)} - \frac{F_s(c)}{f_s(c)} \right] f_s(c) f_b(\pi) dc d\pi.$$

Finally, note that it follows from Lemma 7 that interim individual rationality requires $U_s(\overline{c}) + U_s(\underline{\pi}) - s - b \geq 0$. ■

Lemma 10 *The first best disagreement payoff $d_{FB} = \overline{d}$ and the first best trading rule*

$$q_{FB}(c,\pi) = \begin{cases} 1 & \text{if } \pi - c \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

do not satisfy the participation constraint (IR^{**}).

Proof: This proof only differs from that derived in Myerson and Satterthwaite (1983)[22] by allowing explicitly for non-zero disagreement payoffs. Evaluating (IR^{**}) for the first best disagree first best disagreement payoffs and trading rule gives

$$E_{c,\pi}[(\pi - c - \overline{d} - \frac{1 - F_b(\pi)}{f_b(\pi)} - \frac{F_s(c)}{f_s(c)}) q_{FB}(c,\pi)] = -\int_{\underline{\pi}}^{\overline{d}+\overline{c}} (1 - F_b(\pi)) F_s(\pi - \overline{d}) d\pi < 0.$$

■

[22] See Myerson and Satterthwaite (1983), p.272.

Lemma 11 *For any $d \in (\underline{\Delta}, \overline{\Delta})$ there exists a $\lambda(d) > 0$ such that*

$$\frac{\partial L(q(c,\pi,d,\lambda(d)),d,\lambda(d))}{\partial\lambda} = 0$$

Proof: This proof only differs from that derived in Myerson and Satterthwaite (1983)[23] by allowing explicitly for non-zero disagreement payoffs. Let

$$G(\lambda) \equiv \frac{\partial L(q(c,\pi,d,\lambda),d,\lambda)}{\partial\lambda}$$

First, note that

$$G(0) = -\int_{\underline{\pi}}^{d+\overline{c}} (1-F_b(\pi))F_s(\pi-d)d\pi$$

is strictly negative if $d \in (\underline{\Delta}, \overline{\Delta})$. Second, note that $\lim_{\lambda\to\infty} G(\lambda) \geq 0$ since, as λ becomes very large trade takes place if and only if $\pi - c - d - \frac{1-F_b(\pi)}{f_b(\pi)} - \frac{F_s(c)}{f_s(c)} \geq 0$. Third, to see that $G(\lambda)$ is increasing consider any λ' and λ'' such that $\lambda' > \lambda'' > 0$. The only difference between $q(c,\pi,d,\lambda')$ and $q(c,\pi,d,\lambda'')$ is that the latter realizes the trades for which

$$\frac{\lambda'}{1+\lambda'}[\frac{1-F_b(\pi)}{f_b(\pi)} + \frac{F_s(c)}{f_s(c)}] > \pi - c - d \geq \frac{\lambda''}{1+\lambda''}[\frac{1-F_b(\pi)}{f_b(\pi)} + \frac{F_s(c)}{f_s(c)}]$$

while the former does not. Since for any such trades $\pi - c - d - \frac{1-F_b(\pi)}{f_b(\pi)} - \frac{F_s(c)}{f_s(c)} < 0$ it follows that $G(\lambda') \geq G(\lambda'')$. Finally, to see that $G(\lambda)$ is continuous note that, because of the monotone hazard rate condition, $c + \frac{\lambda}{1+\lambda}\frac{F_s(c)}{f_s(c)}$ is strictly increasing in c for any $\lambda \in (0,\infty)$. Thus $\frac{\lambda}{1+\lambda}[\frac{1-F_b(\pi)}{f_b(\pi)} + \frac{F_s(c)}{f_s(c)}] = \pi - c - d$ has at most one solution. Note that this solution is continuous in λ, d, and π. Therefore I can rewrite $G(\lambda)$ as

$$G(\lambda) = \int_{\underline{\pi}}^{\overline{\pi}} \int_{\underline{c}}^{g(\lambda,\pi,d)} \left[\pi - c - d - \frac{1-F_b(\pi)}{f_b(\pi)} - \frac{F_s(c)}{f_s(c)}\right] f_s(c) f_b(\pi) dc d\pi,$$

where $g(\lambda,\pi,d)$ is continuous in λ and in π. Thus, since $G(0) < 0$, $\lim_{\lambda\to\infty} G(\lambda) \geq 0$, and $G(\lambda)$ is continuously increasing in λ it must be the case that there exists a $\lambda(d) > 0$ such that $G(\lambda(d)) = 0$. ■

[23] See Myerson and Satterthwaite (1983), p.275-276.

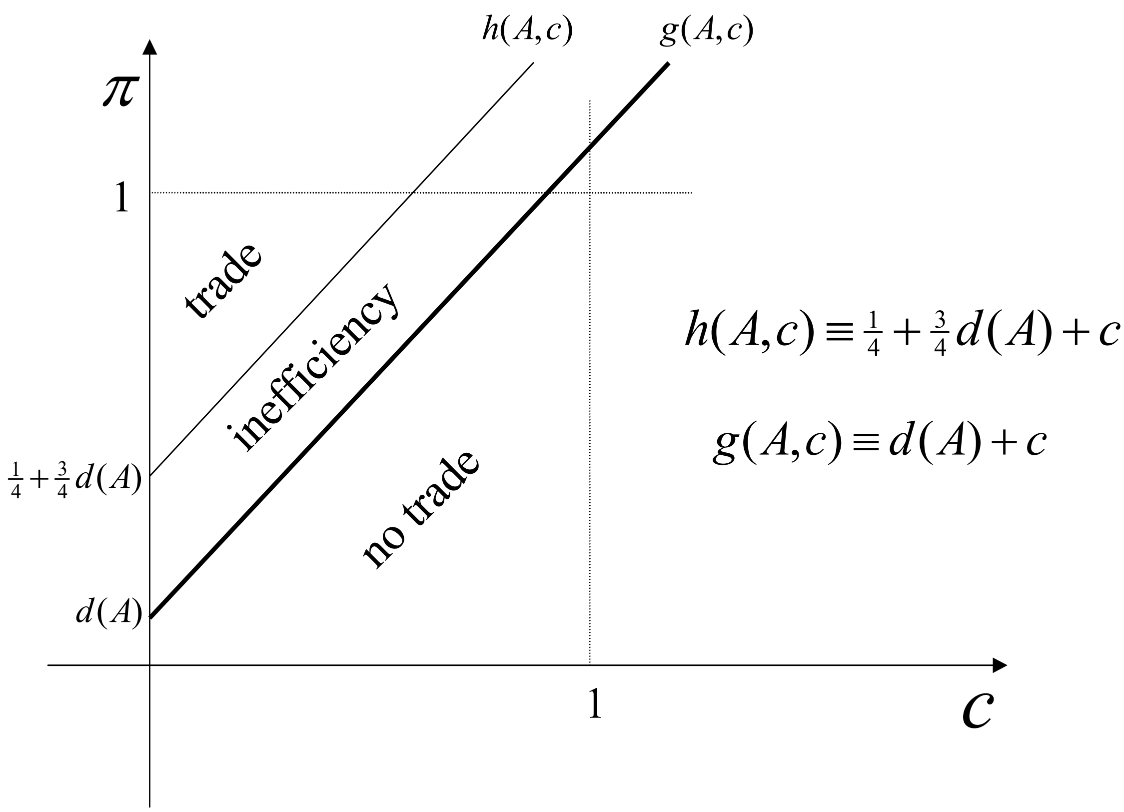

Figure 1: Bargaining inefficiency in a double auction

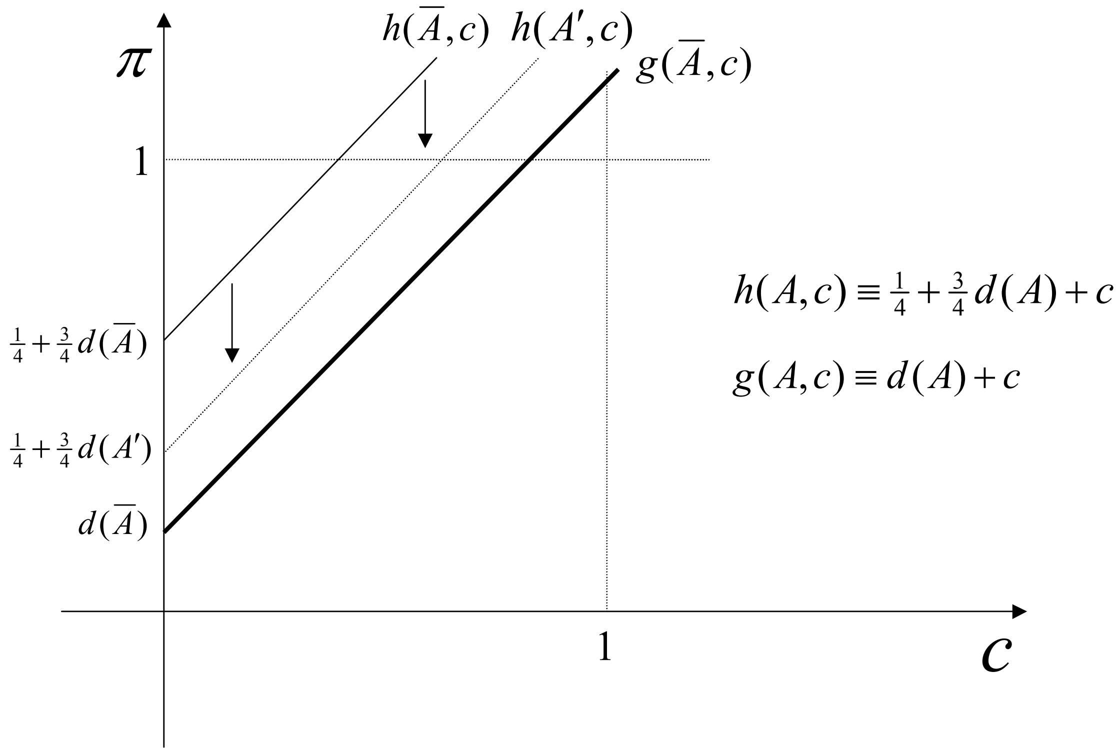

Figure 2: The effect of a small reduction in the aggregate disagreement payoff

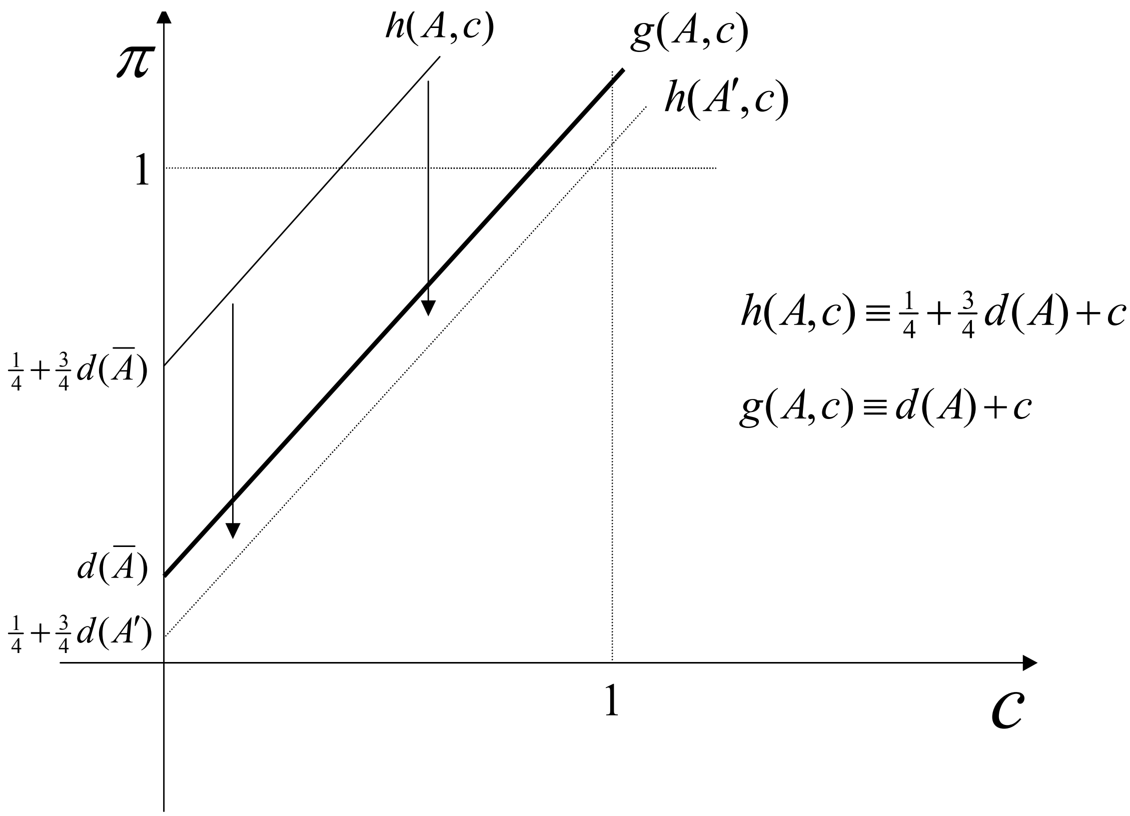

Figure 3: The effect of a large reduction in the aggregate disagreement payoff

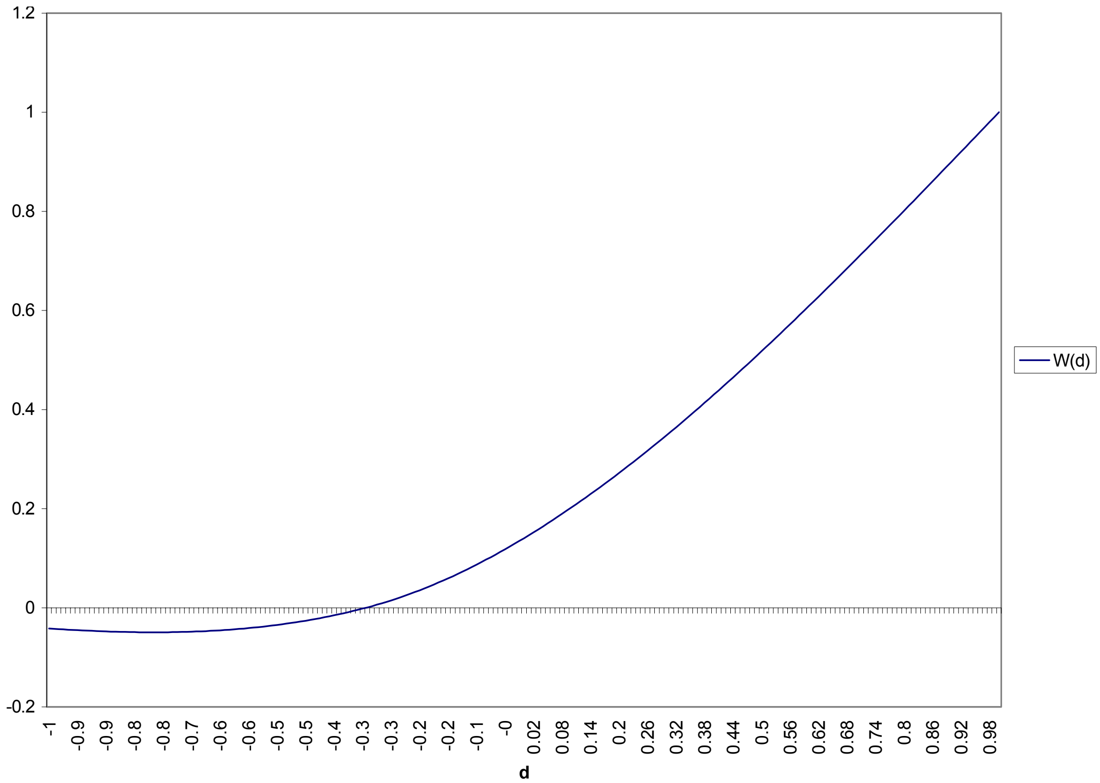

Figure 4: Expected welfare in the double auction

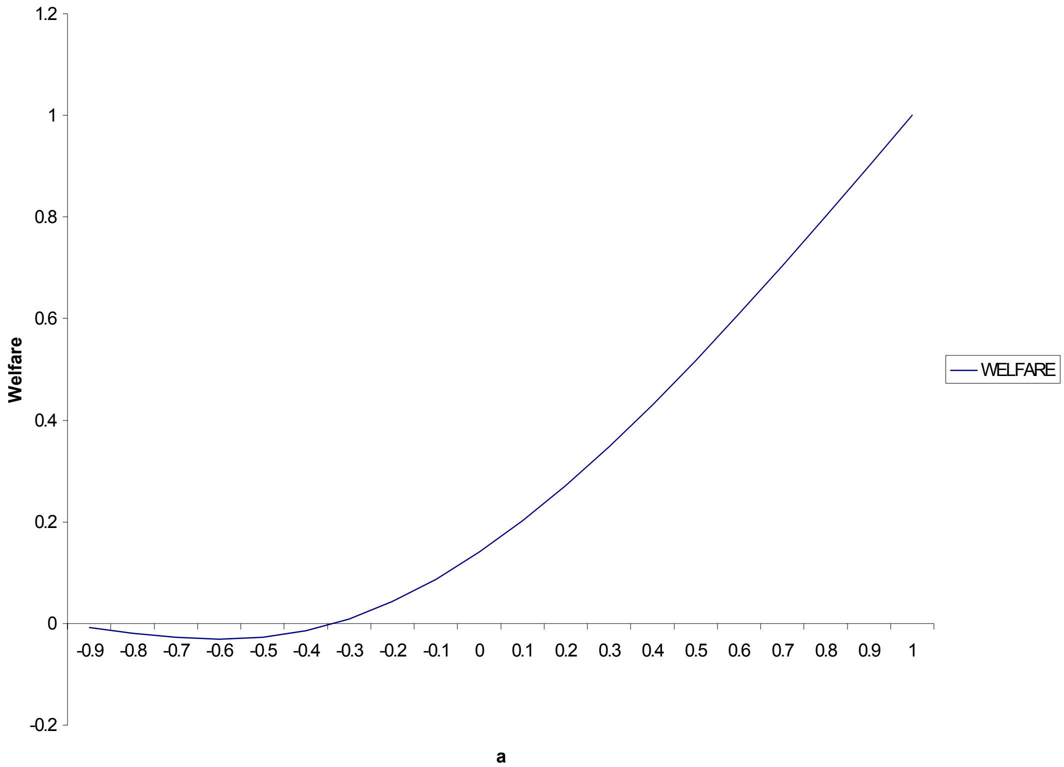

Figure 5: Expected social welfare for the most efficient bargaining game with uniform distributions