Group Lending with Adverse Selection

Jean-Jacques Laffont and Tchéché N’Guessan

USC Center for Law, Economics & Organization
Research Paper No. C02-23
Group Lending with Adverse Selection

Jean-Jacques Laffont†
and
Tchétché N’Guessan‡

October 26, 1999

*We are grateful to Beatriz Armendariz for attracting our interest to this topic. We thank Patrick Rey for his comments. Support of the World Bank to CREMIDE is gratefully acknowledged.

†ARQADE and IDEI, Université des Sciences Sociales de Toulouse, France.
‡CREMIDE and CIRES, Université Cocody, Abidjan, Côte d'Ivoire.
Abstract

We focus on adverse selection as a foundation of group lending. In a simple static model we show that there is no collateral effect if borrowers do not know each other. If the borrowers know each other, group lending implements efficient lending. However, it is not robust to collusive behavior, when transfers are allowed between colluding partners. Finally, we characterize the optimal collusion-proof group contract.
1 Introduction

The Grameen Bank founded by Dr. Yunus in Bangladesh and similar group lending institutions are receiving growing attention as a potential innovative instrument to fight poverty. Development practitioners are very much aware that lending to the poor is a challenge that classical banking has not overcome. Studies from the World Bank have stressed the high recovery rates obtained for the loans granted by these banks (see Khandker, Khalily and Khan (1995)), as well as the positive effects of those loans on social behavior (Pitt and Khandker (1996)). However, some researchers have raised doubts about their sustainability and about the efficient use of the subsidies these institutions benefit from (see Morduch (1997)).

Economic theorists have been interested by the joint-liability lending institutions since Stiglitz (1990). In a recent comprehensive survey Ghatak and Guinane (1998) stress that such institutions can make progress on four problems facing lenders: To ascertain what kind of a risk the potential borrower is (adverse selection), to make sure the borrower will utilize the loan properly so that he will be able to repay (moral hazard), to learn how his project really did in case he declares he cannot pay (auditing), and to find methods to force the borrower to repay the loan if he is reluctant to do so (enforcement). According to these authors the two reasons why these joint liability contracts perform well is because they use the facts that members of a community may know more about one another than a bank and that poor people’s neighbors may be able to impose powerful non-financial sanctions at low cost.

In this paper focused on adverse selection only as a foundation for group lending, we show that, when the investment projects of the members of the group do not know each other, there is no collateral effect of group lending, and that such an effect appears when the borrowers know each other. Contrary to the literature, we consider a monopolistic banking system suffering from adverse selection, but similar results would obtain, if we were maximizing social welfare under incentive constraints and budget balance constraints for the banking sector. Section 2 presents the model and determines the optimal individual loans. Optimal group lending is determined in section 3 when borrowers do not know each other. Section 4 discusses the case of borrowers who know each other and Section 5 characterizes the optimal collusion-proof group lending contract.

2 The Model

There is a continuum of risk neutral borrowers with no personal wealth and limited liability. A proportion $\Pi$ of borrowers, the good type, have sure projects with return $h$ and a proportion $1 - \Pi$, the bad type, have (stochastically independent) projects with return $h$ with probability $p < 1$ and return 0 with probability $1 - p$. All borrowers have outside opportunities valued at $u > 0$ and the type of a borrower is his private (non-verifiable) information.

There is a single bank available for loans which has a refinancing rate of $r$. The bank
offers contracts to maximize its expected profit. For simplicity, we assume that all projects which require one unit of investment are socially valuable, i.e., $ph > r + u$ or $h > \frac{r + u}{p}$.

Let us refer to the good type with the index 1 and to the bad type with the index 2. From the revelation principle, we know that any individual lending strategy is equivalent to a revelation mechanism $(r_1, P_1), (r_2, P_2)$, where $P_i$ is the probability of obtaining a loan and $r_i$ is the interest rate to be paid to the bank if the borrower announces that he is of type $i$, and if his investment succeeds.

The bank maximizes its expected profit under the incentive and participation constraints of the representative borrower, i.e., solves the program:

$$\max \Pi P_1(r_1 \leftrightarrow r) + (1 \leftrightarrow \Pi) P_2(pr_2 \leftrightarrow r)$$

s.t.

$$P_1(h \leftrightarrow r_1) \geq P_2(h \leftrightarrow r_2) \quad (1)$$

$$pP_2(h \leftrightarrow r_2) \geq pP_1(h \leftrightarrow r_1) \quad (2)$$

$$P_1(h \leftrightarrow r_1) \geq u \quad (3)$$

$$pP_2(h \leftrightarrow r_2) \geq u \quad (4)$$

(1) (resp. (2)) is the incentive constraint of type 1 (resp. 2) and (3) (resp. (4)) is the participation constraint\(^1\) of type 1 (resp. 2). We obtain (see appendix 1).

**Proposition 1** Individual lending to both types occurs if

$$h \geq \frac{r + u}{p} + \frac{\Pi \frac{1 \leftrightarrow p u}{p}}{1 \leftrightarrow \Pi} = h^*.$$ 

Lending to the good type only occurs if $h < h^*$.

The bank does not like to give up an information rent to the good type. To limit the cases where it happens, it lends less often to both types that it would be socially efficient.

### 3 Optimal Group Lending

One may wonder if group lending may be a more powerful instrument for the bank. Let us restrict the analysis to groups of two borrowers.\(^2\) Borrowers do not know each other. They know that the matching with other borrowers will be random. We will have to

\(^1\)We assume for simplicity that, when a borrower applies for a loan, he looses its outside opportunity. Similar results obtain if we write the participation constraint of, say, type 1:

$$P_i(h - r_i) + (1 - P_i)u \geq u.$$

\(^2\)Extensions to more than two borrowers are straightforward.
distinguish two cases according to the composition of the set of borrowers applying for loans.

Consider first the case where everybody applies. For a good type to apply requires

$$\Pi(h \leftrightarrow r_0) + (1 \leftrightarrow \Pi)(p(h \leftrightarrow r_0) + (1 \leftrightarrow p)(h \leftrightarrow x)) \geq u$$

(5)

where $r_0$ is the payment if both partners are successful and $x$ is the payment that a succesful partner must make when his partner fails.

For a bad type to apply also requires:

$$\Pi p(h \leftrightarrow r_0) + (1 \leftrightarrow \Pi)(p^2(h \leftrightarrow r_0) + p(1 \leftrightarrow p)(h \leftrightarrow x)) \geq u$$

(6)

or

$$\Pi(h \leftrightarrow r_0) + (1 \leftrightarrow \Pi)(p(h \leftrightarrow r_0) + (1 \leftrightarrow p)(h \leftrightarrow x)) \geq \frac{u}{p}.$$  

(7)

Hence (7) is the binding constraint$^3$.

The bank’s maximization problem is now:

$$\max \Pi^2(r_0 \leftrightarrow r) + \Pi(1 \leftrightarrow \Pi)(2pr_0 + (1 \leftrightarrow p)x \leftrightarrow 2r)$$

$$+ (1 \leftrightarrow \Pi)^2(p^2r_0 + p(1 \leftrightarrow p)x \leftrightarrow r)$$

s.t. (7).

If only the good types apply we need $(h \leftrightarrow r_0) \geq u$ and, by choosing $x$ large enough, one can discourage the bad type to apply. Then, we can show (see appendix 2).

**Proposition 2** These is an infinity of optimal group lending contracts $(r_0, x)$. However, they do not improve the bank’s profit over optimal individual loans which are a special case with $x = r_0$.

The joint liability benefit is fully offset for the bank by the associated decrease of interest rate due to the participation constraint.

Clearly, maximizing expected social welfare under incentive constraints and under the bank’s budget constraint instead would give also efficient lending with different interest rates, but the same conclusion that, when borrowers do not know each other, there is no collateral effect due to grouping.

**Remark:** In Armendariz and Gollier (1998), the projects deliver $h$ with probability 1 and $H$ with probability $p$ such that $h = pH$ and $u = 0$. Then, the incentive constraints are different between the two types and some screening is possible. However if the observability of $h$ and $H$ is not used in both individual and group contracts, the irrelevance of the collateral effect remains.

$^3$As in section 2 we can show that there is no use in differentiating $(r_0$ and $x$) with a message of the borrower.
4 Borrowers Know Each Other

When borrowers know each other, it is easy for the bank to fully extract the surplus by exploiting the fact that a pair of good types can signal themselves easily by accepting very high payments in case the partner fails.

Consider the following offer of contracts \((r_1, x_1)\) and \((r_2, x_2)\) with \(r_1 = h \Leftrightarrow u\) and \(x_1 = h, r_2 \geq h \Leftrightarrow u\) and \(x_2\).

A pair of good types will reveal itself and accept contract \((r_1, x_1)\). Indeed the participation constraint

\[ h \Leftrightarrow r_1 \geq u, \]

is satisfied as well as the incentive constraint

\[ h \Leftrightarrow r_1 \geq h \Leftrightarrow r_2. \]

Similarly for a pair of bad types. The participation constraint is

\[ p^2[h \Leftrightarrow r_2] + p(1 \Leftrightarrow p)(h \Leftrightarrow x_2) \geq u \]

or

\[ pr_2 + (1 \Leftrightarrow p)x_2 \leq h \Leftrightarrow \frac{u}{p}. \]

Since \(r_2 \geq h \Leftrightarrow u\) we must have \(x_2 \leq h \Leftrightarrow (1 + p)u/p\).

In particular, the contract \(r_2 = h \Leftrightarrow u, x_2 = h \Leftrightarrow (1 + p)u/p \) works.\(^4\)

If side contracts with transfers between borrowers are not possible, there is no fear that a pair of a good type and a bad type forms and claims it is a bad type. Indeed, the bad type would then get:

\[ p(h \Leftrightarrow r_2) = pu < u. \]

Note that, contrary to a Grameen Bank contract in which a borrower is penalized when his partner fails \((x_2 > r_2)\), we obtain that he is rewarded \((x_2 < r_2)\). Indeed, for incentive compatibility of the good type, we need \(r_2 \geq r_1\) and therefore \(r_2 \geq h \Leftrightarrow u\) if we extract all the surplus from the good type. But, then, to satisfy the bad type’s individual rationality constraint a payment less than \(h \Leftrightarrow u\) is required when the partner fails.

We summarize the discussion with:

**Proposition 3** If borrowers know each other and if side-contracts with transfers are not possible, group lending enables the monopolist to extract all the informational rent and therefore lending is efficient.

Assortative matching takes place and the bank can fully discriminate between types by exploiting the fact that good pairs are willing to accept very high collaterals for a failing partner since they know that it cannot occur.\(^5\)

\(\text{\textsuperscript{4}}\)If \(h < \frac{(1+p)u}{p}\), this requires a negative \(x_2\).

\(\text{\textsuperscript{5}}\)Group lending appears then as an institutional representation of a Maskin-type mechanism exhibited by the Nash implementation literature.
However, suppose now that side contracts with transfers are possible. A pair of a good type and a bad type would gain by pretending to be a pair of bad types, rather than each matching himself with a borrower of the same type (and achieving the utility level $u$ as we saw above). Indeed, such a coalition would get

$$2p(h \leftrightarrow r_2) + (1 \leftrightarrow p)(h \leftrightarrow x_2) = 2u \left( \frac{(p \leftrightarrow 1)^2 + 2p}{2p} \right) > 2u.$$ 

A good type benefits from the low payment $x_2$ when he is associated with a bad type and is willing to bribe the bad type who loses from such a matching. In the next section we characterize the optimal collusion-proof contracts.

5 Collusion-Proof Contracts

The pair of contracts considered in Section 4 is not robust to a collusion made of a good type and a bad type, when they collude under complete information with costless transfers. The characterization of the best collusion-proof contracts for the bank requires a more careful definition of the collusion games considered. First, we assume that borrowers are distributed in pairs of two good types (with probability $\Pi^2$), of one good type and one bad type (with probability $2(1 \leftrightarrow \Pi)\Pi$) and of two bad types (with probability $(1 \leftrightarrow \Pi)^2$). However, they cannot change their matching.

From the revelation principle we know that we can restrict the analysis to three pairs of contracts.

$r_1, x_1$ for a pair of good types,

$r_2, x_2$ for a pair formed of a good type and a bad type,

$r_3, x_3$ for a pair of bad types.

When contracts are offered to all types, the bank’s expected profit is then proportional to:

$$2\Pi^2 r_1 + 2\Pi(1 \leftrightarrow \Pi)(2pr_2 + (1 \leftrightarrow p)x_2) + 2(1 \leftrightarrow \Pi)^2(2p^2 r_3 + 2p(1 \leftrightarrow p)x_3) \leftrightarrow 2r.$$ 

The participation constraints are:

$$2(h \leftrightarrow r_1) \geq 2u$$ 
$$2p(h \leftrightarrow r_2) + (1 \leftrightarrow p)(h \leftrightarrow x_2) \geq 2u$$ 
$$2p^2(h \leftrightarrow r_3) + 2p(1 \leftrightarrow p)(h \leftrightarrow x_3) \geq 2u.$$ 

The incentive constraints are:

For a pair of good types:

$$2(h \leftrightarrow r_1) \geq 2(h \leftrightarrow r_2)$$ 
$$\geq 2(h \leftrightarrow r_3).$$
For a pair of bad and good types:

\[ 2p(h \equiv r_2) + (1 \equiv p)(h \equiv x_2) \geq 2p(h \equiv r_1) + (1 \equiv p)(h \equiv x_1) \]
\[ \geq 2p(h \equiv r_3) + (1 \equiv p)(h \equiv x_3). \]

For a pair of bad types:

\[ 2p^2(h \equiv r_3) + 2p(1 \equiv p)(h \equiv x_3) \geq 2p^2(h \equiv r_1) + 2p(1 \equiv p)(h \equiv x_1) \]
\[ \geq 2p^2(h \equiv r_2) + 2p(1 \equiv p)(h \equiv x_2). \]

Solving the bank’s maximization problem (see appendix 3) we obtain:

**Proposition 4** The optimal collusion-proof menu of group contracts is composed of a single group contract

\[
\begin{align*}
    r_1 &= r_2 = r_3 = h \equiv u \\
    x_1 &= x_2 = x_3 = h \equiv u(1 + p) \\
    p
\end{align*}
\]

This contract leaves no rent to a pair of good types and to a pair of bad types. However a pair of good and bad types gets the expected rent

\[ u \frac{(p \equiv 1)^2}{p}. \]

Suppose now that matching is endogenous. We can expect matchings of good and bad types. Since the optimal solution above was independent of the proportions of different types, one cannot do better than letting all borrowers regroup in pairs of good and bad types. The optimal contract is unchanged but the expected profit of the bank is lower because of endogenous matching (see Appendix 3 for details).

The main conclusion is that, when borrowers know each other, group lending contracts are useful even if collusion with side contracts takes place. But now the optimal collusion proof group contract does better than individual contracts, but it is not efficient.

# 6 Conclusion

We have used a particularly simple example to show that, in the absence of shared knowledge between would be borrowers, adverse selection cannot be a foundation for group lending. In this example we have shown that group lending with shared knowledge implements efficient lending when side-transfers are not possible between colluding partners. Finally, when such transfers are possible, we have characterized the menu of optimal collusion proof-contracts which is composed of a unique contract which specifies only a payment when both borrowers are successful, gives up rents and therefore is not efficient.
Appendix 1
Proof of Proposition 1

(1) and (2) imply \( P_1(h \Leftrightarrow r_1) = P_2(h \Leftrightarrow r_2) \) and therefore (4) is binding, i.e., \( P_2(h \Leftrightarrow r_2) = \frac{u}{p} \). Type 1 obtains a rent \( P_1(h \Leftrightarrow r_1) \Leftrightarrow u = \frac{1-p}{p} \cdot u \).

If the bank grants loans to both types \( P_1(h \Leftrightarrow r_1) = P_2(h \Leftrightarrow r_2) = \frac{u}{p} \), hence \( r_1 = h \Leftrightarrow \frac{u}{p}r_1 \), \( r_2 = h \Leftrightarrow \frac{u}{p}r_2 \). Substituting in the profit function of the bank, we get:

\[
\Pi P_1(h \Leftrightarrow r) + (1 \Leftrightarrow \Pi) P_2(ph \Leftrightarrow r) \Leftrightarrow \Pi \left( \frac{u}{p} \right) \Leftrightarrow (1 \Leftrightarrow \Pi)u.
\]

Maximizing with respect to \( P_1, P_2 \) in \([0, 1]\) and recalling that \( ph > r \), we get immediately \( P_1 = P_2 = 1 \). The bank’s profit level is then

\[
A_2 = (\Pi + (1 \Leftrightarrow \Pi)p) \left( h \Leftrightarrow \frac{u}{p} \right) \Leftrightarrow r,
\]
i.e.

\[
A_2 \geq 0 \Leftrightarrow h \geq \frac{r}{\Pi + (1 \Leftrightarrow \Pi)p} + \frac{u}{p} = \frac{r + u}{p} \Leftrightarrow \frac{(1 \Leftrightarrow \Pi)p}{p(\Pi + (1 \Leftrightarrow \Pi)p)}.
\]

Alternatively, the bank can offer a loan which is only accepted by the good type. Then, the bank saturates the good type participation constraint \( P_1(h \Leftrightarrow r) = u \) and the profit is

\[
\Pi P_1(h \Leftrightarrow r) \Leftrightarrow \Pi u,
\]
which is maximized for \( P_1 = 1 \). The bank’s profit level is then:

\[
A_1 = \Pi(h \Leftrightarrow r \Leftrightarrow u),
\]
i.e., \( A_1 \geq 0 \Rightarrow h \geq r + u \).

The bank gives loans to both types iff \( A_2 \geq A_1 \) or

\[
ph \geq h^* = r + u + \frac{\Pi}{1 \Leftrightarrow \Pi} \frac{1 \Leftrightarrow p}{p} u.
\]
Appendix 2
Proof of Proposition 2

When lending to both types occurs the bank’s problem can be written:

\[
\max (\Pi + p(1 \Leftrightarrow \Pi))^2 r_0 + (1 \Leftrightarrow \Pi)(1 \Leftrightarrow p)(\Pi + (1 \Leftrightarrow \Pi)p)x \Leftrightarrow r
\]

s.t.

\[
(\Pi + (1 \Leftrightarrow \Pi)p)r_0 + (1 \Leftrightarrow \Pi)(1 \Leftrightarrow p)x \leq h \Leftrightarrow \frac{u}{p}
\]

\[
r_0 \leq h \quad x \leq h.
\]

The iso-profit lines in the space \((r_0, x)\) are parallel to the constrained set. Any pair \((r_0, x)\) satisfying the budget constraint and the limited liability constraints is optimal (see figure 1). In particular we can choose \(x = r_0\), no joint liability; then \(r_0 = h \Leftrightarrow \frac{u}{p}\), i.e., the same contract as obtained in section 2. One can show when this contract is better than lending only to pairs of good type as in Proposition 1.

![Diagram](image)

**Figure 1**

\((h \text{ small})\)

Any contract in \([A, B]\) is a joint liability contract.

8
Appendix 3

Let us rewrite the participation constraints of types 2 and 3 as follows:

\[
\frac{2p}{1}(h \Leftrightarrow r_2) + (h \Leftrightarrow x_2) \geq \frac{2u}{1}(h \Leftrightarrow p),
\]

\[
\frac{p}{1}(h \Leftrightarrow r_3) + (h \Leftrightarrow x_3) \geq \frac{u}{p(1 \Leftrightarrow p)}.
\]

If \(p = \frac{1}{2}\) the status quo levels are identical and we can expect the participation constraint of type 3 to be binding (and similarly if \(p < \frac{1}{2}\)). When \(p\) becomes larger than \(\frac{1}{2}\), we might have countervailing incentives and have the participation constraint of type 2 binding.

Let us conjecture that, as usual, the incentive constraint of the good type, type 2, is binding and the participation constraint of the bad type is binding. Then, we can choose \(x_1 = h\), since it weakens at no cost the incentive constraints:

For type 1, constraints reduce to:

\[
\begin{align*}
    r_1 & \leq h \Leftrightarrow u \\
    r_1 & \leq r_2 \\
    r_1 & \leq r_3.
\end{align*}
\]

For type 2 we have:

\[
\begin{align*}
2pr_2 + (1 \Leftrightarrow p)x_2 & \leq 2pr_1 + (1 \Leftrightarrow p)h \\
& \leq 2pr_3 + (1 \Leftrightarrow p)x_3 \\
& \leq h(1 + p) \Leftrightarrow 2u.
\end{align*}
\]

For type 3 we have:

\[
\begin{align*}
pr_3 + (1 \Leftrightarrow p)x_3 & \leq pr_1 + (1 \Leftrightarrow p)h \\
& \leq pr_2 + (1 \Leftrightarrow p)x_2 \\
& \leq h \Leftrightarrow \frac{u}{p}.
\end{align*}
\]

When the participation constraint of type 3 is binding, and the incentive constraint of type 2 is binding, we have:

\[
\begin{align*}
pr_3 + (1 \Leftrightarrow p)x_3 & = h \Leftrightarrow \frac{u}{p} \\
2pr_2 + (1 \Leftrightarrow p)x_2 & = 2pr_3 + (1 \Leftrightarrow p)x_3 \\
& = pr_3 + h \Leftrightarrow \frac{u}{p}.
\end{align*}
\]

The objective function of the bank becomes:

\[
2\Pi^2r_1 + 2\Pi(1 \Leftrightarrow \Pi)pr_3 + \left[2\Pi(1 \Leftrightarrow \Pi) + 2(1 \Leftrightarrow \Pi)^2p \right] \left( h \Leftrightarrow \frac{u}{p} \right).
\]
So we need to maximize

$$2\Pi^2 r_1 + 2\Pi (1 \equiv \Pi) p r_3$$

under the constraints

$$r_1 \leq r_3 \leq h \equiv u.$$

If the solution satisfies all the other constraints, then we have the optimal contract.

$$r_1 = r_3 = h \equiv u$$

obviously maximizes the objective function. From the participation constraint of type 3 we need

$$x_3 = h \equiv \frac{u(1 + p)}{p}.$$

If we choose $$r_2 = r_1 = r_3$$ and $$x_1 = x_2 = x_3$$ all incentive constraints are satisfied. It remains to see if the participation constraint of type 2 is satisfied. Indeed, type 2 has a non negative expected rent (and therefore there are no countervailing incentives) because

$$2p(h \equiv r_2) + (1 \equiv p)(h \equiv x_2) \equiv 2u = 2u \frac{(1 \equiv p)^2}{p} \geq 0.$$  

Note that this solution is independent of $$\Pi$$. So, if agents reorganize differently, the contract remains optimal. If $$\Pi < 1/2$$, there will be a proportion $$\Pi$$ of pairs of good and bad types who get the above rent, and a proportion $$\frac{1-\Pi}{2}$$ of bad types, having no rent. We can expect the good types to obtain most of the rent in the bargaining process leading to the formation of groups. And the opposite if $$\Pi > 1/2$$.

So, with endogenous matching, the optimal contract is unchanged, but the bank’s expected profit are lower.

It is (for $$\Pi < 1/2$$)

$$[2\Pi p + p^2 (1 \equiv 2\Pi)](h \equiv u)$$

$$+ [\Pi (1 \equiv p) + p (1 \equiv p)(1 \equiv 2\Pi)] \left( h \equiv \frac{u(1 + p)}{p} \right) \equiv 2u,$$

instead of

$$[2\Pi^2 + 4\Pi (1 \equiv \Pi) p + 4p^2 (1 \equiv \Pi)^2](h \equiv u)$$

$$+ [2\Pi (1 \equiv \Pi)(1 \equiv p) + 4p (1 \equiv \Pi)^2(1 \equiv p)] \left( h \equiv \frac{u(1 + p)}{p} \right) \equiv 2u.$$
REFERENCES


