**Skill Accumulation in the Search Unemployment Model**

**Yong Kim**

**USC Center for Law, Economics & Organization**
**Research Paper No. C02-26**

USC
THE LAW
SCHOOL

# CENTER FOR LAW, ECONOMICS AND ORGANIZATION RESEARCH PAPER SERIES

## Sponsored by the John M. Olin Foundation

University of Southern California Law School
Los Angeles, CA 90089-0071

# Skill Accumulation in the Search Unemployment Model

Yong Kim*
University of Southern California
First Version: March 29, 2000

**Abstract**

This paper analyses the correlation between employment duration, unemployment duration and general skill accumulation in a search equilibrium. I show how the level of general skills and the duration of employment are positively correlated, and both variables are negatively correlated with the duration of unemployment spells. With search frictions, general skill accumulation is associated with a hold up problem since it benefits future job vacancies which workers expect to be matched with. However, if vacancies direct their search to workers of different skill levels, workers indirectly internalise this externality. I derive conditions under which skill accumulation is fully efficient. I also show how Becker's general skill finance rule is modified for a search economy. Extensions analyse indirect complementarities between general and job specific skill accumulation, and the decision of whether to accumulate skills through training or education.

**Keywords** Search; Skill accumulation; Unemployment; Efficiency

**JEL Classification** I21; J21; J24; J64; O11

*Acknowledgements:* I wish to thank my advisor Nobu Kiyotaki for numerous discussions. I also wish to thank Chris Pissarides and Alan Manning. *Address*: Department of Economics, University of Southern California, Los Angeles, CA 90089-0253. *Tel.*: 213-740-2098. *Email*: yongkim@usc.edu

# 1 Introduction

This paper analyses the correlation between employment duration, unemployment duration and general skill accumulation in a search equilibrium. A classic topic in development economics is the correlation of labour force participation in the formal, employment sector with the level of development. When richer countries are characterised by lower unit costs of skills and higher skill levels, I characterise the implications for labour market outcomes, and the feedback between such outcomes and skill levels. The duality of labour markets highlighted by Harris and Todaro (1970), has in the past been explained using a labour turnover model by Stiglitz (1974). The need for an updated analysis along these lines has been recently emphasized by Basu (1997). Mortensen and Pissarides (1994) present a canonical model of endogenous job duration in a search unemployment framework. The extension of that analysis to general skill accumulation constitutes the core of my paper.

With search frictions in matching unemployed workers with job vacancies, the marginal benefit of skills is higher in current matches than in future matches since re-matching takes time. This implies that matches become more resilient towards idiosyncratic shocks, and the expected duration of matches is longer, when skill levels are higher. For the same reason, search frictions imply that the bilateral surplus created between workers and vacancies opened by entrepreneurs, is increasing in general skill levels.

Entrepreneurs who open vacancies receive a fixed share of this surplus. Under free entry of vacancies, a higher surplus translates into a higher vacancy to unemployment ratio, as entrepreneurs "compete" for workers by opening job vacancies. For workers this means that unemployment durations on average are shorter. Since general skills are only

productive within matches, longer employment durations and shorter unemployment spells improve the incentives for general skill accumulation. Thus, I show how the level of general skills and the duration of employment are positively correlated, and both variables are negatively correlated with the duration of unemployment spells.

A natural question which arises in analysing the correlations above is (i) who finances the acquisition of general skills and (ii) is the level of general skill accumulation efficient? With search frictions both entrepreneurs and workers have incentives to pay for general skills, since the bilateral surplus is increasing in skill levels. Under Nash Bargaining of the bilateral surplus, the Becker (1975) rule for general skill finance (workers pay for all of it) is modified in an intuitive way. Employers pay for their bargaining share of total skill cost minus the appreciation in the workers outside option resulting from skill accumulation. Skill accumulation has two effects, increasing the bilateral surplus and improving the worker's outside option and bargaining power. Since the entrepreneur cannot appropriate any share of the improvement in the worker's outside option, the level of the entrepreneur's skill finance is exactly his bargaining share of total skill finance less the unappropriable share.

Workers acquire general skills with the first vacancy they are matched with. I assume skills do not depreciate throughout workers' lifetimes. Since the marginal productivity of workers' general skills is shared with future entrepreneurs who workers expect to be matched with, general skill accumulation is subject to a hold up problem. However, if entrepreneurs open vacancies which are *directed* towards workers of different skill levels, workers will indirectly be able to internalise the externality of skill accumulation, through shorter unemployment spells.

Besides the hold up problem associated with general skill accumula-

tion, the search employment economy is subject to further externalities resulting from (i) job separations causing a negative externality to the pool of existing unemployed workers, and (ii) job creations causing a positive externality on the pool of existing unemployed workers. Hosios (1990) identified a condition under which these latter externalities exactly cancel each other out. I show how given the Hosios condition, with free entry of vacancies, and directed search by vacancies to workers of different skill levels, general skill accumulation is fully efficient. Under these conditions, workers are *exactly* internalising the externality generated by skill accumulation on future entrepreneurs that workers expect to be matched with. Meanwhile, if vacancies do not direct their search to skilled and unskilled workers, the economy is subject to coordination failure. Workers are unable to internalize the externality of their skill accumulation decisions. I discuss conditions under which skill accumulation is distorted in different directions.

Compared to the Becker analysis, the search framework predicts a larger range of skills are accumulated through training than education. I assume skills acquired through education (before entering the labour market) and training (acquired upon being matched with first job) are perfect substitutes in production. Then for Becker, skills whose unit costs are lower under education are accumulated through education and otherwise through training. In my analysis, a range of skills whose costs are higher under training are accumulated through training, since (i) entrepreneurs participate in the financing of training and (ii) getting the first job takes time so workers discount the benefits of skills acquired through education.

Individual differences in the unit costs of skill accumulation through education give rise to two distinct classes of workers: "white collar"

workers who accumulate skills through education and "blue collar" workers who accumulate skills through training. Even small differences in unit costs of skill accumulation can lead to discrete differences in skill levels when one group acquires skills through education and another acquires skills through training. White collar workers who accumulate skills through education have *discretely* longer employment duration and lower unemployment duration.

With search frictions, the duration of matches is increasing in job *specific* skills as well. Ceteris paribus, optimal specific skill accumulation maximizes the bilateral surplus of job matches, and minimizes the unemployment duration of workers. In turn, specific skills respond positively to the expected duration of employment. General and job specific skills complement each other indirectly through the expected duration of employment matches.

In a full employment framework, general skill accumulation decisions under search frictions and exogenous job destruction have been studied by Acemoglu (1997). Non-directed search by vacancies was assumed in that paper. Acemoglu and Shimer (1998) have shown how the externality generated by firms' specific (physical) capital decisions can be indirectly internalized, when workers exercise directed search towards vacancies following wage posting by firms. The paper begins by introducing the skill accumulation process through training. The description of equilibrium is followed by efficiency results and analysis of the economy with non-directed search. The extensions cover education and specific skill accumulation. The last section concludes with suggestions for future research.

## 2 Model

There are two types of agents in the economy: entrepreneurs and workers. The share of entrepreneurs in the population is fixed and the size of the labour force who are workers is normalized to 1.[1] A homogenous good with price numeraire, can be produced using a variety of "methods" $j \in 1, ..., J$, and within each method there can be several "ideas" indexed by $i \in 1, ..., I$.

Method specific skills, $k_j$ are embodied in workers and these are general skills since workers can utilize these skills in different matches with ideas within the same method. Entrepreneurs have a large set of ideas they can implement, but they do not have more than one idea from each method. The only shocks in the economy are idea specific ones, and all ideas have an identical and independent productivity distribution at the point of implementation (when idea and worker are first matched).[2]

Productive matches are bilateral (between one worker and one idea), implying the production function is Leontieff. The productivity flow of a particular idea-worker match $i$, in method $j$ is given by,

$$f(k_j) + x_{ij}\sigma \qquad \text{where } f' > 0, f'' < 0 \tag{1}$$

The general and idea specific components of productivity are additive.[3] For $\forall i, j,\ x_{ij} \in [\underline{x}, \bar{x}]$, and at the point of idea implementation $x_{ij} = \bar{x}$. The motivation for the last assumption is that only the latest and most productive ideas are adopted at any given time. Bilateral matches are subject to idea specific shocks at Poisson rate $\lambda$ which give rise to

[1] This paper concentrates on the worker side of the economy. The analysis of entrepreneurs is trivial in this economy.

[2] The dominace of plant or firm specific idiosyncratic shocks in gross job flows is well documented. See the survey by Davis and Haltiwanger (1998).

[3] This formulation is important for the qualitative results. In some past studies, skill levels were multiplicative to the idisyncratic shock component, but it is unclear why general skill levels should increase the variance of idiosyncratic shocks.

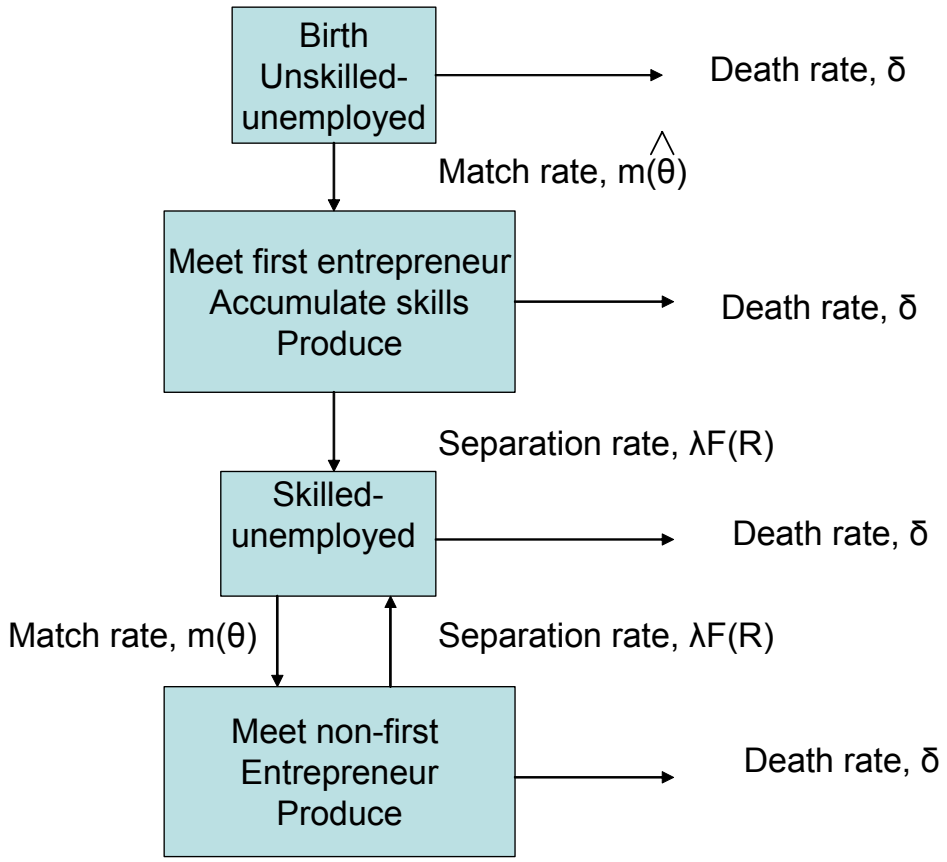


Figure 1: Lifetime of a worker

draws of $x_{ij}$ from a fixed distribution $F(x)$ with $E(x) = 0\ \forall i, j$. When idea specific draws yield a productivity below an optimally determined destruction margin $R$, bilateral matches are terminated.[4]

[Figure 1] documents the possible lifetime paths of workers. All agents are subject to Poisson death shocks at rate $\delta$, which causes their asset values to drop to zero. I assume zero interest rates such that the discount rate of the economy is equal to the death rate. The assumption of zero interest rates is adopted to facilitate the welfare analysis later.

The skill accumulation decision or workers is made when an unskilled

[4] Expected employment tenure is then given by $\frac{1}{\lambda F(R)}$.

worker is matched with his first entrepreneur. Skill accumulation is instantaneous. Then, all production occurs with workers who have been skilled. I assume that the productivity of matches is not verifiable by third parties. This assumption is used to separate out the output sharing decision from the skill finance sharing decision and will allow me to pin down and analyse a particular optimal skill finance sharing rule.

Entrepreneurs direct (idea) vacancies to workers of differentiated skill levels.[5] At any given point in time the distribution of workers with particular skills and the distribution of vacancies searching for particular workers is common knowledge. In equilibrium, only two types of vacancies will be observed, those opened for unskilled workers and those opened for workers skilled at the equilibrium level.

Search frictions in the matching of workers to vacancies are governed by a standard constant returns to scale match function with the measure of vacancies and unemployed workers as arguments. The flow of skilled worker-idea matches is given by $M \equiv M(u_s, v)$,and the flow of unskilled worker-idea matches is given by $\hat{M} \equiv \hat{M}(1-s, \hat{v})$. $u_s$ is the unemployed share of the labour force who are skilled, $v$ the measure of vacancies opened to skilled workers, $s$ the skilled share of the labour force, and $\hat{v}$ the measure of vacancies opened for unskilled workers.

Let $\theta \equiv \frac{v}{u_s}, \hat{\theta} \equiv \frac{\hat{v}}{(1-s)}$ denote the labour market tightness for skilled and unskilled workers respectively. $m(\theta), m(\hat{\theta})$ are the respective arrival rates of entrepreneurs to skilled and unskilled workers. $\frac{m(\theta)}{\theta} \equiv q(\theta), \frac{m(\hat{\theta})}{\hat{\theta}} \equiv q(\hat{\theta})$ are the respective arrival rates of skilled and unskilled workers to entrepreneurs.[6] The elasticity of the arrival rate of workers

[5] This necessarily implies general skill levels are verifiable, and skill contingent transfers can be implemented within matches.

[6] The expected unemployment durations for skilled and unskilled workers are given by $\frac{1}{m(\theta)}, \frac{1}{m(\hat{\theta})}$ respectively.

$\eta \equiv -\frac{q'(\theta)\theta}{q(\theta)} \equiv -\frac{q'(\hat{\theta})\hat{\theta}}{q(\hat{\theta})} > 0$ is assumed constant.

I assume that on-the-job search is less intensive than that off the job, and rule out on-the-job search altogether for simplicity. Everyone is risk neutral and non-wealth constrained.

## 2.1 Entrepreneurs

Given that entrepreneurs can implement several ideas at the same time, ideas are independent and each entrepreneur can only have at most one idea in each method, each idea can be analysed in isolation. Since each method can also be analysed in isolation I drop indices for method for notational convenience.

Entrepreneurial ideas can be in three different states: matched with a skilled worker, searching for a skilled worker (skilled vacancy) and searching for an unskilled worker (unskilled vacancy). The steady state asset value of idea $i$ matched with a skilled worker of skill level $k$ is given by,

$$\delta J_i(k) = f(k) + x_i\sigma - w_i(k) + \lambda \int [\max\{\tilde{J}(k), 0\} - J_i(k)]dF(\tilde{x}) - \delta J_i(k) \tag{2}$$

The flow of expected output to entrepreneurs consists of the product net of wage plus the capital appreciations following productivity shocks minus the capital depreciations following the death of the worker.[7] The outside option of the idea once it has been matched is zero since entrepreneurs weakly prefer to open new ideas as opposed to re-opening old ones. Assume that old ideas are never re-opened.

The asset value of any skilled vacancy searching for a worker of skill level $k$ is given by,

$$\delta V(k) = -c + q(\theta)(J(k) - V(k)) \tag{3}$$

[7] The other asset equations are straightforward.

$c$ is the per period cost of recruitment and its level is assumed the same for all skill levels. During the match process, the identity of skilled workers who will be matched to particular vacancies is unknown so entrepreneurs cannot commit to skill contingent transfers before matches occur. Once matches are realized the incentive for vacancies to offer such contracts disappears.

The asset value of any unskilled vacancy searching for an unskilled worker is given by,

$$\delta\hat{V} = -c + q(\hat{\theta})(J(k) - \gamma k_e - \hat{V}) \tag{4}$$

$J - \gamma k_e$ is simply the initial asset value of an idea matched with an unskilled worker, where $\gamma$ is the unit cost of skill accumulation. $k = k_e + k_w$, skill accumulation consists of that financed by entrepreneurs and that financed by workers.

## 2.2 Workers

Workers can find themselves in three different states: skilled and employed, skilled and unemployed and unskilled. The asset value of a skilled worker matched with idea $i$ is given by,

$$\delta W_i(k) = w_i(k) + \lambda \int [\max\{\tilde{W}(k), U(k)\} - W_i(k)]dF(\tilde{x}) + \delta(U(k) - W_i(k)) \tag{5}$$

The asset value of a skilled worker searching for a match is given by,

$$\delta U(k) = \max\{\delta\hat{U}, a + m(\theta)(W(k) - U(k))\} \tag{6}$$

$a$ is the income derived from labour outside the employment sector. Workers skilled in a particular method are unskilled in all other methods so always have an option to search for unskilled worker vacancies. Throughout the analysis, I assume that $a + m(\theta)(W(k) - U(k) > \delta\hat{U}$ $\forall k > 0$, and later verify this must be true in equilibrium.

The asset value of an unskilled worker searching for a match is given by,

$$\delta \hat{U} = a + m(\hat{\theta})(W(k) - \gamma k_w - \hat{U}) \tag{7}$$

$W(k) - \gamma k_w$ is the initial asset value of an unskilled worker matched with his first entrepreneur. Although skills can be acquired outside matches, I assume that it is optimal to delay the skill accumulation decision until the first match is realized. The conditions under which this assumption is valid are verified in the extension on education.

## 2.3 Bargaining

Nash Bargaining is adopted throughout. In a match between an entrepreneur's idea and a skilled worker the Nash Bargaining Rule for Wages is given by,

$$w_i^*(k) = \arg\max (J_i(k))^{1-\beta}(W_i(k) - U(k))^{\beta} \text{ given } x_i, k \tag{8}$$

where $\beta$ is the workers' bargaining share. Productivity contingent transfers cannot be made and as a result the wage rule determines a renegotiation proof rule for the division of output. Wages are bargained over given $k$ since the skill investment has already been sunk either in a previous match of the worker or at the beginning of the current match.[8]

Let $S_i(k) \equiv (J_i(k) + W_i(k) - U(k))$ denote the bilateral match surplus between a skilled worker and idea. The first order condition (FOC) of the wage bargaining rule implies that ideas and skilled workers receive their bargaining share of the match surplus,

$$J_i(k) = (1 - \beta)S_i(k) \qquad W_i(k) - U(k) = \beta S_i(k) \tag{9}$$

Further bargaining considerations are made when an unskilled worker is matched with an idea for the first time. The Nash Bargaining Rule

[8] Allowing for productivity contingent transfers would mean that this sharing rule relates to one of an infinite set of possible optimal sharing rules.

for Skill Accumulation and Finance is given by,

$$\max_{k,k_e}(J(k)-\gamma k_e)^{1-\beta}(W(k)-\hat{U}-\gamma(k-k_e))^{\beta} \text{ given } x_i=\bar{x} \qquad (10)$$

Let $T(k) \equiv (J(k)+W(k)-\hat{U}-\gamma k)$ denote the bilateral match surplus between an unskilled worker and entrepreneur idea. The (partial) FOC for $\hat{k}_e$ implies that ideas and unskilled workers receive their bargaining share of the match surplus,

$$J(k)-\gamma k_e^* = (1-\beta)T(k) \qquad W(k)-\hat{U}-k_w^* = \beta T(k) \qquad (11)$$

Due to non-verifiable productivity, the division of output and the division of skill finance become independent problems. Substituting in the wage bargaining rule (8) for $x_i = \bar{x}$, gives us the Skill Finance Rule,

$$\gamma k_e^* = (1-\beta)(\gamma k - (U(k)-\hat{U})) \qquad (12)$$

Skill accumulation has two effects, increasing the bilateral surplus and improving the workers outside option and bargaining power. Since the entrepreneur cannot appropriate any share of the improvement in the worker's bargaining power, the level of the entrepreneur's skill finance is exactly his bargaining share of total skill finance less the unappropriable share. This modifies the Becker general skill finance rule for the search employment economy.

Since I assume that skill contingent transfers are contractible (between well defined contractual parties, namely a matched worker and entrepreneur), skill levels are set to maximize the asset value of the worker and entrepreneur's idea. However, this optimal skill accumulation rule can also be interpreted as the outcome of Nash Bargaining.[9]

[9] So the verifiablity of skills is not a necessary assumption for the Skill Accumulation Rule.

The Skill Accumulation Rule is given by,

$$k^* \equiv \arg\max\{J(k) + W(k) - \hat{U} - \gamma k\} = \arg\max\{J(k) + W(k) - \gamma k\} \tag{13}$$

Skill accumulation is made taking $\hat{U}$ as given since this is a function of skill accumulation outside the current match.

# 3 Equilibrium

## 3.1 Skilled Worker Sector

The model is solved as follows. First take the skilled worker endogenous variables: the skill accumulation level $k$, the job destruction margin $R$, the vacancy to unemployment ratio for skilled workers $\theta$, and the wage rate for workers $w_i$ (note all wage earners are skilled). These are solved using the following four rules.

Job Destruction Rule $S(x_i = R) \equiv J(R) + W(R) - U(R) \equiv 0$

Free Entry Rule $V(k) = 0$

Nash Bargaining Rule for Wages

$w_i^*(k) = \arg\max(J_i(k))^{1-\beta}(W_i(k) - U(k))^{\beta}$ given $x_i, k$

Skill Accumulation Rule $k^* = \arg\max\{J(k) + W(k) - \gamma k\}$

A feature of the Skill Accumulation Rule is that the decision internalizes the effect that skill levels have on the labour market tightness $\theta$, faced by the worker. This is a feature of vacancies exercising directed search to workers of different skill levels.

The following equilibrium conditions are derived in the Appendix.

The Job Destruction Equation is,

$$[f(k) + R_i^*\sigma] + \frac{\lambda\sigma}{2\delta + \lambda}\int_{R_i^*}^{\bar{x}}(1 - F(\tilde{x}))d\tilde{x} = [a + m(\theta)\beta\frac{(\bar{x} - R_{h\neq i}^*)\sigma}{2\delta + \lambda}] \tag{14}$$

This says the lowest acceptable productivity of a match plus the option value of retaining the existing match in anticipation of productivity

improvements equals the opportunity cost of employment.

Differentiating the Job Destruction Equation with respect to $k$ we get,

$$f'(k)+\sigma\frac{\partial R}{\partial k}-\frac{\lambda\sigma(1-F(R))}{2\delta+\lambda}\frac{\partial R}{\partial k}=-m(\theta)\beta\frac{\sigma}{2\delta+\lambda}\frac{\partial R}{\partial k}+m'(\theta)\beta\frac{\sigma(\bar{x}-R)}{2\delta+\lambda}\frac{\partial\theta}{\partial k} \tag{15}$$

If the opportunity cost of employment is constant, a higher skill level which increases the productivity of matches implies that the destruction margin must fall. This is for two reasons, one the productivity within the current match is higher than elsewhere (second term of LHS), and two the option value of retaining the current match is higher (third term of LHS). The opportunity cost of employment does respond positively to the skill level since as in the current match, skills reduce the destruction margin of future matches and improve the rematch probabilities of workers. Since both these effects are conditional on skills lowering the destruction margin of the current match, overall the destruction margin must respond negatively to skill levels.

From the Job Destruction Rule $S_i - S(R) = S_i$, the bilateral surplus created by a match between an idea and a skilled worker can be rewritten as a direct function of the destruction margin only,

$$S_i = \frac{\sigma(x_i - R)}{2\delta+\lambda} \tag{16}$$

Note that $-\frac{\sigma}{2\delta+\lambda}\frac{\partial R}{\partial k} = \frac{\partial S_i}{\partial k} = \frac{\partial S}{\partial k}$.

So from the equation for $S_i$ we know that $\frac{\partial R}{\partial k} < 0 \Leftrightarrow \frac{\partial S_i}{\partial k} > 0$. The destruction margin responding negatively with respect to skill levels is equivalent to the statement that the match surplus between an entrepreneurial idea and skilled worker is increasing in skill levels, given the exogenous parameters of our model.

The Job Creation Equation is,

$$\frac{c}{(1-\beta)q(\theta^*)} = \frac{\sigma(\bar{x}-R)}{2\delta+\lambda} = S \tag{17}$$

Entrepreneurs receive fixed shares of the bilateral surplus. Increases in the surplus which occur through reductions in the destruction margin, will invite entry into the competitive vacancy market and increase $\theta$.

Differentiating the Job Creation Equation with respect to $k$ one gets,

$$-\frac{\partial R}{\partial k} = \frac{2\delta+\lambda}{\sigma}\frac{\partial S}{\partial k} = \frac{\eta}{\theta}(\bar{x}-R)\frac{\partial \theta}{\partial k} \tag{18}$$

Since entrepreneurs appropriate a share of the match surplus, higher skill levels will invite entry of vacancies into the market for skilled workers and increase the tightness of the market for skilled workers.

The comparative statics imply $\frac{\partial R}{\partial k} < 0, \frac{\partial \theta}{\partial k} > 0$.

The Skill Accumulation Equation is,

$$\frac{1}{f'(k^*)} = \frac{\delta + m(\theta)\frac{\beta}{\eta}}{\gamma\delta[2\delta+\lambda F(R)+m(\theta)\frac{\beta}{\eta}]} \tag{19}$$

This equation implies $\frac{\partial k}{\partial R} < 0, \frac{\partial k}{\partial \theta} > 0$. The marginal productivity of skills in improving the option value of retaining existing matches is decreasing in the destruction margin. This implies that the marginal productivity of skills in improving the match surplus is decreasing in the destruction margin. The $m(\theta)\frac{\beta}{\eta}$ term in the numerator captures the fact that when unemployment durations are shorter the marginal productivity of general skills is higher during the worker's lifetime. The same term in the denominator captures the fact that shorter unemployment durations reduce the expected duration of any match.

Comparative statics thus reveal a feedback effect between the skill level and labour market variables. Skill levels are positively correlated with the match durations and negatively correlated to unemployment

durations of skilled workers. The higher job turnover rate (implied by a higher destruction margin) in countries with lower skill accumulation has been documented by several studies [OECD (1994), Roberts and Tybout (1996)]. The longer duration of unemployment for formal sector skilled and unskilled workers in poorer countries provides an explanation for their lower formal sector labour force participation rates.

The Wage Equation is,

$$w_i^*(k) = \beta\left(f(k) + x_i\sigma - \left(a + m(\theta)\beta\frac{\sigma(\bar{x}-R)}{2\delta+\lambda}\right)\right) + a + m(\theta)\beta\frac{\sigma(\bar{x}-R)}{2\delta+\lambda} \tag{20}$$

Wages consist of the opportunity cost of employment plus the workers' share of the flow surplus from the match. Wages are increasing in $\theta$ which is increasing in the skill level, reflecting that as skills become less de facto specific, wages approach the marginal product of the match[10].

## 3.2 Unskilled Worker Sector

The remaining endogenous variables, the vacancy to unemployment ratio for unskilled workers $\hat{\theta}$, and the entrepreneur share of general skill finance $k_e$, are solved by backwards induction using the solutions from the skilled worker sector and the following two rules,

Skill Finance Rule $\gamma k_e^* = (1-\beta)(\gamma k^* - (U(k^*) - \hat{U}))$[11]

No Skill Arbitrage Rule $V(k^*) = \hat{V}$

The second rule says the value of a vacancy searching for workers of the equilibrium skill level, is equivalent to that searching for unskilled workers.

Combining these rules with the Free Entry Rule yields the Job Cre-

[10] This Wage Equation for skilled workers defines the implicit labor contract offered by a vacancy to the skilled worker it is matched with.

[11] This rule in conjunction with the Wage Equation for skilled workers defines the contract offered by a vacancy to a unskilled worker it is matched with.

ation Condition for Unskilled Workers,

$$\frac{c}{q(\hat{\theta}^*)(1-\beta)} = (J(k^*) + W(k^*) - \hat{U} - \gamma k^*) = T^* \tag{21}$$

Increases in the vacancy to unemployment ratio for unskilled workers reflect increases in the initial surplus of a match between an idea and an unskilled worker.

Both the solved asset equation for unskilled workers and unemployed skilled workers are linear in their respective labour market tightness measures. The difference in these asset values is given by,

$$U(k) - \hat{U} = \frac{\beta c(\theta - \hat{\theta})}{\delta(1-\beta)} \tag{22}$$

Substituting this into the Skill Finance Rule yields the equilibrium level of entrepreneur skill finance.

I claimed earlier that $U(k) > \hat{U}\ \forall k > 0$. The Skill Finance Rule and No Skill Arbitrage Rule imply,

$$\frac{c}{q(\theta)} - \frac{c}{q(\hat{\theta})} = (1-\beta)(\gamma k - (U(k) - \hat{U})) \tag{23}$$

Combining the last two equations, for all $k > 0$ we must have $\theta > \hat{\theta} \Rightarrow U(k) > \hat{U}, k_e > 0$. The expected unemployment duration for skilled workers is shorter. Unemployed skilled workers never prefer to behave like an unskilled worker. The entrepreneur share of skill finance is never negative.

**A Justification for Job separation**

Why don't entrepreneurs use workers in other ideas once one idea specific job is destroyed, i.e. why is there job separation? It is reasonable to believe that the match frictions between entrepreneurs and unskilled workers occur due to idea specific idiosyncrasies, so why don't worker-entrepreneur matches re-train the worker in another method to pursue

other ideas together? In this model this will never occur under the following condition,

$$U(k_j^*) > J(k_{l\neq j}^*) + W(k_{l\neq j}^*) - \gamma k_{l\neq j}^* > \hat{U} \tag{24}$$

The asset value of the worker's search option is greater than the net asset value of re-training the worker and pursuing another idea in a different method. Under this condition, the entrepreneur must earn negative profits to convince the worker to stay with him. What drives this story is the assumption that individual entrepreneurs are never endowed with more than one idea from each method, although each has a large number ideas from different methods. This contrasts from conventional search models which assume workers are matched with 'firms', which are endowed at most with a single job.

## 3.3 Unemployment

The steady state share of skilled workers out of total workers is only a direct positive function of the vacancy to unemployment ratio for unskilled workers,

$$s = \frac{m(\hat{\theta})}{m(\hat{\theta}) + \delta} \tag{25}$$

The steady state share of skilled and unemployed workers out of total workers is a positive function of the vacancy to unemployment ratio for skilled workers and a negative function of the job destruction margin,

$$u_s = \frac{\delta + \lambda F(R)}{m(\theta) + 2\delta + \lambda F(R)} s \tag{26}$$

So the share of workers who are unemployed is given by one minus the share of workers who are employed,

$$u \equiv 1 - (s - u_s) = 1 - \frac{m(\theta) + \delta}{m(\theta) + 2\delta + \lambda F(R)} \frac{m(\hat{\theta})}{m(\hat{\theta}) + \delta} \tag{27}$$

Higher equilibrium skill levels are associated with lower $R$, and higher $\theta, \hat{\theta}$. Skill levels are thus correlated with a larger share of the workforce who are skilled and a larger share of the skilled who are employed.

## 3.4 Efficiency

Here I show how workers indirectly internalise the positive externality that their skill accumulation decisions have on future entrepreneurs they expect to be matched with. Given $\theta$, skill accumulation by workers generates a positive externality on the share of the match surplus $(1-\beta)S$, enjoyed by future entrepreneurs who workers expect to be matched with. The marginal externality of skill accumulation is given by,

$$(1-\beta)\frac{m(\theta)}{\delta}\frac{\partial S(k)}{\partial k}$$

Where $(1-\beta)\frac{\partial S(k)}{\partial k}$ is the flow of marginal externalities to future entrepreneurs which arrive at rate $m(\theta)$, and this is normalised by the discount rate of the economy to yield a stock measure.

If vacancies undertake directed search to workers of different skill levels, workers internalise the effect that skill accumulation has on their outside option through improved matching prospects $m(\theta)$. The flow of marginal product of skills through improved match prospects is given by $m'(\theta)\frac{\partial \theta}{\partial k}\beta S$. Using (18), and the definition of the elasticity of the arrival rate of workers $\eta$, the flow of this marginal product can be rewritten as $\frac{\beta}{\eta}(1-\eta)m(\theta)\frac{\partial S(k)}{\partial k}$. The stock of marginal productivity through improved job prospects is then given by,

$$\frac{\beta}{\eta}(1-\eta)\frac{m(\theta)}{\delta}\frac{\partial S(k)}{\partial k}$$

Under the Hosios condition $\eta = \beta$, the externality of skill accumulation is exactly internalised through improved match prospects. The free entry of vacancies and directed search by vacancies, together with the Hosios

condition ensure general skill accumulation is efficient. By inspection, when $\eta > \beta$, there will be under-investment in skills, and when $\eta < \beta$ there will be over-investment in skills.

More generally one can define the social planner's problem and show how under the assumed conditions, the outcomes under a competitive equilibrium coincide exactly with those of the social planner. The social planner's problem is given by,

$$\max_{s,u_s,v,\hat{v},k} Y = \int_0^\infty \left\{ \begin{array}{l} [(1-s)+u_s]a - [v+\hat{v}]c + (s-u_s)f(k) \\ +(s-u_s-b)\bar{x}\sigma + bE(\tilde{x}|\tilde{x} \geq R)\sigma - \hat{M}\gamma k \end{array} \right\} e^{-\delta t} dt \tag{28}$$

subject to the search friction constraints,

$$\begin{aligned} \dot{s} &= \hat{M} - \delta s \\ (s-u_s) &= \hat{M} + M - (2\delta + \lambda F(R))(s-u_s) \\ \dot{b} &= \frac{\lambda(1-F(R))}{2\delta+\lambda}(\hat{M}+M) - (2\delta+\lambda F(R))b \end{aligned} \tag{29}$$

The social planner maximizes the discounted flow of net income streams. $b$ denotes the measure of workers who are in matches where idea specific productivity is not at the top level.

Given the Hosios Condition for no search externalities, the free entry of vacancies and directed search by vacancies, the solution to the social planner's problem is identical to outcomes under the competitive equilibrium. The (rather long) proof of this statement is found in the Appendix. Assuming the discount rate is equal to the death rate of the economy simplifies the welfare analysis by allowing us to directly compare steady state solutions of the social planning and real economy rather than having to determine the discounted value of the change in some variable along a convergent path from one solution to the other. Otherwise the assumption that the interest rate is zero is inessential.

Acemoglu (1997) identifies a hold up problem which arises from workers being unable to directly internalize the benefits that their skill accumulation has on future entrepreneurs they expect to be matched with. This is because the identity of the future entrepreneurs or vacancies is unknown and contracts cannot specify parties to the contract. Here, this underinvestment is mitigated since workers are able to indirectly internalize this externality through higher expected match rates given the Free Entry Condition: competition solves the hold up problem.[12]

## 3.5 Equilibrium with Non-directed Search

When the same type of vacancy is opened for workers of every skill level we have non-directed search. By construction then, $\theta \equiv \hat{\theta} \equiv \frac{v}{1-s+u_s}$. The asset equation for vacancies becomes,

$$\delta V = -c + q(\theta)(J - \frac{1-s}{1-s+u_s}k_e - V) \tag{30}$$

The expected capital gain through matches is a weighted average of the gains through being matched with a skilled worker and being matched with an unskilled worker. The skill accumulation rule is modified to,

$$k^* \equiv \arg\max\{J(k) + W(k) - \gamma k\} \text{ given } \theta \tag{31}$$

Under non-directed search, the tightness of the market becomes an aggregate variable the change of which is not internalized by individual worker-entrepreneur matches. The new skill accumulation equation,

$$\frac{1}{f'(k^*)} = \frac{\delta + m(\theta)\beta}{\gamma\delta[2\delta + \lambda F(R) + m(\theta)\beta]} \tag{32}$$

reflects the underinvestment which results from this externality. Qualitatively, the feedback between $k$ and $R, \theta$ is unchanged, but some of the feedback between $k$ and $\theta$ is not captured.

[12] Acemoglu and Shimer (1998) have discussed a similar mechanism for the case of entrepreneurs making match specific ex ante physical investments before opening vacancies.

The externality that workers cannot collectively internalize consists of the effect of skill levels on future re-match probabilities. This has a positive and negative feedback effect. Higher rematch probability implies workers should invest more (this is represented by the second term on the numerator), but the same effect implies that workers exaggerate the effect of skills on lowering the destruction margin such that workers should invest less (this is represented by the third term in brackets on the denominator). Overall there is under-investment by workers, and the positive feedback implies the existence of co-ordination failure.

# 4 Extensions

## 4.1 Education

In the classic Becker (1975) analysis, the decision to acquire skills through either training or education follows a simple rule. For those skills the marginal cost of which is lower in training, acquire them on the job, and otherwise acquire them through education.[13] This formulation cannot justify the observation that a given set of skills are often first acquired through training and then acquired through education along the development process. I provide a justification here.

The education decision is set to solve,

$$h^* = \arg\max\{\hat{U}(h) - \alpha\gamma h\} \text{ s.t. } h \geq 0 \tag{33}$$

Where $0 < \alpha \leq 1$. For Becker, workers would accumulate all these skills through education. $\alpha < 1$ implies that it is cheaper to buy skills through education than on the job.

The education decision is solved by backward induction given the solutions from the unskilled sector, which in turn are solved by backward

[13] This is the case with instantaneous skill accumulation.

induction given the solutions from the skilled sector. Define the level of on the job skill accumulation $t \equiv k - h \geq 0$. The new Job Creation Condition for Unskilled Workers is,

$$\begin{aligned} \frac{c}{q(\hat{\theta}^*)(1-\beta)} &= J(k^*) + W(k^*) - \hat{U}(h) - \gamma k^* + \gamma h \quad \text{for } h < k^* \quad (34) \\ &= J(h) + W(h) - \hat{U}(h) \quad \text{for } h \geq k^* \end{aligned}$$

When the level of education exceeds the optimising training level $h \geq k^*$, there is no longer a distinction between skilled and unskilled workers in the search market, i.e. $\hat{\theta} = \theta, \hat{U} = U$. Substituting in for equilibrium $\hat{U}, U$ this condition maps a continuous monotonic increasing relationship between equilibrium $\hat{\theta}$ and $h$.

The marginal benefit of education is given by,

$$\begin{aligned} \frac{\partial \hat{U}(h)}{\partial h} &= \gamma \frac{m(\hat{\theta})\beta}{\eta\delta + m(\hat{\theta})\beta} > 0 \quad \text{for } h < k^* \quad (35) \\ &= \frac{\partial (J(h) + W(h))}{\partial h} \frac{m(\hat{\theta})\beta}{\eta\delta + m(\hat{\theta})\beta} > 0 \quad \text{for } h \geq k^* \end{aligned}$$

Since $\frac{\partial (J(h)+W(h))}{\partial h} = \gamma$ when $h = k^*$, this marginal benefit function is continuous in $h$. Since $\frac{\partial (J(h)+W(h))}{\partial h} \leq \gamma$ when $h \geq k^*$, $\frac{\partial \hat{U}(h)}{\partial h} < \gamma \; \forall \theta < \infty$. So when $\alpha = 1$, workers strictly prefer to accumulate skills through training, and buy no education; $h^* = 0$ the corner solution. Due to discounting and the sharing of skill finance, training is superior to education. Education has some marginal benefit in improving the match probabilities of workers virgin to the labour market but, this benefit is strictly dominated in the case of equal unit costs of training and education.

There exists a critical $\alpha^* < 1$ defined by,

$$\hat{U}(h^*) - \alpha^* h^* = \hat{U}(h = 0) \;\; \text{where } h^* > 0 \quad (36)$$

For a particular method of skills $j$, if $\alpha_j \geq \alpha^*$ workers prefer to accumulate these skills through training, and otherwise they prefer to accumulate skills through education. So compared to the Becker analysis, the set of skills accumulated through training is larger in a labour market characterized by search frictions.

If $h < k^*$, since $\hat{\theta}$ is monotonically increasing in $h$, the marginal benefit of education is increasing in the level of education. If $h \geq k^*$, this process slows down and eventually the marginal benefit is falling in education, although it remains positive. Given that the marginal cost of education is constant, these statements imply that $h_j^* > k_h^*$ given $\alpha_j < \alpha^*$. When education is optimally purchased for a particular skill, it is never 'topped-up' by training following the first job match.

Both improvements in the productivity of matches and lower unit cost of skills $\gamma$, are channels through which economic development can occur. Both effects raise the marginal productivity of education schedule relative to its marginal costs schedule which implies that the cut off $\alpha^*$ increases. The range of skills accumulated through education increases through the development process.

Overall, the analysis of education in a search economy highlights the 'backwards induction' feature of education incentives. The decision to educate or train is subtle: the incentives to buy education are derived from the employment opportunities available in the job market and the degree to which the unit costs of education are cheaper than that of training.

**White versus Blue Collar Workers**

The analysis on education suggests sharp predictions about the formation of distinct classes of workers in the economy. Workers with different unit costs of skills may exhibit qualitatively different patterns

of skill accumulation. The cut off margin for relative costs of education will be higher for workers with lower unit costs of skill accumulation, $\alpha_L^* > \alpha_H^*$ given $\gamma_L < \gamma_H$. If important skills are characterized by relative costs of education $\hat{\alpha}$, and $\alpha_L^* > \hat{\alpha} \geq \alpha_H^*$, workers with the higher unit costs (blue collar workers) buy no education and become skilled only once matches with entrepreneurs are realized. Blue collar workers train within matches to skill level $k^*$, whereas workers with low unit costs (white collar workers) accumulate all their skills through education, to a level $h^* > k^*$ (strictly higher skill levels). White collar workers will have longer average tenure, lower unemployment duration and higher specific skill accumulation. These distinctions have been established in the empirical literature.

## 4.2 Specific Skill Accumulation

The productivity flow of a particular entrepreneur-worker match, $i$ in method $j$ is modified as follows,[14]

$$g(z_{ij}) + x_{ij}\sigma \qquad \text{where } g' > 0, g'' < 0 \tag{37}$$

As before, I shall drop indices for method for simplicity. It is also convenient to allow agents to live forever by getting rid of death shocks, so the death rate should now be interpreted as the interest rate. For vacancies, all searching workers are now identical, so the issue of whether search is directed or not is irrelevant. The asset value of a matched idea $i$, after specific skill investments have been sunk is given by,

$$\delta J_i = g(z_i) + x_i\sigma - w_i + \lambda \int [\max\{\tilde{J}, 0\} - J_i] dF(\tilde{x}) \tag{38}$$

The asset value of an idea searching for a worker is given by,

$$\delta V = -c + q(\theta)(J - \nu z_{h \neq i}^e - V) \tag{39}$$

[14] Again we assume that specific skill levels do not increase the variance of idiosyncratic shocks.

The asset value of a worker matched with idea $i$, after specific skill investment has been sunk is given by,

$$\delta W_i = w_i + \lambda \int [\max\{\tilde{W}, U\} - W_i] dF(\tilde{x}) \tag{40}$$

The asset value of a worker searching for a job is given by,

$$\delta U = a + m(\theta)(W - \nu z^w_{h \neq i} - U) \tag{41}$$

Where $z_i \equiv z^e_i + z^w_i$.

Again productivity contingent transfers are ruled out and wages are set to maximize the Nash Product for Wages once the specific investment is sunk,

$$w^*_i = \arg\max (J_i)^{1-\beta}(W_i - U)^{\beta} \text{ given } x_i, z_i \tag{42}$$

Specific Skill Accumulation and Finance are determined by maximizing the Nash Product,

$$\max_{z_i, z^e_i} (J - \nu z^e_i)^{1-\beta}(W - \nu(z_i - z^e_i) - U)^{\beta} \text{ given } x_i = \bar{x}, w_i = w \tag{43}$$

The FOCs yield the Specific Skill Accumulation Rule and Skill Finance Rule,

$$z^*_i \equiv \arg\max\{J + W - \nu z_i\} = \arg\max\{S - v z_i\} \tag{44}$$

$$z^{e*}_i = (1 - \beta) z_i \tag{45}$$

Unlike general skills, specific skills do not improve the outside option of workers so entrepreneurs simply pay their bargaining share of the costs of specific skill accumulation.

From the FOC for the Skill Accumulation Rule we get the Specific Skill Accumulation Equation,

$$\frac{1}{g'(z^*_i)} = \frac{1}{\nu(\delta + \lambda F(R_i))} \tag{46}$$

Intuitively, the incentives for specific skill accumulation are independent of the matching possibilities within the economy and only a function of the expected duration of the current match.

From the Job Destruction Rule the Job Destruction Equation is,

$$[g(z_i) + R_i^*\sigma] + \frac{\lambda\sigma}{\delta+\lambda}\int_{R_i^*}^{\bar{x}}(1-F(\tilde{x}))d\tilde{x} = [a + m(\theta)\beta\frac{\sigma(\bar{x}-R_{h\neq i})}{\delta+\lambda}] \quad (47)$$

Unlike general skills, idea specific skills have no effect on the vacancy to unemployment ratio and no effect on the destruction margin of future matches of the worker.

This implies that,

$$\frac{\partial R_i}{\partial z_i} = -\frac{g'(z_i)}{\delta+\lambda F(R_i)}\frac{\delta+\lambda}{\sigma} < 0 \quad (48)$$

$$= -v\frac{\delta+\lambda}{\sigma} < 0 \qquad \text{at } z_h = z_h^* \quad (49)$$

From the Free Entry Rule the Job Creation Equation is given by,

$$\frac{c}{q(\theta^*)} = J - \nu z_h^e = (1-\beta)\left(\frac{\sigma(\bar{x}-R_h)}{\delta+\lambda} - \nu z_h\right) \quad (50)$$

Taking differentials with respect to $z_h$,

$$\frac{c\eta}{\theta q(\theta)}\frac{\partial\theta}{\partial z_h} = (1-\beta)\left(\frac{g'(z_h)}{\delta+\lambda F(R_h)} - \nu\right) > 0 \qquad \forall z_h < z_h^*$$

$$= 0 \qquad \text{at } z_h = z_h^*$$

In the economy where specific skill accumulation is undertaken, the equilibrium specific skill level maximizes the tightness of the market.

Combining these results with the analysis for general skill accumulation, general and specific skill are indirect complements. Higher general skills complement specific skills through a lower destruction margin. Optimally determined specific skills complement general skills through a lower destruction margin and higher labour market tightness for skilled workers.

# 5 Conclusion

I have identified the feedback linkages between general skills and specific skills, and between each of these skills with labour market variables. The modified Becker rule for general skill accumulation in a search economy together with the conditions for efficient skill accumulation provide analytical benchmarks for future research in this area. The endogenous determination of skill accumulation through training or education provides insights into the relationship between education and per capita income levels which should be explored further.

The propagation mechanisms developed here are confined to the worker side of the economy. The perfectly elastic supply of vacancies assumed in the paper needs to be modified in an integrated story of worker and entrepreneurial dynamics. The fact that efficient skill accumulation is conditional on free entry of vacancies suggests that the efficiency implications of entrepreneurial dynamics will not be straightforward.

The most natural extension of this work is to apply it to issues of youth unemployment and training, and long term unemployment and training. This could be accommodated by adopting a life-cycle version of the perpetual youth over-lapping generations model, as explored by Gertler (1999). Workers could be in two states young and old where the young face a constant transition probability to become old and the old face a constant probability of death. The model should predict that the longest unemployment durations are suffered by the old unskilled, the shortest by the young skilled, although as a group there is more unemployment among the young then among the old.

A further application of this framework is to consider the importance of the first job for young labour market entrants. Recall the quality of

the first job of workers will determine initial levels of general training and thus future job prospects. When ex ante identical young workers within a cohort are matched with first jobs of different quality, it is likely to create substantial within-cohort earnings inequality which persists throughout the careers of the cohort members. Meanwhile, were one to compare between cohorts who entered the labour market at different times, the performance of the aggregate economy at the time of labour market entry is likely to have a persistent effect on the earnings outcomes of different cohort members' careers.

## References


[1] Acemoglu, Daron (1997), "Training and Innovation in an Imperfect Labour Market," *Review of Economic Studies*, vol. 64, pp. 445-464.

[2] Acemoglu, Daron and Robert Shimer, (1999), "Holdups and Efficiency in Search Equilibrium," *International Economic Review*, forthcoming.

[3] Basu, Kaushik (1997), "*Analytical Development Economics,*" MIT Press.

[4] Becker, Gary (1975), "*Human Capital: A Theoretical and Empirical Analysis, With Special Reference to Education*", NBER: New York.

[5] Davis, S. and J. Haltiwanger (1998), "Gross Job Flows,' *Handbook of Labour Economics*," eds. Ashenfelter O. and D. Card, Volumes 3 and 4, forthcoming.

[6] Gertler, Mark (1999), "Government Debt and Social Security in a Life-cycle Economy," *Carnegie Rochester Series in Public Policy*, vol. 50:1, pp. 61-110.

[7] Harris, J. and M. Todaro (1970), "Migration, Unemployment and Development: A Two Sector Analysis," *American Economic Review*, vol. 60, pp. 126-42.

[8] Hosios, Arthur (1990), "On the Efficiency of Matching and Related Models of Search and Unemployment," *Review of Economic Studies*, vol. 57:2, pp. 279-98.

[9] Mortensen, Dale and Chris Pissarides, (1994), "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies*, vol. 61, pp. 397-415.

[10] OECD (1994), *Employment Outlook*.

[11] Roberts, Mark and John Tybout (1996), "*Industrial Evolution in Developing Countries: Micro Patterns of Turnover, Productivity and Market Structure*," New York: Oxford University Press.

[12] Stiglitz, Joseph (1974), "Alternative Theories of Wage Determination and Unemployment in LDCs: The Labour Turnover Model," *Quarterly Journal of Economics*, vol. 88, pp.194-227.

# 6 Appendix A: Skilled Sector Equilibrium

The per period income flow to an entrepreneur from idea $i$ is,

$$(2\delta + \lambda)J_i = f(k) + x_i\sigma - w_i + \lambda \int \max\{\tilde{J}, 0\}dF(\tilde{x})$$

The per period surplus income flow to a skilled worker matched with idea $i$ is,

$$(2\delta + \lambda)(W_i - U) = w_i + \lambda \int \max\{\tilde{W} - U, 0\}dF(\tilde{x}) - (r + \delta)U$$

The steady state joint bilateral surplus income flow is simply the sum of these two equations and by integrating we get,

$$(2\delta + \lambda)S_i = f(k) + x_i\sigma + \frac{\lambda\sigma}{2\delta + \lambda} \int (1 - F(\tilde{x}))d\tilde{x} - (r + \delta)U$$

Setting $x_i = R$ and $S(R) \equiv 0$ we get the Job Destruction Rule. The Free Entry Condition implies a positive correlation between the size of the asset value from a newly matched idea and the vacancy to unemployment ratio for skilled workers,

$$J = \frac{c}{q(\theta)}$$

Combining this with the Surplus Division Rule, we get the Job Creation Equation.

From the function for the bilateral surplus and the wage bargaining rule, the asset value from being unemployed can be rewritten as,

$$\delta U = a + m(\theta)\beta\frac{\sigma(\bar{x} - R)}{2\delta + \lambda}$$

The joint asset value of a new idea matched with an unskilled worker is,

$$J + W - \gamma k = S + U - \gamma k = \frac{a}{\delta} + \left(1 + \frac{m(\theta)\beta}{\delta}\right)\frac{\sigma(\bar{x} - R)}{2\delta + \lambda} - \gamma k$$

The FOC for the Skill Accumulation Rule is initially given by,

$$\frac{m'(\theta)\beta}{\delta}\frac{\sigma(\bar{x} - R)}{2\delta + \lambda}\frac{\partial\theta}{\partial k} - \left(1 + \frac{m(\theta)\beta}{\delta}\right)\frac{\sigma}{2\delta + \lambda}\frac{\partial R}{\partial k} = \gamma$$

The marginal productivity and marginal cost of skills are equated. The first term in the LHS captures the effect that skill levels have on the workers individual rematch probability (conditional on directed search) and

the second term captures the effect that skill levels have on improving the size of current and future expected match surpluses. Substituting in the equilibrium equations for $\frac{\partial R}{\partial k}, \frac{\partial \theta}{\partial k}$ we get the Skill Accumulation Equation.

The wage equation is solved by substituting in equilibrium $J, W, U$ into the surplus division rule.

# 7 Appendix B: Efficiency

The current value Hamiltonian of the social planner's problem is given by,

$$\begin{aligned} H(s, u_s, v, \hat{v}, k) = & [(1-s)+u_s]a - [v+\hat{v}]c - (s-u_s)f(k) \\ & +(s-u_s-b)\bar{x}\sigma + bE(\tilde{x}|\tilde{x}\ R)\sigma - \hat{M}\gamma k \\ & +\mu(\hat{M} + M - (2\delta + \lambda F(R))(s-u_s)) \\ & +\phi(\hat{M} - \delta s) \\ & +\pi\left(\frac{\lambda(1-F(R))}{2\delta+\lambda}(\hat{M}+M) - (2\delta+\lambda F(R))b\right) \end{aligned}$$

The partial FOCs imply,

$$\frac{\partial H}{\partial u_s} = 0 \Rightarrow \mu = \frac{f(k) + \bar{x}\sigma - a - \pi M_{u_s}\frac{\lambda(1-F(R))}{2\delta+\lambda}}{2\delta + \lambda F(R) + M_{u_s}}$$

$$\frac{\partial H}{\partial b} = 0 \Rightarrow \pi = \frac{(E(\tilde{x}|\tilde{x} \geq R) - \bar{x})\sigma}{2\delta + \lambda F(R)}$$

$$\frac{\partial H}{\partial v} = 0 \Rightarrow \frac{c}{M_v} = \mu + \pi\frac{\lambda(1-F(R))}{2\delta+\lambda}$$

The equivalence of this with the Job Creation Equation for skilled workers implies that decentralized entry of vacancies for skilled workers is efficient.

$$\frac{\partial H}{\partial u_s}|_{\overline{(s-u_s)}} = a + M_{u_s}\left(\mu + \pi\frac{\lambda(1-F(R))}{2\delta+\lambda}\right)$$

The equivalence of this with the RHS of the Job Destruction Equation implies that decentralized job destruction decisions are efficient.

$$\frac{\partial H}{\partial s} = 0 \Rightarrow \phi = \frac{f(k) + \bar{x}\sigma - a - \mu(\hat{M}_{(1-s)} + 2\delta + \lambda) - \pi\hat{M}_{(1-s)}\frac{\lambda(1-F(R))}{2\delta+\lambda} + \hat{M}_{(1-s)}\gamma k}{\delta + \hat{M}_{(1-s)}}$$

$$\frac{\partial H}{\partial \hat{v}} = 0 \Rightarrow \frac{c}{\hat{M}_{\hat{v}}} = \phi + \mu + \pi \frac{\lambda(1 - F(R))}{2\delta + \lambda} - \gamma k$$

The equivalence of this with the Job Creation Equation for unskilled workers implies that decentralized entry of vacancies for unskilled workers is efficient.

$$\frac{\partial H}{\partial (1-s)}|_{\bar{s}} = a + \hat{M}_{(1-s)} \left( \phi + \mu + \pi \frac{\lambda(1 - F(R))}{2\delta + \lambda} - \gamma k \right)$$

The equivalence of this with the flow value of unskilled workers implies that births into the unskilled worker pool are efficient.

$$\frac{\partial H}{\partial k} = 0 \Rightarrow \frac{1}{f'(k)} = \frac{(s - u_s)}{\gamma \hat{M}} = \frac{\delta + m(\theta)}{\gamma \delta [2\delta + \lambda F(R) + m(\theta)]}$$

Immediate inspection shows that skill accumulation is efficient under the Hosios Condition.

Under the Hosios Condition, we have $M_{u_s} = m(\theta)\beta, M_v = q(\theta)(1 - \beta), \hat{M}_{(1-s)} = m(\hat{\theta})\beta, \hat{M}_{\hat{v}} = q(\hat{\theta})(1 - \beta)$ and $S = \mu + \pi \frac{\lambda(1-F(R))}{2\delta+\lambda}, T = \phi + \mu + \pi \frac{\lambda(1-F(R))}{2\delta+\lambda} - \gamma k$. These imply that the economy is fully efficient subject to search frictions as long as there are no search externalities.