Evidence Disclosure and Verifiability

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Abstract

We explore the conceptual basis of "verifiability" by explicitly modeling the process of evidence production in contractual relationships of complete information. We study how the contracting parties' incentives to disclose evidence (in the form of documents) narrows the set of enforceable contracts and affects what can be considered verifiable. Our *Full Disclosure Result* characterizes verifiability purely on the basis of whether documents exist in various contingencies. This result identifies conditions under which "message game phenomena" cannot arise, so that the only useful evidence is "hard" in the sense of ruling out some contingencies. The required conditions include opportunities for parties to engage in side-dealing and renegotiation during the enforcement phase. We also prove a version of the revelation principle — our *Honest Disclosure Result* — that clarifies the meaning of "truthful reporting." We briefly discuss the relevance of our results to the functioning of legal institutions. *JEL Classification: C70, D74, K10.*

The notion of "verifiability" — that a court can observe a given aspect of a contractual relationship — is at the heart of the contract theory literature. Economic models of contract generally start with a specification of things considered verifiable and then assume that court-enforced contracts can condition transfers arbitrarily on (and only on) these things.[1] In reality, however, court action (and, hence, implementability) is a function not of what *can* be observed by the court but what evidence is *actually* presented to the court by the contracting parties. Thus, verifiability critically depends on the parties' incentives to submit evidence.

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[1]Often constraints are added to reflect an opportunity for the parties to renegotiate before or after court enforcement.

This paper offers a foundation for the notion of verifiability, by explicitly modeling evidence disclosure and contract enforcement in contractual relationships of complete information. In our model, players first agree to a contract, then engage in productive interaction, and afterward can voluntarily submit evidence in a court-administered enforcement phase. We represent evidence as physical documents, the existence of which depends on the outcome of productive interaction.[2] We study the agents' incentives to disclose documents and we characterize how incentives in the enforcement phase impose limits on the extent to which court action (external enforcement) can be conditioned on the outcome of productive interaction. In other words, our results show how verifiability is sensitive to the nature of documented evidence.

Our main theoretical goal is to provide conditions under which verifiability can be characterized in terms of "hard evidence." At the heart of our analysis is a theorem we call the *Full Disclosure Result*; it establishes that, when evaluating the scope of external enforcement, one can restrict attention to full disclosure behavioral rules. Full disclosure means that, at the time of contract enforcement, every player always submits *all* of the documents in his possession.

The Full Disclosure Result holds in an environment in which the court only compels monetary awards (rather than productive actions) and the players pursue their individual and joint incentives in the enforcement phase. Specifically, given a contract and productive contingency (the *state*), individuals selectively produce feasible documents to maximize their court-enforced transfers. In addition, for settings of more than two players, we assume that coalitions of players can engage in externally-enforced side contracting on the documents each is to disclose.[3] We require in any equilibrium that no coalition of players can gain by deviating from the prescribed disclosure rules to spot contract over their own documents. That is, equilibria of the evidence disclosure phase must be *impervious to side contracting*. This condition applies Bernheim, Peleg, and Whinston's (1987) notion of coalition-proof Nash equilibrium in a setting in which players can write externally-enforced side contracts. Overall, side-dealing and renegotiation conditions add some "balancedness" restrictions that constrain implementation, yielding our characterization.

The Full Disclosure Result simplifies the analysis of evidence disclosure and contract enforcement, leading to two main insights. The first insight is that the players' ability to side-contract precludes so-called "message game" schemes that tie transfers to coordinated messages the players send to the court. For example, consider a situation with three players (1, 2 and 3) and two states (a and b); the players jointly observe the state, while the court can only condition transfers on voluntary messages the players send. Suppose that there are two messages each player can send to the court: "a" and "not a." Think of the first of these as a document called d (which can be sent regardless of the state) and the second as silence. Further, suppose the

[2] "Documents" here include any testimony, statements, objects, etc. that can be submitted as evidence.

[3] Such a side deal may simply amount to joint disclosure of evidence, which is feasible in reality.

parties wish to achieve the transfer $(1, 1, -2)$ — that is, player 3 is forced to transfer one unit to each of players 1 and 2 — in state a and the transfer $(0, 0, 0)$ in state b.

As the literature has recognized, such a transfer schedule can be implemented as the outcome of a Nash equilibrium. One way of doing this is to write a contract specifying that the court will impose transfer $(1, 1, -2)$ as long as two or three players send the document d; otherwise, the court imposes $(0, 0, 0)$. Then it is an equilibrium for the players to each submit document d if and only if the state is a. No player has an incentive to unilaterally deviate. However, such a contract is *not* impervious to side contracting. In state a, for instance, players 2 and 3 have the joint incentive to enter into a side deal whereby they agree to a transfer of 1.5 from player 3 to player 2 conditional on neither player submitting d. With this side deal in place (in addition to the original contract), players 2 and 3 each have the incentive to send the "not a" message, disrupting the intent of the original contract. The document in this example is *cheap* because it can be produced in either state. In general, cheap documents cannot be the basis of verifiability when side contracts are enforced.

The second insight is that court-enforced transfers are sensitive to whether evidence is *positive* or *negative* in nature. For an illustration, consider a setting with two players (1 and 2) and two states (a and b). Suppose document d exists in state a but does not exist in state b (an example would be a completed product). We say that disclosure of d is positive evidence of a, while nondisclosure of d is negative evidence of b. There is a sense in which the disclosure or nondisclosure of d verifies whether a or b occurred. However, arbitrary transfers conditional on the state are not always possible.

For example, imagine that the players wish to force a transfer t from player 1 to player 2 in state a and no transfer in state b. To achieve this transfer schedule, the players must write a contract instructing the court to compel transfer t if and only if document d is disclosed. The contract implements the desired transfer schedule as long as d is actually submitted in state a. But if player 1 possesses the document then he has no interest in submitting it unless $t \leq 0$, since only in this case is he rewarded for disclosing d. Thus, if only player 1 possesses the document then there is no externally-enforced contract that supports $t > 0$. On the other hand, $t > 0$ can be induced if player 2 possesses the document, since she has the incentive to submit it in this case. Further, it is possible to implement $t > 0$ if player 1 possesses a document d' that only exists in state b. This can be done by giving player 2 a transfer of t *unless* d' is disclosed by player 1.

Overall, we show that (i) court-enforced transfers must be measurable with respect to a partition reflecting the distribution of documents over states and (ii) additional restrictions are implied by the extent to which states are distinguished only by negative evidence. Conclusion (i) implies a coarser partition than is generally assumed in the literature — in particular, for settings with more than two players. Conclusion (ii) implies that it is sometimes inappropriate to treat verifiability merely as a partition of the state space, as is typically done in the literature.[4] Our analysis indicates where the standard methodology is appropriate. Specifically, we describe evidence environments for which the set of implementable court-enforced transfers is unconstrained by incentives in the disclosure phase.

Our document-based approach to studying evidence and verifiability allows us to clarify the notion of "truthful reporting" that plays such a major role in the mechanism design literature. For a very general contracting environment (not necessarily assuming what is needed for our Full Disclosure Result), we prove a revelation principle that we call the *Honest Disclosure Result*.[5] This theorem establishes a predefined meaning of "honest disclosure" — in terms of the required documented evidence — that can be applied independently of the players' implementation goal. We also demonstrate that any document-based model can be legitimately translated into a more abstract mechanism design model. This analysis provides a justification, and foundation, for Green and Laffont's (1986) modeling approach and their "nested range condition."

In a modest sense, our model also helps organize one's thinking about actual legal institutions. In particular, the U.S. legal system is sensitive to the distinction between positive and negative evidence. The theory identifies positive evidence as the strongest form of evidence and, in fact, positive evidence presented to substantiate a claim is always given weight by the legal system. Further, although U.S. courts recognize the significance of negative evidence, they generally treat it as less compelling than positive evidence. Wigmore (1935) suggests that failure by a party to disclose evidence that would be favorable to her claim if it existed is "open to the inference that it does not exist."[6] That is, in our example above, it is known that document d exists in state a but does not exist in state b. As suggested, a contract can be structured to give player 2 the incentive to disclose d when this document is available. Thus, under such a contract (and assuming rational behavior), her failure to disclose d can be taken to mean that d does not exist. However, this conclusion may not be reached if the original contract failed to provide player 2 with the incentive to disclose d. A classic case, *State v. Simons* (1845) provides a good example of a setting where the court's decision is based upon negative evidence.[7]

[4]In the standard approach, one takes a partition as given and then looks for an externally-enforced contract that is measurable with respect to this partition and which induces a particular productive action profile. Examples of this approach include Holmstrom (1982), Legros and Matthews (1993), and more recently Bernheim and Whinston (1999).

[5]The Honest Disclosure Result is very similar to an independently-produced theorem of Deneckere and Severinov (2001).

[6]Wigmore (1940) notes that uncertainty exists as to the exact "nature of the inference and conditions in which it may legitimately be drawn."

[7]Simons was accused of selling spirits without a license to do so. The court held that the failure of Simons to present a license, even though State presented no evidence showing lack of a license, was sufficient to conclude that he did not possess such license. The court's opinion suggests that evidence "need not be of the most direct and positive kind."

The modeling exercise reported herein combines elements of several noteworthy papers in the literature. Our notion of documents existing in different contingencies is much like the "truthful reporting" constraints studied by Milgrom and Roberts (1986), Okuno-Fujiwara, Postlewaite and Suzumura (1990), Shin (1994), Lipman and Seppi (1995), and Seidmann and Winter (1997); and it resembles the "sending a message cannot be verified" setting studied by Hart and Moore (1988). Following the large body of work on Nash implementation and implementation with renegotiation, we study the case of complete information between contracting parties. See, for example, Holmstrom (1982), Legros and Matthews (1993), Maskin and Moore (1999), and Maskin and Tirole (1999).[8] As noted above, we also utilize a version of Bernheim, Peleg, and Whinston's (1987) coalition-proofness concept.

Our approach differs from most other models of court decision making and evidence production.[9] The influence-cost literature studies models in which the probability of a litigant winning at trial is determined by an exogenous function which depends upon litigants' effort or expenditure (typically on evidence production).[10] There are also studies of the court as a Bayesian decision maker who receives signals of the defendant's type. The signals are influenced by effort or expenditure in evidence production.[11] The strategic search literature models each party's evidence production as a costly random draw of evidence from a distribution of evidence.[12] A party may draw as many times as she desires. The most favorable evidence drawn is assumed to be presented in court. This allows for incomplete information between the parties.[13] An advantage of this costly sampling approach is that it allows for the

[8]Also relevant is Miller (1997), who studies a version of Legros and Matthews' (1993) model in which a single player observes the actions of the others and can send a message to the enforcement authority.

[9]Classic legal treatments of evidence are Bentham (1827) and Wigmore (1940).

[10]See Tullock (1980), Danzon (1983), Braeutigam, Owen, and Panzar (1984), Hause (1989), Katz (1988), Plott (1987), Cooter and Rubinfeld (1989), and Landes (1993). Skaperdas (1996) develops an axiomatic foundation for this type of model. Sanchirico (1999) models the court's decision as depending upon the evidence presented at trial. In his model, the cost of producing evidence depends upon the state, and evidence can be forged. The analysis there focuses on the relationship, in a tort setting, between sustaining a particular level of care and the cost of producing evidence. He uses this framework to provide an explanation of the English transition toward a more passive fact-finding jury. Bernardo, Talley, and Welch (2000), who study a standard principal-agent model with the added feature of the possibility of litigation following a low productive outcome. Using an interesting twist on the influence-cost approach, they study a setting where the agent's cost of evidence production depends upon her level of productive effort. They use this framework to analyze legal presumptions.

[11]See for, example, Rubinfeld and Sappington (1987). In some settings, such as Sobel (1985) and Shin (1994 and 1998), the decisionmaker is allowed to reallocate the burden of proof. Modeling the court as a Bayesian decision maker may not be appropriate in all settings, however (Daughety and Reinganum 2000a).

[12]See Reinganum (1982), who studies research and development by firms in a duopoly.

[13]Examples of this approach include Daughety and Reinganum (2000b), Froeb and Kobayashi (2000), and Gong and McAfee (2000).

consideration of evidence costs. However, it does not address multiple dimensions of evidence production, nor the cost of individual pieces of evidence. Distinct from the contributions in these related literatures, our modest goal is to push the contract theory literature a bit closer to reality by providing a foundation for "verifiability" and "honest reporting" in terms of the opportunities that contracting parties have to present documented evidence.

The paper is organized as follows. In Section 1 we describe the model of evidence disclosure and discuss the concept of equilibrium that is impervious to side contracting. In Section 2, we characterize implementable court-enforced transfers and prove the Full Disclosure Result. Section 3 analyzes the relationship between the manner by which players can distinguish one state from another and the relevant partition of the state space. In Section 4, we turn our attention to incentives in productive interaction and we characterize, given an induced partition, those productive action profiles that can be induced. In Section 5 we present general analysis on honest disclosure. Appendix A contains proofs not found elsewhere in the text. Appendices B and C present extensions noted in the text.

1 A Model of Contract, Evidence Disclosure, and External Enforcement

We consider a contractual relationship between n players (also called agents), who interact over four periods of time. In the first period, the players form a contract. This contract has an externally-enforced component m which specifies monetary transfers to be compelled by the court in period 4, conditional on evidence presented to the court in period 3.

In the second period, all productive interaction occurs, leading to an outcome a which we call the *state of the relationship.* The state is commonly observed by the players. We let A denote the set of possible states and we assume A is finite. Players receive an immediate payoff given by $u : A \to \mathbf{R}^n$. Since most of our analysis addresses how behavior later in the game is conditioned on the state, we defer to Section 4 further discussion of the details of the production phase.

External contract enforcement occurs in periods 3 and 4. Specifically, in period 3 the players simultaneously and independently submit documents to the court. Documents represent evidence on which the court conditions transfers. Denote by $D_i(a)$ the set of documents that can be presented by player i in state a. Since not all documents may be available in all states, $D_i(a) \neq D_i(b)$ is generally, but not necessarily, the case. Let $D_i \equiv \bigcup_{a\in A} D_i(a)$ denote the set of documents available to player i over all states. We assume D_i is finite. We also assume $D_1, D_2, \ldots, D_n$ are disjoint sets and, defining $N \equiv \{1, 2, \ldots, n\}$, we let $D \equiv \bigcup_{i\in N} D_i$ and $D(a) \equiv \bigcup_{i\in N} D_i(a)$ for each a. For any set of documents $E \subset D$, we write E_i as those documents disclosed by player i. The feasible sets of disclosed documents are given by $\mathcal{D} \equiv \{E \mid E \subset D(a),\ \text{for some } a \in A\}$.

Note that the empty set (no documents disclosed) is an element of $\mathcal{D}$.

In period 4 the court imposes the transfer m as a function of the documents disclosed by the players.[14] Formally, $m : \mathcal{D} \to \mathbf{R}^n$, so for any evidence set $E \subset \mathcal{D}$, $m_i(E)$ is the monetary transfer made to player i. Thus, player i's total payoff in the contract game is $u_i(a) + m_i(E)$. We assume $\sum_{i \in N} m_i \leq 0$. Remember that m is jointly selected by the players in period 1.

Regarding document disclosure, we apply the term *disclosure rule* to any function $\beta : A \to \mathcal{D}$, satisfying $\beta(a) \subset D(a)$ for each $a \in A$, which describes how the players behave in period 3, conditional on the state.[15] For example, in state a the players disclose documents $\beta(a)$. Let $\beta_i(a)$ denote the documents presented by player i in state a and, for any $J \subset N$, define $\beta_J(a) \equiv \bigcup_{i \in J} \beta_i(a)$.[16]

We call any function $g : A \to \mathbf{R}^n$ a *transfer function*. An externally-enforced contract m and a disclosure rule β imply the transfer function g defined by $g(a) = m(\beta(a))$ for every $a \in A$. The following terminology will be handy.

Definition 1 *A transfer function g is called* **implemented by externally-enforced contract** m **and disclosure rule** β *if $g(a) = m(\beta(a))$ for every $a \in A$. Also, g is called* **implemented by disclosure rule** β *if there is an externally-enforced contract m such that m and β implement g.*

On a technical level, our main goal is to characterize the degree to which court-compelled transfers can be indirectly conditioned on the state, given that the players behave rationally in the disclosure phase. In other words, we seek to understand whether any given transfer function g can be implemented by an externally-enforced contract m and a disclosure rule β that is consistent with rationality.

Interpretation

Our model associates the common notion of "verifiability" with the court's observation of actual documents disclosed by the players. As an example, consider a relationship between a creditor and a debtor. Suppose there are two states, one representing that the debtor has paid the creditor and the other representing that no payment was made. Three documents can be presented by the debtor. The first, denoted d_d^1, is a receipt given to the debtor by the creditor certifying payment. The second, d_d^2, is a canceled check that exists only if payment was made. The third document, d_d^3, is written testimony of payment made by the debtor's friend, who is brought forth as a witness. Two documents may be presented by the creditor. The

[14] Our model applies equally well to other external enforcement systems. Note that all productive interaction occurs before period 4, so external enforcement occurs via monetary transfers.

[15] For simplicity, we focus here on pure disclosure rules. In Appendix B, we show that consideration of mixed rules (where players submit documents with nondegenerate probabilities) adds little to the analysis; our main results are unchanged when mixed disclosure rules are allowed.

[16] Note that an individual player's behavior can easily be deduced from $\beta(a)$ since the players' document spaces are disjoint.

	d_d^1	d_d^2	d_d^3	d_c^1	d_c^2
Paid	X	X	X		X
Did not pay			X	X	X

Figure 1: Description of $D_i(a)$ sets in the debtor/creditor example.

first, d_c^1, is a bank notice demonstrating that the debtor's check was returned due to insufficient funds. The second, d_c^2, is testimony of non-payment made by the creditor's colleague, who is brought forth as a witness.[17]

Figure 1 describes how the availability of these documents depends on the state (that is, the specification of $D_i(a)$ sets) in this example. In the figure, an X indicates that a particular document exists and can feasibly be disclosed in a given state. For example, the debtor can produce a canceled check (document d_d^1) or a receipt (document d_d^2) only if she actually paid the creditor. However, it may be that she can find a witness to testify (document d_d^3) regardless of whether she paid. Similarly, the creditor can only produce the bounced check (document d_c^1) if the debtor did not pay, but the creditor can always find a witness to say that payment has not occurred (document d_c^2).

One way of thinking about documents is that players can present any particular document at a *cost* which depends on the state. We assume that, given a, documents in $D(a)$ can be disclosed at zero cost, while documents outside of this set can be produced only at infinite — or sufficiently prohibitive — cost; for example, a player cannot convincingly forge a canceled check.

Readers comfortable with the contract literature will naturally be inclined to view verifiability in terms of partitions of the state space. It is important to realize that there is no such partition at the foundation of our theory, since what can be deduced about the state from disclosed documents depends on the players' disclosure rule. It can be helpful, though, to keep in mind how the evidence environment (characterized by the sets $D_i(a)$) and disclosure rules relate to partitions of the state space. We can start by calculating the set of states under which the same evidence set E is produced by the players. Formally, if we know the players' disclosure rule β and we observe evidence set E, where $E = \beta(a')$ for some state a', then we can deduce that the state is in the set $\{a \in A \mid \beta(a) = E\}$. Thus, β suggests a partition of the state space given by

$$\{\{a \in A \mid \beta(a) = \beta(a')\} \mid a' \in A\}.$$

This partition *only* reflects what can be deduced from the disclosure rule; it is ob-

[17] One can imagine other documents that might be relevant and could easily be included in this example. In addition, "document" should be interpreted broadly to include items such as videotapes, audio recordings, oral statements made in court, etc.

viously not a fundamental of the players' contracting problem. The disclosure rule itself is subject to availability of documents (from sets $D_i(a)$) as well as the players' incentives given the externally-enforced contract m. The fineness of the induced partition corresponds to the extent that the players disclose distinct sets of evidence in different states.

Our assumption that the D_i sets are disjoint does not limit application of the model. For example, suppose in reality both the creditor and debtor can produce the canceled check in some state. Then, in fact, it will be the case that one of them can produce the actual canceled check while the other can produce *a copy* of the canceled check, with these two items considered as different documents.

Equilibrium Concept

To analyze behavior in period 3, we use the standard Nash equilibrium concept. Given m, we call β an *equilibrium disclosure rule* if β specifies a Nash equilibrium for each a; that is,

$$m_i(\beta(a)) \geq m_i(E_i \cup \beta_{-i}(a))$$

for each $i \in N$, $E_i \subset D_i(a)$, and $a \in A$. In addition to equilibrium, we add constraints reflecting the players' ability to renegotiate between periods 3 and 4 and to make side deals between periods 2 and 3.[18] On the former, suppose the players can renegotiate the externally-enforced contract m between periods 3 and 4. If their outstanding contract is such that $\sum_{i \in N} m_i(E) < 0$ for some evidence set E, then following disclosure of E the players would re-specify m before the court compels transfers. This justifies restricting attention to *balanced* externally-enforced contracts, which are functions of the form $m : \mathcal{D} \to \mathbf{R}^n_0$, where $\mathbf{R}^n_0 \equiv \{x \in \mathbf{R}^n \mid \sum_{i \in N} x_i = 0\}$.

Regarding side deals made between periods 2 and 3, we shall impose a version of Bernheim, Peleg, and Whinston's (1987) *coalition-proofness* concept that assesses whether a sub-group of players can benefit from side contracting. We examine whether a coalition can gain from writing a side contract m' to be externally enforced along with the players' original contract m. To illustrate, suppose that a coalition of players $J \subset N$ wishes to write an additional contract m' specifying transfers between members of this coalition as a function of the documents disclosed in period 3. It must be that $m'_i = 0$ for each $i \neq J$ since coalition J cannot force a side contract on players outside of the coalition. In addition, m' must be balanced due to the specter of renegotiation between periods 3 and 4. We assume that m' is enforced, either by the court or by some other means.[19] With the side contract, externally-enforced transfers in period 4 are given by $m + m'$.

[18]The constraints represent free renegotiation and side-dealing. For a discussion of costly renegotiation, see Schwartz and Watson (2001).

[19]In practical terms, a side contract may amount to joint disclosure of documents, which is generally possible in reality and, as we shortly demonstrate, is an appropriate interpretation of our mathematical constraint. For a discussion of civil procedure, see, for example, Teply, L. and R. Whitten (2000).

Players outside the coalition do not observe the side contract until after documents are disclosed. Thus, the point of writing a side contract is to induce members of the coalition to change their behavior in the disclosure phase in a way that benefits the coalition. To evaluate whether this is possible, define $M_J(E) \equiv \sum_{j \in J} m_j(E)$ and let $M'_J(E)$ be defined analogously. Suppose that in state a the players would coordinate on disclosure of documents $\beta(a)$ in the absence of side contracting. Further suppose that, by side contracting, a coalition J can induce its members to disclose documents E_J. Then the coalition strictly gains from the side deal if and only if

$$M_J(\beta_{-J}(a) \cup E_J) + M'_J(\beta_{-J}(a) \cup E_J) > M_J(\beta(a)) + M'_J(\beta(a)),$$

which is equivalent to $M_J(\beta_{-J}(a) \cup E_J) > M_J(\beta(a))$ since $M'_J = 0$.

Between periods 2 and 3, a J coalition can always find a side contract that induces its members to disclose any particular set of documents, as long as the specified documents exist given the state. This is because the coalition writes a side contract *after players observe the state* — knowing which documents exist. The coalition can write a *forcing* side contract that punishes individuals for not disclosing exactly those documents desired by the coalition (by way of arbitrarily large transfers to the other members of the coalition). The side contract can also implement any desired split of its gains between the coalition members. Thus, between periods 2 and 3, coalitions can effectively spot contract on which documents to disclose. Mathematically, in state a, a coalition J can side contract to force its members to disclose any set of documents $E_J \subset D_J(a) \equiv \bigcup_{j \in J} D_j(a)$.

A side contract between members of a coalition J may be undermined by a subsequent side contract between members of a sub-coalition $K \subset J$. Following Bernheim, Peleg, and Whinston (1987), we view a side contract as viable only if it is immune to disruption by sub-coalitions (who have to pass the same test). In fact, the issue of sub-coalitions is easily handled in our model, because forcing contracts can always be designed to stifle any further side dealing by sub-coalitions. Specifically, a coalition J can specify m' so that any player $j \in J$ who does not disclose a specified set of documents must pay an amount y to each of the other players in the coalition. Then any sub-coalition $K \subset J$ will lose at least y when one or more of its members deviates from the prescription of m'. The number y can be set large enough so that this loss is greater than any gain the sub-coalition can get by way of the original contract m.[20]

We look for specifications of behavior that are coalition-proof with respect to externally-enforced side contracts. Given the discussion above, it is sufficient to simply evaluate whether coalitions can gain from spot contracting on disclosure of documents. With reference to a state a and a disclosure rule β, we say that $E \in \mathcal{D}$ is a *J-deviation from $\beta(a)$ at a* if $E \subset D(a)$ and $E_i = \beta_i(a)$ for all $i \notin J$. In words, E is

[20]One may wonder whether the players could effectively deter all side contracting by writing this kind of y-punishment contract initially in period 1. This is not the case, because the state (and thus the set of documents that should be disclosed) is not realized until period 2.

a set of documents disclosed when only the players in J deviate from the prescription $\beta(a)$. Coalition-proofness is defined as follows.

Definition 2 *Given an externally-enforced contract m, a disclosure rule β is called* **impervious to side contracting (ISC)** *if $M_J(\beta(a)) \geq M_J(E)$ for each $a \in A$, each coalition $J \subset N$, and each $E \subset D(a)$ that is a J-deviation from $\beta(a)$ at a.*

We use the term *impervious to side contracting* instead of *coalition-proof Nash equilibrium* because the latter is defined for self-enforced contracts (Nash equilibria) of standard non-cooperative games, while we require a version that examines externally-enforced contracts. That is, ISC is Bernheim, Peleg, and Whinston's (1987) definition applied to externally-enforced contracts. Note that ISC includes the self-enforced component, since for each $a \in A$, $\beta(a)$ must be a Nash equilibrium with payoffs defined by m.[21] Because ISC includes constraints for single-player coalitions, clearly ISC implies Nash equilibrium in the evidence disclosure phase. Also, it is important to note that ISC coincides with Nash equilibrium when there are only two players.

Lemma 1 *Every ISC disclosure rule is an equilibrium disclosure rule. Further, in a two player setting, every equilibrium disclosure rule is an ISC disclosure rule.*

Thus, the ISC requirement, which we think of as rather strong yet intuitive and worth studying as a benchmark, is only necessary for our results if there are more than two players. We apply the ISC concept to study rational behavior in the evidence disclosure and enforcement phases of the contractual relationship.

Regarding transfer functions, recall the definition of g implemented by m and β. In the same vein, we say that g is *implemented by an ISC disclosure rule* if there are functions m and β such that (i) g is implemented by m and β, and (ii) β is an ISC disclosure rule with respect to m. We shall also use the term "g implemented by ISC disclosure rule β," meaning there is an m such that (i) and (ii) are satisfied. Since any externally-enforced contract is required to be balanced, we can restrict attention to transfer functions that are also balanced; that is, $g : A \to \mathbf{R}_0^n$.

2 Implementable Transfer Functions

In this section, we characterize the set of transfer functions that are implemented by ISC disclosure rules. An important part of our analysis concerns whether implementing a transfer function relies on players strategically withholding documents that exist.

Definition 3 *The* **full disclosure rule** *$\overline{\beta}$ is defined by $\overline{\beta}(a) \equiv D(a)$ for all $a \in A$.*

With full disclosure, each player submits all of the documents in his possession in every state.

[21] In Section 4, we further discuss self-enforced components of contract.

Definition 4 *A transfer function g is called* **implemented by an ISC/full disclosure rule** *if and only if there exists an externally-enforced contract m such that (i) the full disclosure rule $\overline{\beta}$ is ISC and (ii) g is implemented by m and $\overline{\beta}$.*

Our main technical result is:

Theorem 1 (Full Disclosure Result) *If a transfer function g is implemented by an ISC disclosure rule, then g is implemented by an ISC/full disclosure rule.*

Before proceeding to the characterization of implementable transfer functions and the proof of Theorem 1, we comment on the significance of the Full Disclosure Result. Note that the result is similar in theme to the classic revelation principle of mechanism design theory.[22] Like the revelation principle, it simplifies the analysis of implementation/evidence production by allowing us to focus on the full disclosure rule. As demonstrated later in this paper, the Full Disclosure Result streamlines the analysis of implementable transfer functions.

To understand how the Full Disclosure Result differs from the revelation principle, note that the former pertains to the disclosure of *real* documents, rather than to abstract messages that are theoretical constructs. This distinction may seem minor at first blush, but it is quite significant considering how the existence of documents (or feasible messages) depends on the state. Suppose, for example, that, in both states a and b, the players are able to write "a occurred" on a piece of paper and submit this document. Suppose also that, in both states, the players can write and submit "b occurred." The revelation principle establishes that, to calculate the implementable transfer functions in this setting, one can restrict attention to behavioral rules in which the players submit exactly "a occurred" in state a and "b occurred" in state b. The Full Disclosure Result, on the other hand, justifies restricting attention to behavioral rules in which *both* documents are submitted in *both* states. In other words, in both states a and b, the players present "a occurred" and "b occurred."

As this example demonstrates, the Full Disclosure Result produces a different insight into implementability than does the revelation principle. At the heart of the comparison is the notion of "truthful reporting." With respect to the revelation principle, truthful reporting means that the players honestly name the state, even though they can lie. In regard to the Full Disclosure Result, truthful reporting means disclosing all existing documents. Thus, externally-enforced transfers rely on distinguishing between states via the *existence* of documents (or lack thereof), which is our "hard evidence" foundation for the concept of verifiability. Section 3 elaborates on this point and discusses the implication for "message game phenomena." We pick up on the issue of "truthful reporting" in Section 5, where we prove a version of the revelation principle and compare our model with that of Green and Laffont (1986).

[22] See, for example, Dasgupta, Hammond, and Maskin (1979).

Characterization of Implementability Given β

We begin the analysis with a simple characterization of the ISC condition. For each $a \in A$, $E \subset D(a)$, and $E' \in \mathcal{D}$, we define the function $R(a; E, E')$ as follows. If $E' \not\subset D(a)$, then we let $R(a; E, E') \equiv N$. If $E' \subset D(a)$ then we let $R(a; E, E') \equiv \{i \in N \mid E_i \neq E'_i\}$. That is, $R(a; E, E')$ is the minimum set of players that would be needed to deviate from E in order to achieve E' in state a.

Lemma 2 *Given m, β is an ISC disclosure rule if and only if $m_i(\beta(a)) \leq m_i(E)$, for each $a \in A$, $E \in \mathcal{D}$, and $i \notin R(a; \beta(a), E)$.*

The proof of Lemma 2 is intuitive and straightforward. Suppose for some equilibrium disclosure rule $\beta(a)$ and some state a, there is a set of documents E and a player i such that $i \notin R(a; \beta(a), E)$ and $m_i(\beta(a)) > m_i(E)$. This means that if the players in group $R(a; \beta(a), E)$ achieve disclosure of E by altering what documents they present, player i is strictly worse off. Since m is balanced, this implies that the other players $(-i)$ are collectively strictly better off when E is disclosed rather than $\beta(a)$. Further, since $R(a; \beta(a), E) \subset -i$ we know that E is a $-i$-deviation from $\beta(a)$ at a, which means $\beta(a)$ could not be ISC. In the other direction the condition of the lemma obviously implies that no coalition can strictly gain by deviating from β. *Q.E.D.*

Next we provide a necessary and sufficient condition for a transfer function g to be implemented by an ISC disclosure rule, where we focus on a given disclosure rule β. The analysis examines the different ways in which coalitions of players can deviate to induce disclosure of an arbitrary set of documents. For example, consider some set of documents E and states a and b. Disclosure rule β prescribes presentation of documents $\beta(a)$ in state a and $\beta(b)$ in state b. It may be that E is a J-deviation from $\beta(a)$ in state a, while E is a K-deviation from $\beta(b)$ in state b. If there is a function m with respect to which β is ISC, then it must be that m deters the J coalition from deviating to E in state a and also deters the K coalition from deviating to E in state b. We use Lemma 2 to translate this constraint into an inequality defined for E and β.

For any disclosure rule β, each $i \in N$, and $E \in \mathcal{D}$, let

$$B(i; \beta, E) \equiv \{a \in A \mid i \notin R(a; \beta(a), E)\}.$$

This is the set of states at which player i is *not* needed to induce disclosure of E by deviation from the prescription of β. Note that $E \not\subset D(a)$ implies $a \notin B(i; \beta, E)$. For any $a \in B(i; \beta, E)$, using Lemma 2, we know that player i's transfer when E is disclosed must be at least as great as his transfer when $\beta(a)$ is disclosed; further, the latter transfer is supposed to be $g_i(a)$, given the transfer function g. Examining all states in $B(i; \beta, E)$, we have the following lower bound on player i's transfer conditional on E.

$$z_i(E; \beta, g) \equiv \begin{cases} \max_{a \in B(i;\beta,E)} g_i(a) & \text{if } B(i; \beta, E) \neq \emptyset \\ -\infty & \text{if } B(i; \beta, E) = \emptyset \end{cases}.$$

Our characterization result is

Theorem 2 (Characterization) *Take as given a disclosure rule β and a transfer function g. There exists an externally-enforced contract m such that (i) g is implemented by β and m, and (ii) β is an ISC disclosure rule, if and only if*

$$\sum_{i \in N} z_i(E; \beta, g) \leq 0, \text{ for every } E \in \mathcal{D}.$$

To generate intuition, recall that $z_i(E; \beta, g)$ is a lower bound on the transfer to player i when E is the set of documents disclosed. Since the externally-enforced contract m must be balanced, it is possible to meet the lower bounds for all of the players only if the sum of z_i's is non-positive. Note that $z_i(E; \beta, g)$ equals negative infinity when player i is needed to deviate to E from every state. In this case, deviations to E can be easily deterred by punishing player i severely.

Proof of the Full Disclosure Result

We use the characterization theorem to prove Theorem 1. Suppose transfer function g is implemented by disclosure rule β. We define $\overline{\beta}$ to be the full disclosure rule; that is, $\overline{\beta}(a) = D(a)$ for all $a \in A$. Using Theorem 2, to ascertain whether g is implemented by $\overline{\beta}$ we need to check whether

$$\sum_{i \in N} z_i(E'; \overline{\beta}, g) \leq 0, \text{ for all } E' \in \mathcal{D}. \tag{1}$$

To do this, consider any set of disclosed documents E'. If $z_i(E'; \overline{\beta}, g) = -\infty$ for some i, then

$$\sum_{i \in N} z_i(E'; \overline{\beta}, g) \leq 0$$

is assured. Therefore, suppose $z_i(E'; \overline{\beta}, g) \neq -\infty$, for all $i \in N$. Then for each player i, let a^i be a state such that a^i maximizes $g_i(a)$ over all $a \in B(i, \overline{\beta}, E')$. Then we can define E so that $E_i = \beta_i(a^i)$ for all i. This implies that player i is not needed to deviate to E from $\beta(a^i)$ at a^i, unless we have the non-feasibility case where $E \not\subset D(a^i)$. However, we can rule out $E \not\subset D(a^i)$ by the definition of B and since $a^i \in B(i, \overline{\beta}, E')$. This is because $E' \subset D(a^i)$ for every $i \in N$ and, by the definition of E', it must be that $E'_i = D_i(a^i)$. Thus, $E_i \subset E'_i$ for all $i \in N$, which implies that $E \subset D(a^i)$ for all $i \in N$. We conclude that $a^i \in B(i, \beta, E)$. Thus

$$z_i(E; \beta, g) \geq z_i(E'; \overline{\beta}, g) \text{ for all } i \in N,$$

which implies that

$$\sum_{i \in N} z_i(E'; \overline{\beta}, g) \leq \sum_{i \in N} z_i(E; \beta, g) \leq 0,$$

for all $E' \in \mathcal{D}$. That g is implemented by ISC disclosure implies (1). *Q.E.D.*

Note how the Full Disclosure Result simplifies checking whether a given transfer function can be implemented by an ISC disclosure rule. One need only verify the condition of Theorem 2 for the full disclosure rule. That is, defining $\overline{\beta}(a) \equiv D(a)$ for each $a \in A$, one evaluates whether

$$\sum_{i \in N} z_i(E; \overline{\beta}, g) \leq 0, \text{ for every } E \in \mathcal{D}. \tag{2}$$

A transfer function g is implementable by ISC disclosure if and only if condition (2) holds.

3 Evidence and the Relevant Partition

The examples in the Introduction show that, regarding whether a given transfer function can be implemented, it matters how the players can distinguish between states by submitting documents. In this section we refine the characterization of implementable transfer functions on the basis of the manner in which players can differentiate between states. We begin by noting that meaningful differentiation between states a and b cannot occur unless players disclose in at least one state a document that *does not exist* in the other state.

Theorem 3 *Suppose g is implemented by m and ISC disclosure rule β. If $\beta(a), \beta(b) \subset D(a) \cap D(b)$, then $g(a) = g(b)$.*

The intuition behind Theorem 3 is that, since document sets $\beta(a)$ and $\beta(b)$ are feasible in both states a and b, it is easy for coalitions of players to pretend that one state occurred when in fact the other occurred. To prove the theorem, suppose any player i deviates to disclose $\beta_i(b)$ in state a, when the others are disclosing $\beta_{-i}(a)$. Since i must be deterred from doing this, we have $m_i(\beta(a)) \geq m_i(\beta_i(b) \cup \beta_{-i}(a))$. Next suppose the $-i$ coalition were to deviate to disclose $\beta_{-i}(a)$ in state, b when i discloses $\beta_i(b)$. Then it must be that $M_{-i}(\beta(b)) \geq M_{-i}(\beta_i(b) \cup \beta_{-i}(a))$, which implies $m_i(\beta(b)) \leq m_i(\beta_i(b) \cup \beta_{-i}(a))$. Combining the inequalities yields $m_i(\beta(a)) \geq m_i(\beta(b))$. The same argument can be repeated starting with i deviating in state b, yielding $m_i(\beta(b)) \geq m_i(\beta(a))$. Thus, $m_i(\beta(a)) = m_i(\beta(b))$ for all i. *Q.E.D.*

Theorems 1 and 3 suggest that "cheap talk" is of little use for contract enforcement when players can make side deals. To formalize this idea, let us use the term *cheap document* for a document that can be provided in any state. In the implementation literature, "message game phenomena" refers to a situation in which (i) players distinguish between various states by disclosing different cheap documents, and (ii) different transfers are implemented in various states.[23] Theorem 3 establishes that,

[23]The literature takes as fact that message game schemes solve contracting problems when there is complete information and more than two players.

under ISC, these cheap documents *cannot* serve to distinguish between states in any instrumental manner. Specifically, an ISC disclosure rule that differentiates between states only on the basis of cheap documents must imply the same transfer values over the states. We summarize this discussion with:

Corollary 1 *Message game phenomena using cheap documents cannot occur under ISC disclosure.*

Clearly, this result is due to the combination of the ISC condition and the zero-sum aspect of externally-enforced transfers (given renegotiation possibilities). The conclusion does not necessarily hold for settings in which some productive actions are taken after documented messages are sent, since then continuation payoffs may not be zero-sum. Yet our result does demonstrate that, in general, the opportunity for players to side contract can significantly constrain message game phenomena.

Partitions and Implementability

Next we focus on how documents can be used to distinguish between states. We show how different forms of evidence imply partitions of the state space and we examine how the partitions relate to the set of implementable transfer functions. Denote by F^D the set of transfer functions implemented by ISC disclosure rules; that is, $g \in F^D$ if and only if g satisfies condition (2). Consider an arbitrary partition P of the state space. We use the convention that, for any state a, $P(a)$ denotes the element of the partition containing a; that is, $b \in P(a)$ if and only if a and b are in the same element of the partition P. Let us denote by $G(P)$ the set of transfer functions that are measurable with respect to P.

When considering how evidence is provided by documents, it is important to differentiate between "positive" and "negative" evidence.[24] Suppose there is a document d that can be presented in state a (that is $d \in D(a)$) but it does not exist in state b ($d \notin D(b)$). Then disclosure of d is considered *positive evidence* of a since a player can present this document in state a but not in some other state. Further, *non*disclosure of d is considered *negative evidence* of b because a player cannot disclose d in state b but can disclose the document in some other state. For example, consider the canceled check in the debtor/creditor story. When the canceled check is presented, it is positive evidence that payment has been made. However, when the canceled check is not presented, this is negative evidence that payment has not been made.

We say that a player can *distinguish between states a and b* if she has positive evidence of at least one of these states to the exclusion of the other (which is thus identified by negative evidence). That is, player i can distinguish between a and b if and only if $D_i(a) \neq D_i(b)$. In the debtor/creditor example, the debtor can distinguish "paid" from "not paid" and vice versa. Let P^D denote the partition of A induced by

[24] Lipman and Seppi (1995) study similar notions. Saltzburg (1978) studies negative inferences resulting from the absence of evidence and contrasts this with positive evidence.

the notion of distinguishing between states. Formally P^D is defined so that $a \in P^D(b)$ if and only if $D(a) = D(b)$. Theorem 3 implies:

Corollary 2 *Every transfer function g that is implemented by an ISC disclosure rule has the property that $g(a) = g(b)$ for states a and b that cannot be distinguished. That is, $F^D \subset G(P^D)$.*

This result establishes that measurability with respect to P^D is necessary for implementability. However, it is generally not sufficient.

We say that player i can *positively distinguish a from b* if there exists a document $d \in D_i(a)$ such that $d \notin D_i(b)$. Whether action profiles can be positively distinguished is relevant in considering the transfer functions that can be implemented. As the next lemma shows, lack of positive evidence limits enforceable transfers, since players must be given the incentive to disclose documents.

Lemma 3 *Take a subset of the players $K \subset N$. Suppose the K players cannot positively distinguish b from a ($D_K(b) \subset D_K(a)$) and the $N \backslash K$ players cannot positively distinguish a from b ($D_{N\backslash K}(a) \subset D_{N\backslash K}(b)$). If g is implemented by an ISC disclosure rule then $\sum_{i\in K} g_i(a) \geq \sum_{i \in K} g_i(b)$.*

This lemma establishes that lack of positive evidence implies $F^D \neq G(P^D)$. Intuitively, if the K players as a whole did worse in state a then they would prefer to behave as though state b had occurred; further, the $N\backslash K$ group is unable to counter this by providing positive evidence of a.

We say that player i can *fully distinguish between a and b* if neither $D_i(a) \subset D_i(b)$ nor $D_i(b) \subset D_i(a)$. To fully distinguish between a and b, a player must have positive evidence of a, to the exclusion of b, as well as positive evidence of b, to the exclusion of a. For any player i, we define the partition $\overline{P}^{D_i}$ as the finest partition satisfying

$$D_i(a) \subset D_i(b) \text{ implies } a \in \overline{P}^{D_i}(b).$$

This partition reflects the notion of player i fully distinguishing between states, which the next result associates with a lower bound on the set of implementable transfer functions.

Theorem 4 *Fix some $i \in N$. If transfer function g satisfies $g(a) = g(b)$ for all states a and b between which player i cannot fully distinguish, then g is implemented by an ISC disclosure rule. That is, $G(\overline{P}^{D_i}) \subset F^D$.*

As Theorem 4 shows, if a player can fully distinguish between states then a contract exists that essentially forces her to disclose evidence isolating one state from the others. This is accomplished by severely punishing the player for not disclosing one of these critical documents.

Implications of a Legal Rule

We now consider an implication of our theory for the design of legal institutions, in particular with regard to rules of evidence and discovery. This is not meant to be an exhaustive study of legal institutions, but suggests that further research would be valuable. Recall that we have assumed players have common knowledge of documents. Suppose that rules of discovery allow a player to force another player to provide all documents of a particular type. We term this a setting of *enforced discovery requests*. Since our model assumes disjoint document spaces, we capture the setting of enforced discovery requests by assuming that, essentially, each player has *copies* of the documents in the possession of other players. Mathematically, this means that for $i, j \in N$ and each document $d \in D_i$, there exists a document $d' \in D_j$ such that $d \in D_i(a)$ if and only if $d' \in D_j(a)$. In words, if one player can positively distinguish a from b, then all players can do so.

In such an environment, F^D is not constrained by the failure of positive evidence.

Theorem 5 *In the setting of enforced discovery requests, the difference between positive and negative evidence is not critical, and* $G(P^D) = F^D$.

The intuition behind this result is that no constraint is created by an individual player wanting to suppress a document that lowers her transfer, since then another player will benefit from the document being disclosed and thus disclose it.

We acknowledge that in practice enforced discovery requests may be difficult to implement. This is due to the possibility of players destroying or suppressing evidence, and incentives in the discovery process. Brazil (1978) and Shapiro (1979) suggest that in practice the discovery process does not result in the intended open exchange of information since parties seek to suppress evidence. Cooter and Rubinfeld (1994), in a setting of costly evidence production, characterize an efficient level of discovery requests, while Cooter and Rubinfeld (1995) show that current federal law does not adequately prevent the problem of excessive discovery requests. Although Theorem 5 does not address these issues, it does lend support to those who suggest that more attention should be given to improving the workings of the discovery process. However, we maintain that enforced discovery requests are more plausible in today's society than during earlier periods of legal history. For example, in *State v. Simons* (1845), noted in the Introduction, the state's difficulty in keeping accurate, accessible records made the issue of whether Simons presented the license relevant.

4 Productive Interaction

Thus far, our analysis has been geared toward understanding behavior in periods 3 and 4 of the contractual relationship. In this section, we turn our attention to interaction in periods 1 and 2 and we take a broader perspective on the components of contract. Let us presume a simple specification of productive interaction. Suppose that in period 2 the players simultaneously and independently select actions. Player i's action space is denoted A_i; we define $A \equiv A_1 \times A_2 \times \cdots \times A_n$. In other words, in period 2 the agents play a one shot "production" game with action profiles A and payoffs given by u plus the continuation value from period 3. The resulting action profile $a = (a_1, a_2, \ldots, a_n)$ represents the state of the relationship at the end of period 2.

In period 1, before playing the production game, the players jointly agree to a contract. The contract has two components: an externally-enforced part m and a self-enforced part which describes the disclosure rule β and behavior in the production phase. As modeled in the preceding sections, we suppose that β is an ISC disclosure rule with respect to m. Thus, interaction in periods 3 and 4 can be summarized by the implied transfer function g. In period 1, the players indirectly select a transfer function g through their selection of m and their coordination on β. This justifies viewing interaction in period 2 as a game with action profiles A and payoffs given by $u+g$, where $g \in F^D$ is selected by the players in period 1. We analyze this contracting game by determining whether there is a transfer function in F^D that facilitates self-enforcement of a given a^* — in other words, that makes a^* a Nash equilibrium of period 2 interaction.[25]

Given $a^* \in A$, let $F(a^*)$ be the set of (balanced) transfer functions that induce a^* as a Nash equilibrium of the production phase. Mathematically, $g \in F(a^*)$ if and only if g is balanced and

$$u_i(a^*) + g_i(a^*) \geq u_i(a_i, a^*_{-i}) + g_i(a_i, a^*_{-i}),$$

for each player i and each $a_i \in A_i$. Clearly, there is a contract that yields action profile a^* if and only if $F(a^*) \cap F^D \neq \emptyset$. Also note

Lemma 4 *If $g \in F^D$ and, for some balanced vector $\phi \in \mathbf{R}^n$, g' is defined by $g'(a) = g(a) + \phi$ for each a, then $g' \in F^D$ as well.*

In words, adding a constant transfer between the players does not disrupt implementability, since incentives in period 3 are not altered. Thus, in period 1 players implement a transfer function associated with the solution of

$$\max_{a' \in A^*} \sum_{i \in N} u_i(a'), \text{ where } A^* \equiv \{a^* \in A \mid F(a^*) \cap F^D \neq \emptyset\}.$$

This maximization problem yields the highest joint value that can be attained in the contractual relationship, given the incentive constraints in the production and evidence disclosure phases. The joint value can be arbitrarily allocated between the players, given Lemma 4.

[25] Here we focus on enforcing pure action profiles. Appendix C presents analysis of mixed action profiles and moves of nature. One can easily extend the analysis to more complicated period 2 interaction, using the appropriate equilibrium concept. For example, production in period 2 may be modeled as a dynamic game. We simply require that all productive activity takes place in period 2, prior to evidence disclosure.

As explored in the preceding section, sometimes it is helpful to analyze the set of transfer functions that are measurable with respect to a particular partition of the state space. Given a partition P, we can provide a necessary and sufficient condition for a^* to be induced by a transfer function that is measurable with respect to P. The condition relates to the function

$$w_i(a, a^*) \equiv \begin{cases} -\infty & \text{if } Q^i(a^*) \cap P(a) = \emptyset \\ \max\{u_i(a_i', a_{-i}^*) \mid (a_i', a_{-i}^*) \in P(a)\} - u_i(a^*) & \text{if } Q^i(a^*) \cap P(a) \neq \emptyset \end{cases},$$

where $Q^i(a^*) \equiv \cup_{a_i \in A_i} P(a_i, a_{-i}^*)$. The function w_i represents the maximal increase in productive payoff that can be achieved by player i by unilaterally deviating from a^* in a way that yields an action profile in the partition element $P(a)$.

Theorem 6 *Consider any partition P of the action space. Then $F(a^*) \cap G(P) \neq \emptyset$ if and only if*

$$\sum_{i \in N} w_i(a, a^*) \leq 0, \text{ for all } a \in A. \tag{3}$$

Theorem 6 is closely related to Theorem 1 of Legros and Matthews (1993).[26] The intuition for necessity is simple. As transfers must be balanced due to renegotiation, summing each player's Nash equilibrium condition gives the result. In the sufficiency direction, we consider separately each element of the partition of A. If players generally do better by deviating to the element $P(a)$ than by being in $P(a^*)$ (i.e., $\sum_{i \in N} w_i(a, a^*) > 0$), then we cannot expect to induce play of a^* as a Nash equilibrium with any transfer function g as we require $\sum_{i \in N} g_i = 0$. That is, there does not exist a transfer function g that sufficiently punishes all players for unilateral deviation. However, when $\sum_{i \in N} w_i(a, a^*) \leq 0$ there does exist a transfer function that is measurable with respect to P and prevents deviation to the element of the partition $P(a)$.

5 An Honest Disclosure Result and The Revelation Principle

In this section, we digress from our program of representing verifiability purely in terms of the existence and non-existence of documents. We now determine the extent to which sets of documents can be interpreted as a player's declaration of the state. More precisely, for each player i, we look for a collection of evidence sets $\{E_i^a\}_{a \in A}$ such that (i) $E_i^a \subset D_i(a)$ for each $a \in A$, and (ii) $E_i^a \neq E_i^b$ for all $a, b \in A$ with $a \neq b$. We then can interpret the evidence set E_i^a as "player i says the state is a." We shall

[26]By specifying that an individual player appropriates the verifiable partnership output and confining attention to finite actions Legros and Matthew's result is seen as a special case of ours. See Appendix C for analysis of mixed productive actions in our model.

prove that implementability can be characterized in terms of such fixed definitions. The key is that each state must be associated with a distinct set of documents, so that the states can be individually identified. Cheap documents play a role in separating the states, in particular when the same set of documents is available in two or more states.

We study a more general model here than we did in previous sections, because the analysis does not require some of the restrictions needed for the Full Disclosure Result.[27] A more general setting is defined as follows. As with the basic model, the state is given by $a \in A$ and the feasible documents in state a are in the set $D(a)$. But now we allow the court's decision to be more general than a balanced transfer. Suppose the court makes a public decision s, taken to be an element of a set S. In state a, court decision s leads to the payoff vector $v(s|a)$. The function v can represent renegotiated outcomes, if one wishes to assume that the players renegotiate after disclosure of documents.

The agents' externally-enforced contract is a function $\mu : \mathcal{D} \to S$. Behavior in period 3 is given by β, as with the basic model. The function g we now call an *outcome function* and it maps states into public decisions; that is, $g : A \to S$. We say that g is *implemented* by contract μ and disclosure rule β if $g(a) = \mu(\beta(a))$ for every $a \in A$.

Our equilibrium and ISC definitions extend to this setting as follows. Given externally-enforced contract μ, disclosure rule β is an equilibrium disclosure rule if

$$v_i(\mu(\beta(a))|a) \geq v_i(\mu(E_i \cup \beta_{-i}(a))|a)$$

for each player i, every $a \in A$, and every $E_i \subset D_i(a)$. To define the ISC condition, let us assume that the players can arbitrarily transfer utility as a part of side contracting, so that we can represent conditions on side deals using utility sums. Let $V_J(s|a) \equiv \sum_{j \in J} v_j(s|a)$. Then a disclosure rule β is impervious to side contracting if

$$V_J(\mu(\beta(a))|a) \geq V_J(\mu(E)|a),$$

for each $a \in A$, each coalition $J \subset N$, and each $E \subset D(a)$ that is a J-deviation from $\beta(a)$ at a.

We begin with a simple example to show that the Full Disclosure Result fails to hold in the general setting. Suppose there are two players, two states a and b, two documents d and d', and two public decisions s^a and s^b. The existence of documents is given by: $D_1(a) = \emptyset$, $D_1(b) = \{d\}$, and $D_2(a) = D_2(b) = \{d'\}$. Note that d' is a cheap document that player 2 can present in both states. The payoffs are given by: $v(s^a|a) = (0, 0)$, $v(s^b|a) = (1, -1)$, $v(s^a|b) = (1, -1)$, and $v(s^b|b) = (0, 0)$. Note that the payoffs sum to zero, so there is no scope for renegotiation even with transferable utility.

[27] In particular, we can drop the balancedness and side-deal conditions, as well as the assumption that the players' preferences over the court's decision are independent of the state.

Suppose we wish to implement $g(a) = s^a$ and $g(b) = s^b$ in equilibrium, so that the payoff vector is $(0, 0)$ in both states. This can be done by specifying $\mu(\emptyset) = s^a$, $\mu(E) = s^b$ for all $E \neq \emptyset$, $\beta(a) = \emptyset$, and $\beta(b) = \{d, d'\}$. It is easy to check that neither player has an incentive to deviate from the prescribed disclosure rule in either state. In particular, note that player 1 cannot get outcome s^b in state a because document d is not available to him in this state.

In this example, there is no way of implementing $g(a) = s^a$ and $g(b) = s^b$ using the full disclosure rule. This is because, holding fixed player 2's disclosure of d' (in both states), player 1 controls the public decision — it is s^a if he does not produce d, whereas it is s^b if he does produce d. Player 1 would then not disclose d in state b. Clearly, the desired outcome function could also not be implemented if document d' were completely absent.

The example shows that, in some settings, a cheap document can be critical to implementation. Further, it demonstrates the importance of differentiating between states, even if only via cheap documents. We are then led to ask whether one can *fix* how the players differentiate between the states — in a way that is valid for the general implementation exercise.[28] In particular, we focus on disclosure rules in which the players fully disclose all non-cheap documents, while selectively disclosing the cheap documents so as to ensure separating the states.

Let D_i^C be the cheap documents and let D_i^N be the non-cheap documents for player i. Let D^C, D^N, $D^N(a)$, and $D_i^N(a)$ be defined accordingly. We say that the document set is *rich* if, for each player i, $|D_i^C| \geq |A|$ (there are at least as many cheap documents for player i as there are states). In addition, in this case, we shall assume that $|A|$ of the cheap documents are labelled with the state names — d_i^a for each state a — so that $\{d^a\}_{a \in A} \subset D_i^C$. The idea here is that cheap document d_i^a is player i voluntarily stating "the state is a."

Definition 5 *The* **honest disclosure rule** $\hat{\beta}$ *is defined by* $\hat{\beta}_i(a) = D_i^N(a) \cup \{d_i^a\}$ *for every* $a \in A$ *and each player* i*, meaning that each player discloses all of his non-cheap documents and the cheap document naming the state.*

Definition 6 *An outcome function* g *is called* **implemented by an equilibrium/ honest disclosure rule** *if and only if there is an externally-enforced contract* μ *such that (i) the honest disclosure rule* $\hat{\beta}$ *is an equilibrium and (ii)* g *is implemented by* μ *and* $\hat{\beta}$*.*

Where we use the ISC condition in place of equilibrium, we say "implemented by an ISC/honest disclosure rule."

[28] Though the discussion here is directed primarily towards the general implementation exercise, this approach suggests an interesting direction for future research concerning how legal institutions treat various types of evidence.

Theorem 7 (Honest Disclosure Result) *Suppose the document set is rich. If an outcome function g is implemented by an equilibrium disclosure rule, then g is implemented by an equilibrium/honest disclosure rule. Furthermore, if an outcome function g is implemented by an ISC disclosure rule, then g is implemented by an ISC/honest disclosure rule.*

The Honest Disclosure Result is proved by showing that evidence sets in the context of honest disclosure can be translated into evidence sets relative to a given equilibrium. We demonstrate that the ways in which the players can deviate from the honest disclosure rule, and the implied incentive conditions, are subsumed by those of the given equilibrium. Whereas this method is similar to that employed to prove the Full Disclosure Result, here cheap documents play two substantive roles: (i) ensuring that each state can be linked to a distinct set of documents and (ii) guaranteeing the existence of evidence sets that are associated with none of the states.

The Honest Disclosure Result ensures that there is a meaningful notion of "truthful reporting," independent of whatever outcome function the players wish to implement. For player i, the set of documents $D_i^N(a) \cup \{d_i^a\}$ is interpreted as "player i says the state is a." In legal terms, the set $D_i^N(a) \cup \{d_i^a\}$ is the required evidence of state a from player i. Importantly, this standard of evidence can be *fixed* by the legal system, without artificially constraining the players' contractual alternatives. Further, in line with the Full Disclosure Result, honest disclosure incorporates full disclosure of all non-cheap documents.

In technical terms, the Honest Disclosure Result is an extended revelation principle. It not only justifies focusing attention on truthful reporting (the standard revelation principle), but it *defines* what truthful reporting is in terms of documents submitted as evidence. To further establish the link to a more abstract version of the revelation principle, let us consider translating a setting with arbitrary documents into a "direct-report" setting in which the documents are just the names of the states (and no more), meaning the players simply declare the state. Our question is: Can we perform such a translation, preserving the implementability conditions?

Definition 7 *The contracting environment is said to be a* **direct-report** *setting if $D_i \equiv \{(a, i) \mid a \in A\}$ for each player i. In this setting,* **truthful disclosure** *means $\beta_i(a) = \{(a, i)\}$ for each state a and each player i.*

We use the terms "equilibrium/truthful" and "ISC/truthful" when referring to truthful-reporting disclosure rules that are equilibria or ISC.

Theorem 8 *Consider any n-player contracting environment defined by a set of states A, a set of public alternatives S, a payoff function v, and state contingent document sets $D_1(\cdot), D_2(\cdot), \ldots, D_n(\cdot)$. Suppose the document set is rich. Define a direct-report setting by A, S, v, and document sets $D_i' \equiv \{(a, i) \mid a \in A\}$ for each player i, with*

$$D_i'(a) \equiv \{(b, i) \mid b \in A \text{ such that } D_i(b) \subset D_i(a)\}.$$

Outcome function g is implemented by an equilibrium/honest disclosure rule in the given setting if and only if it is implemented by an equilibrium/truthful disclosure rule in the direct-report setting. Furthermore, g is implemented by an ISC/honest disclosure rule in the given setting if and only if it is implemented by an ISC/truthful disclosure rule in the direct-report setting.

The key to the proof of Theorem 8 is how document sets are translated between the two settings for purposes of defining externally-enforced contracts. For each player, we think of the "honest report" sets as equivalent to the "truthful report" sets. Other, off-equilibrium, sets in either setting are translated into the empty set in the other setting.

Theorem 8 establishes the formal relation between our Honest Disclosure Result and the revelation principle of Green and Laffont (1986). Indeed, the theorem provides an evidence-based foundation for Green and Laffont's approach and generalizes their analysis. Green and Laffont study a model that is equivalent to our direct-report setting, with two alterations. First, they study a principal-agent setting, which corresponds to $n = 1$ here. Second, Green and Laffont assume that the agent can disclose just *one* document.[29]

Presumably, Green and Laffont constrain the agent to one document because of their interest in proving a revelation principle, whereby the agent discloses exactly one document in equilibrium. However, then their message restrictions and key "nested range condition" are difficult to interpret. Our theory shows how the direct report of the state in an abstract model can be interpreted as a set of real documented evidence. Further, it is easy to verify that, in translating a given model of real documents into a direct-report model, Green and Laffont's nested range condition holds by construction in the direct-report setting. In other words, Green and Laffont's approach is valid as an abstract version of a model of real documents, but violations of their nested range condition make little intuitive sense.[30]

Deneckere and Severinov (2001) study the revelation principle, and critique Green and Laffont's (1986) model, from a perspective that is very similar to ours in this section. Deneckere and Severinov study a principal-agent setting along the lines of Green and Laffont (1986), but they allow the agent to send multiple messages. Independently from us, they proved a revelation principle based on the idea that the agent sends all possible messages (starting with the truthful report) in each state. Technically, their Theorem 1 is a special case of our Honest Disclosure Result. The distinguishing characteristic of our approach is that we start from a foundation of modeling real documents, so our comparison with Green and Laffont (1986) is in

[29] Green and Laffont (1986) assume that the agent reports the state. Thus, the agent can say "the state is a" or "the state is b," but he *cannot* make both statements (saying "the state is a or b").

[30] There may be a real setting in which the one-document assumption is valid — if disclosing one document destroys the ability of an agent to disclose another — but this does not seem to be common. Technically, such a constraint can be studied in terms of general disclosure costs (Bull 2001).

terms of how real evidence sets can be represented by abstract direct reports. Our Theorem 8, along with the Honest Disclosure Result, gives the formal connection.[31]

6 Conclusion

We have presented a model of contracting in which external enforcement depends on the players' disclosure of documented evidence. The state of the contractual relationship determines whether or not any given document is available to be disclosed by a player. Our Full Disclosure Result provides a foundation for the concept of verifiability that is solely in terms of the system of documents. The Result justifies focusing on behavioral rules in which all documents are disclosed in every state. The Result demonstrates how side contracting can disrupt message game phenomena and it reveals how external-enforcement is constrained by the manner in which states can be distinguished from one another. To obtain the Full Disclosure Result, we assumed that the court compels balanced transfers, that the players' preferences over the court's action are state-independent, and, for settings of more than two players, that players can engage in side deals over the documents they disclose.

Our Honest Disclosure Result puts the revelation principle in the context of real documents. It also reveals the existence of a standard of evidence that transcends the players' particular goal for implementation. Further, it provides the basis for comparing our document-based approach with the more abstract approaches taken by other researchers. Our structured, game-theoretic model of contract and enforcement (whereby evidence production and enforcement are made explicit) has an advantage over mechanism design models because it can more easily incorporate technological and institutional features. Directly analyzing these features produces new insights and reveals new avenues for productive research. Also, as Watson (2001) proves in the context of renegotiation and the technology of trade, some abstract mechanism design models are misguided.

Our approach offers an alternative viewpoint than is generally taken in the law and the law and economics literatures — allowing us to address issues, such as positive and negative evidence, and details of the evidence environment that cannot be studied in the more abstract and restrictive models that are typical of the literature. On the practical front, we are motivated by the view that contract enforcement hinges on the extent to which documents exist in some states and not in others; our model abstracts from the intermediate costs of producing evidence.[32] We see several

[31] Squintani (2001) studies contract enforcement in settings where verifiability may be represented by a non-partitional information correspondence. These information correspondences arise because of implicit assumptions about how agents present evidence and how the external enforcement authority responds. As our modeling exercise shows, however, one can develop an understanding of implementability and technological constraints only by studying evidence and external enforcement directly.

[32] In reality, litigants know that witnesses are subject to cross-examination, and this may lead them

promising directions for related research: (a) examining litigation procedures that involves sequential document disclosure, (b) studying settings in which the players have private information about available documents, and (c) adding more institutional details and comparing different litigation systems (for example, default rules, burden of proof considerations, and evidence admissibility rules).

A Proofs Not in the Text

Proof of Theorem 2

(Necessity) Suppose β is an *ISC* disclosure rule, but

$$\sum_{i \in N} z_i(E; \beta, g) > 0, \text{ for some } E \in \mathcal{D}.$$

ISC disclosure requires $m_i(\beta(a)) \leq m_i(E)$, for every $i \notin R(a; \beta(a), E)$, for any $E \in \mathcal{D}$, for all $a \in A$. But if $\sum_{i \in N} z_i(E; \beta, g) > 0$, for some $E \in \mathcal{D}$, then (by $\sum_{i \in N} m_i = 0$) for some a, β is not an ISC disclosure.

(Sufficiency) Take any $E \in \mathcal{D}$. $\sum_{i \in N} z_i(E; \beta, g) \leq 0$, for any $E \in \mathcal{D}$ implies the existence of an enforced contract $m : \mathcal{D} \to \mathbf{R}_0^n$ such that $m_i(\beta(a)) \leq m_i(E)$, for every $i \notin R(a; \beta(a), E)$, for any $E \in \mathcal{D}$, for all $a \in A$. Since it must be that $\sum_{i \in N} m_i(E) = 0$ for all $E \in \mathcal{D}$ implies there exists an enforced contract m such that $M_J(\beta(a)) \geq M_J(E)$, for each $J \subset N$, for any $E \in \mathcal{D}$, for every $a \in A$, where E is any J deviation from $\beta(a)$. *Q.E.D.*

Proof of Lemma 3

By the Full Disclosure Result we need to only consider disclosure rules such that $\overline{\beta}(a) = D(a)$ for all $a \in A$. That g is implemented implies there exists an m such that $M_K(\overline{\beta}_K(b) \cup \overline{\beta}_{N \setminus K}(a)) \leq M_K(\overline{\beta}(a))$, and that $M_{N \setminus K}(\overline{\beta}_K(b) \cup \overline{\beta}_{N \setminus K}(a)) \leq M_{N \setminus K}(\overline{\beta}(b))$, which can be written alternatively as $M_K(\beta_K(b) \cup \overline{\beta}_{N \setminus K}(a)) \geq M_K(\overline{\beta}(b))$. Combining these two expressions, we get $M_K(\overline{\beta}(a)) \geq M_K(\overline{\beta}(b))$, or equivalently $\sum_{i \in K} g_i(a) \geq \sum_{i \in K} g_i(b)$. *Q.E.D.*

Proof of Theorem 4

By the Full Disclosure Result we need to only consider disclosure rules such that $\overline{\beta}(a) = D(a)$ for all $a \in A$. Consider a contract m as follows. If $E_i \in \{E_i \mid E_i = D_i(a) \text{ for some } a \in A\}$, then $m_i(E_i) = g(a)$ for that a. If $E_i \notin \{E_i \mid E_i = D_i(a) \text{ for some } a \in A\}$, then $m_i(E_i) = -x$, where $x < \min_{a \in A} g_i(a)$. As $g(a) = g(b)$ for those states a, b between which i cannot fully distinguish, and $D_i(a') \not\subset D_i(b')$ and $D_i(b') \not\subset D_i(a')$ for any other states a' and b', i will always fully disclose. *Q.E.D.*

to call only those witnesses whose testimony will stand up to cross-examination. This is discussed in Posner (1999).

Proof of Theorem 5

By the Full Disclosure Result, we need only consider disclosure rules which specify $\overline{\beta}(a) = D(a)$ for all $a \in A$. Take any g and $E \in \mathcal{D}$. We consider first the case where E is such that $K(a) \subset N$ $D_{K(a)} = E_{K(a)}$ and $E_{N \setminus K(a)} \subset D_{N \setminus K(a)}(a)$ (strictly) for some $a \in A$. We proceed by finding all a's such that for some $K(a) \subset N$ $D_{K(a)} = E_{K(a)}$ and $E_{N \setminus K(a)} \subset D_{N \setminus K(a)}(a)$ (strictly). Let $\tilde{A}$ denote the set of such a's. It can be shown that $\cap_{a \in \tilde{A}} N \setminus K(a) \neq \emptyset$. We show this and then continue the proof. Suppose that for some E, $\cap_{a \in \tilde{A}} N \setminus K(a) = \emptyset$. This implies there exists $i, j \in N$ and $a, b \in \tilde{A}$ such that $E = (\overline{\beta}_i(a) \cup \overline{\beta}_j(b) \cup E_{-i,j})$. So it must be that at a, j is required to deviate, but at b, i is required to deviate. In other words, $\overline{\beta}_i(b) \subset \overline{\beta}_i(a)$ and $\overline{\beta}_j(a) \subset \overline{\beta}_j(b)$. But this contradicts the assumption of full discovery requests. Thus, $\cap_{a \in \tilde{A}} N \setminus K(a) \neq \emptyset$, and the court can always tell that at least one player has deviated at any such E. This implies that for any such E, it is always the case that $\sum_{i \in N} z_i(E; \beta, g) \leq 0$.

We must also rule out all players deviating from $\overline{\beta}(a)$. If all players deviate to some $E' \neq \overline{\beta}(b)$ for any b, we can easily construct an m such that some player i has incentive to disclose $\overline{\beta}_i(a)$. However, suppose that all players deviate from $\overline{\beta}(a)$ to $\overline{\beta}(b)$. If this is possible, it must be that $\overline{\beta}(b) \subset \overline{\beta}(a)$. However, unless $g(a) = g(b)$, $\sum_{i \in N} g_i = 0$ implies there exists some player i such that $g_i(a) > g_i(b)$. Thus player i will gain by disclosing $D_i(a)$. *Q.E.D.*

Proof of Theorem 6

(Necessity) If player i deviates to $P(a)$, he gains $w_i(a, a^*) + g_i(a) - g_i(a^*)$. That a^* is a Nash equilibrium in the induced game implies that $w_i(a, a^*) + g_i(a) - g_i(a^*) \leq 0$, for all $i \in N$, for all $a \in A$. As transfers are balanced, this implies

$$\sum_{i \in N} w_i(a, a^*) \leq 0, \text{for all } a \in A.$$

(Sufficiency) Suppose (3) holds for some a^*. We need to construct a transfer function such that a^* is a Nash equilibrium. Take $a \notin Q^i(a^*)$ for some i. Specify

$$g_j(a) = -w_j(a, a^*), \text{ for all } a \in P(a), j \text{ such that } a \in Q^j(a^*).$$

Let

$$g_k(a) = \frac{1}{(n-K)} \left[\sum w_j(a, a^*) \mid j \text{ such that } a \in Q^j(a^*) \right],$$

for each k such that $a \notin Q^k(a^*)$, where K is the number with $a \in Q^j(a^*)$. Next take $a \in \cap_{i \in N} Q^i(a^*)$. For players $j = 2, ..., n$, define $g_j(a) = -w_j(a, a^*)$, for all $a \in P(a)$. For player 1, let $g_1(a) = \sum_{j \in N \setminus 1} w_j(a, a^*)$, for all $a \in P(a)$. Note that by construction, players $2, ..., n$ have no incentive to deviate from a^*. If player 1 deviates to a, he gains

$$w_1(a, a^*) + g_1(a) = \sum_{i \in N} w_i(a, a^*).$$

However, by assumption

$$\sum_{i \in N} w_i(a, a^*) \leq 0, \text{ for all } a \in A. \text{ } Q.E.D.$$

Proof of Theorem 7

Take a contract μ and a disclosure rule β that is either an equilibrium or ISC, such that μ and β implement an outcome function g. We need to show that g can be implemented by some other contract μ' and the honest disclosure rule $\hat{\beta}$, such that $\hat{\beta}$ is either an equilibrium or ISC, as appropriate.

For each player i, there must be a set of cheap documents $\underline{E}_i \subset D_i^C$ such that $\underline{E}_i \neq \beta_i(a)$ for all $a \in A$. That is, in no state does player i disclose exactly $\underline{E}_i$. This is true because there are more subsets of cheap documents for player i than there are states. For each possible evidence set E_i for player i, define

$$\gamma_i(E_i) \equiv \begin{cases} \beta_i(a) & \text{if } E_i = D_i^N(a) \cup \{d_i^a\} \text{ for some } a \in A \\ \underline{E}_i & \text{otherwise} \end{cases} .$$

Let $\gamma(E) \equiv \gamma_1(E_1) \times \gamma_2(E_2) \times \cdots \gamma_n(E_n)$.

Define $\mu' : \mathcal{D} \rightarrow S$ so that $\mu'(E) \equiv \mu(\gamma(E))$, for all $E \in \mathcal{D}$. Note that $E \subset D(a)$ implies that $\gamma(E) \subset D(a)$. If β is an equilibrium with respect to μ, then it must be that

$$v_i(\mu(\beta(a))|a) \geq v_i(\mu(\underline{E}_i \cup \beta_{-i}(a))|a)$$

for each player i, and every $a \in A$. This implies that

$$v_i(\mu'(\hat{\beta}(a))|a) \geq v_i(\mu'(E_i \cup \beta_{-i}(a))|a)$$

for each player i, every $a \in A$, and every $E_i \subset D_i(a)$. If β is ISC with respect to μ, then it must be that

$$V_J(\mu(\beta(a))|a) \geq V_J(\mu(\underline{E}_J \cup \beta_{-J}(a))|a)$$

for each $a \in A$, each coalition $J \subset N$, and each $\underline{E}_J \subset D_J(a)$. This implies that

$$V_J(\mu'(\hat{\beta}(a))|a) \geq V_J(\mu'(E)|a)$$

for each $a \in A$, each coalition $J \subset N$, and each $E \subset D(a)$ that is a J-deviation from $\beta(a)$ at a. Thus, if μ and β implement g, then μ' and $\hat{\beta}$ also implement g. *Q.E.D.*

Proof of Theorem 8

Suppose g is implemented by an equilibrium/honest disclosure rule in the given setting and let μ be the externally-enforced contract. For the direct-report setting, we define a contract μ' as follows. Define $\hat{E}_i(\{(a, i)\}) \equiv D_i^N(a) \cup \{d_i^a\}$, for each player i

and every state a. For every other document set E_i' for player i, define $\hat{E}_i(E_i') \equiv \emptyset$. Then define

$$\mu'(E') \equiv \mu(\hat{E}_1(\{E_1'\}) \cup \hat{E}_2(\{E_2'\}) \cup \cdots \cup \hat{E}_n(\{E_n'\}),$$

for every evidence set $E' \in \mathcal{D}'$ (in the direct-report setting). It is not difficult to verify that truthful disclosure is an equilibrium in the direct-report setting, and, by definition of μ', g is implemented by the truthful disclosure rule and μ'. The argument extends to ISC implementation. The reverse implication can be proved similarly, with off-equilibrium document sets in the given setting translated into the empty set in the direct-report setting. *Q.E.D.*

B Analysis of Random Disclosure Rules

In this section we show that our main results are not changed when mixed disclosure rules are allowed. We denote a random disclosure rule by $\sigma : A \to \Delta^u\mathcal{D}$, where Δ^u denotes that the randomization is uncorrelated across players. We use $\mathcal{E}\left[m_i(E) \mid \sigma(a)\right]$ to denote the expected court-enforced transfer for player i in state a when σ is played. Let θ denote a deviation from $\sigma(a)$. In this setting, the ISC condition is as follows.

$$\mathcal{E}\left[M_J(E) \mid \sigma(a)\right] \geq \mathcal{E}\left[M_J(E) \mid \theta\right],$$

where θ is a J deviation from $\sigma(a)$ and θ_J is degenerate. We are justified in considering only degenerate deviations because if J cannot gain from a degenerate deviation, then J cannot gain by using a mixture.

We define $\hat{R}(a; \sigma(a), \theta)$ to be the minimum set of players that would be needed to deviate from $\sigma(a)$ in order to achieve θ in state a. The analysis proceeds as in the text and we have:

Lemma 2′ *σ is ISC if and only if $\mathcal{E}\left[m_i(E) \mid \sigma(a)\right] \leq \mathcal{E}\left[m_i(E) \mid \theta\right]$, for all $i \notin \hat{R}(a; \sigma(a), \theta)$, θ, and a.*

The proof is straightforward. Suppose the lemma does not hold. Then it must be that $\mathcal{E}\left[M_{-i}(E) \mid \sigma(a)\right] < \mathcal{E}\left[M_{-i}(E) \mid \theta\right]$. However, note that $\hat{R}(a; \sigma(a), \theta) \subset -i$. So $-i$ players can reach θ. Further, there must be a θ' such that θ'_{-i} is degenerate and $\theta_i' = \theta_i$, with $\mathcal{E}\left[M_{-i}(E) \mid \sigma(a)\right] < \mathcal{E}\left[M_{-i}(E) \mid \theta'\right]$. But this violates ISC. *Q.E.D.*

To generalize the Full Disclosure Result to mixed disclosure rules we first define the following notation. For any disclosure rule σ, each $i \in N$, and θ, let

$$\hat{B}(i, \sigma, \theta) \equiv \{a \in A \mid i \notin \hat{R}(a; \sigma(a), \theta)\}.$$

We define

$$\hat{z}_i(\theta; \sigma, g) \equiv \begin{cases} \max_{a \in \hat{B}(i,\sigma,\theta)} g_i(a) & \text{if } \hat{B}(i, \sigma, \theta) \neq \emptyset \\ -\infty & \text{if } \hat{B}(i, \sigma, \theta) = \emptyset \end{cases}.$$

We now show that if a transfer function g is implemented by a mixed ISC disclosure rule, then g is implemented by an ISC/full disclosure rule. Take any ISC rule σ, with respect to m, that implements g. Let $\overline{\beta}(a) \equiv D(a)$ for all $a \in A$. Consider any E'. As in the proof for the pure disclosure case, let a^i be a state that maximizes $g_i(a)$ over all $a \in B(i, \overline{\beta}, E')$. Define θ such that $\theta_i \equiv \sigma_i(a^i)$ for all $i \in N$. We have $a^i \in \hat{B}(i, \sigma, \theta)$. This implies that $\hat{z}_i(\theta; \sigma, g) \geq z_i(E'; \overline{\beta}, g)$ for all $i \in N$.

C Analysis of Random Productive Actions

In this section we study mixed strategies in productive interaction. Let A_0 denote the action space of nature, let $A \equiv A_0 \times A_1 \times \cdots A_n$, and consider mixtures $\Delta^u A$, where Δ^u denotes that randomization is uncorrelated across players. We take nature's mixed action α_0 as given. We say that $\alpha^* \in \Delta^u A$ is enforced when there exists a transfer function g such that α^* is a Nash equilibrium of the induced game as in Section 4. Nash equilibrium requires

$$\sum_{a_{-i} \in A_{-i}} [u_i(a_i', a_{-i}) + g_i(a_i', a_{-i})] \left[\Pi_{k \neq i} \alpha_k^*(a_k)\right] \leq \sum_{a \in A} [u_i(a) + g_i(a)] \left[\Pi_{j \in N} \alpha_j^*(a_j)\right].$$

There is no simple extension of Theorem 6 for this setting because the analysis of equilibrium involves a system of inequalities corresponding to the different elements of the partition P that are encountered with positive probability when any given action profile is played. However, we can provide a result indicating conditions under which the analogy of $w_i = -\infty$ holds, in which case some productive actions can be disregarded when checking whether a given profile can be enforced. Given an action profile α and an element p of partition P, we say that α *leads to p with positive probability* if there is a pure action profile a such that $a \in p$ and a is in the support of α.

Lemma 5 *Consider whether α^* is a Nash equilibrium with respect to some transfer function that is measurable with respect to P. Suppose that there is an element p of P and a player j such that, for each $a_j' \in A_j$, (a_j', α_{-j}^*) does not reach p with positive probability. Then if player i's action a_i is such that (a_i, α_{-i}^*) reaches p with positive probability then a_i can be disregarded when checking whether α^* can be induced.*

This lemma is easily proved. Only by deviation of a player other than player j can p be reached with positive probability. Thus, for each $a \in p$, $g_i(a)$ we specify an arbitrarily large transfer from each of the players in $-j$ to player j (the same thing for all states in p), which ensures that no player has the incentive to play an action that leads to p with positive probability. *Q.E.D.*

Lemma 5 generalizes the main component of Legros and Matthews' (1993) Theorem 2.

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