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USC Center for Law, Economics & Organization
Research Paper No. C02-8

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University of Southern California Law School
Los Angeles, CA 90089-0071

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First Version: December 3, 2000
This Version: September 19, 2001

Abstract

This paper focuses on implementation issues in environments where it may be costly for the players to send certain messages. We develop an approach allowing to characterize the set of implementable outcomes in such environments, and then apply it to derive optimal mechanisms. The key elements of our approach are the absence of any restrictions on the communication structure in a mechanism and the ability of the principal to screen the agents not only on the basis of their preferences over the outcomes, but also on the basis of their communication abilities. A number of interesting implications for the monopoly regulation, signaling and screening is derived. In particular, we show that a monopoly may not want to exclude low-valuation consumers if some consumers in the population are not able to misrepresent their valuations, and why the employers may prefer to screen applicants via multiple rounds of interviews rather than via menus of contracts. Our findings also provide a justification for privacy laws.
JEL Nos: C72, D82
Keywords: mechanism design, implementation, communication, signaling, screening.

1 Introduction.

This paper focuses on mechanism design and implementation issues in environments where agents participating in a mechanism sometimes incur communication costs when they send messages to the principal. A large body of literature on mechanism design assumes that communication in a mechanism is costless, so that an agent can send all possible messages and, in particular, can misrepresent her private information in any way she likes. At the same time, the view that sending certain messages may be costly is also accepted and has been explored in a number of contributions. Such view is well grounded in common intuition that an agent's ability to manipulate information may be limited, even if one abstracts from endogenous costs that a principal can impose on the agent after receiving a message from the latter. Several reasons for this have been discussed in the literature.

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First, it may be costly for an individual to lie for psychological reasons. (S)he may experience stress or discomfort when doing so, and a incur a disutility from providing false information. For example, Erard and Feinstein (1994) argue that "some taxpayers appear to be inherently honest, willing to bear their full tax burden even when faced with financial incentives to underreport their income. Evidence for such inherently honest taxpayers derives not just from casual introspection; it is also supported by econometric evidence and survey findings..." A similar view is advocated by Alger and Ma (1998). They consider a health care environment, where some physicians with stronger ethical views are not able to exaggerate the medical problems of a patient when requesting coverage from an HMO, while other physicians are willing to do so. Clearly, an honest person can also be viewed as someone whose cost of misrepresenting her private information is very high.

Second, in some contexts a person may be able to make credible claims i.e., claims the validity of which can be verified. For example, an agent may have some hard evidence to supplement her message. Existence of such verifiable claims makes imitation harder. Indeed, failure of an agent to produce claims containing evidence which is known to be available in a certain state of the world should convince the principal that this state of the world has not occurred. The 'credible claims' model has been studied by Lippman and Seppi (1995) who show that very little verifiability is required for the principal to be able to elicit the truth from a group of agents with conflicting interests. The model of evidence disclosure in court studied by Bull and Watson (2001) can also be seen as a process where 'credible claims' are submitted. Green and Laffont (1986) consider a situation where an agent can claim to be of a certain type only if that type belongs to a subset of the type space which depends on the agent's true type. This situation also admits interpretation in the spirit of the 'credible claims' model. Che and Gale (2000) study an optimal mechanism for selling a good to a buyer who may be budget constrained. In this case the ability of a buyer to misrepresent her willingness to pay is limited if the seller can ask her to post a bond and thus credibly disclose some information about her budget.

Finally, misrepresenting the truth may require costly physical actions. Lacker and Weinberg (1989) argue that 'there are many instances in which lying about the state of nature requires more than simply sending a false signal regarding one's private information. Often, costly actions must be taken to lend credence to the signals being sent'. For example, a sharecropper who underreports the crop, may have to hide part of the output. It is reasonable to believe that there is a cost to hiding and storing the crop or borrowing it from a third party. A similar situation is explored by Maggi and Rodriguez-Clare (1995).

In environments with communication costs, communication becomes an implementation tool. It allows to relax a number of incentive constraints that otherwise would have to be imposed on an allocation. To see this, consider the following example. Suppose that it is very (infinitely) costly for an agent of the first type to send some set of messages A, while an agent of the second type can send this set of messages at zero cost. If sending the set of messages A in the mechanism is necessary to obtain the allocation that is designed for the agent of the second type, then the allocation designed for the first type does not have to be better for her than the allocation designed for the second type.

This argument, which can be attributed to Green and Laffont (1986)), explains why the presence of communication costs has an effect on the set of implementable outcomes. Alger and Ma (1998) and Green and Laffont (1986) show that mechanisms in which an agent does not always report her information truthfully allow to implement allocation profiles that are not implementable via any mechanism with truthful reporting. Similarly, Maggi and Rodriguez-Clare (1995) demonstrate that it may be optimal for the mechanism designer to induce the agent to incur some falsification costs.

However, the existing contributions focus on specific situations, while the issues of characterizing the set of implementable outcomes and identifying the optimal mechanisms in environments with communication costs have not been sufficiently explored. So the first goal of this paper is to suggest a general approach to implementation in such environments, and use it to characterize the set of implementable outcomes. We then apply our results to study the effects of costly communication and construct optimal mechanisms in a number of situations. We are also able to integrate the existing models within a common framework and to show whether the mechanisms that have been exhibited in the literature are, indeed, optimal.

We consider two classes of environments. The first class includes environments where the set of costless messages typically includes more than just truthful announcement of an agent's type (state of the world). The agent can make a number of statements costlessly, including such statements that misrepresent her type. For tractability, we make an important simplification in this case that the cost structure is binary, with the cost of sending any particular message being either zero or infinity. The environments studied by Alger and Ma (1998), Green and Laffont (1986), Lipman and Seppi (1995), and Erard and Feinstein (1994) belong to this class.

The second class includes environments where the only statements that an agent can make costlessly are truthful type announcements, i.e. truthful statements about the state of the world. We allow for non-binary cost structure in this case: the cost of sending a non-truthful message is finite and may be large or small depending on the message itself and the true state of the world. Lacker and Weinberg (1989) and Maggi and Rodriguez-Clare (1995) study situations of this kind.

The key element of our approach is the absence of any restrictions on the communication structure in a mechanism. In particular, we assume that an agent can be asked to send any number of messages or announcements. Furthermore, we also assume that the cost which an agent incurs when she sends a particular message does not depend on the nature or the number of other messages that she also sends. So an agent incurs a separate cost associated with each message or announcement.

In contrast, most existing contributions adopt an assumption (often, implicitly) that an agent can send only one message, or make only one announcement/claim.[1] Although the revelation principle implies that this assumption is without loss of generality in environments with costless communication, it is not so when communication is costly. Limits on the number of messages or announcements affect the set of implementable outcomes.

In fact, we show that in environments with binary communication costs the set of implementable outcomes is maximal in a mechanism where an agent sends all messages that are feasible for her (i.e. that can be sent at zero cost), and some allocation profiles are imple-

[1]This applies to Green and Laffont (1986), Alger and Ma (1998) and Erard and Feinstein (1994). In Maggi and Rodriguez-Clare (1995) an agent can send only one costly signal, which is justifiable given their interpretation of the model. Agents can send several messages in Lipman and Seppi (1995) and Bull and Watson (2001), but these papers focus on a different set of issues.

mentable only via such mechanism. This result can be seen as an extension of the revelation principle to the costly communication case. In general, the set of messages that an agent can send costlessly is her private information and, therefore, can be regarded as part of her type. By reporting this set an agent essentially demonstrates her full communication abilities. This point strengthens the analogy between our result and the revelation principle[2].

To understand this result, note that because communication costs limit an agent'a ability to mimic messages that can be sent by agents of other types, some standard incentive constraints need not be imposed on an implementable allocation profile. A mechanism in which an agent sends all feasible messages and makes all claims that she is able to make fully exploits the communication cost structure and allows to eliminate the maximal possible number of incentive constraints which otherwise have to be imposed on the allocation profile.

Then, an implementable allocation profile must satisfy all standard incentive constraints that cannot be taken care of in the communication stage and, additionally, new incentive constraints that prevent agents with better communication abilities from imitating agents whose communication abilities are more modest.

Imposing these incentive constraints we can characterize the set of implementable allocation profiles. This set is larger than the set of implementable outcomes in the standard case (with no communication costs) or when an agent makes only one announcement. To demonstrate this, we reconsider the environments studied by Green and Laffont (1986) and Alger and Ma (1998). We show that mechanisms in which an agent is asked to make several announcements about her type allow the principal to screen the agents better than the mechanisms exhibited by these authors. We also extend the basic Alger and Ma (1998) model -which we recast as a nonlinear pricing model with consumers who have privately known valuations and may be either 'honest' (unable to misreport their valuations) or 'strategic' (can misrepresent them in any way they like) - to the case with a continuous distribution of types, and characterize the optimal mechanism in this case. This problem shares many features of the multidimensional mechanism design problem studied by Armstrong (1996) and Choné and Rochet (1998). However, one of the central results obtained in the multidimensional screening context -that a subset of types of a positive measure is rationed i.e., get a zero quantity allocation -does not hold in our case. We show that all 'honest' and 'strategic' agents with positive valuations are assigned a positive quantity allocation in the optimal mechanism.

Mechanisms in which each agent sends all messages that she is able to send provide an analytical tool allowing to characterize the set of implementable outcomes. A separate issue is whether some of these outcomes, and especially the desirable ones, can be implemented via simpler mechanisms. The answer to this question, of course, depends on the particular context under consideration. In certain classes of environments it is possible to reduce the number of messages that an agent has to send.

In particular, in the above mentioned non-linear pricing model with 'honest' and 'strategic' consumers, we characterize the set of implementable allocation profiles by using our general result which relies on mechanisms where agents are asked to make all announcements that are feasible for them. Then we show that any implementable allocation profile can also be

[2]Another way to establish a link between our result and the revelation principle is to consider additional announcements to be part of the allocation space. Note that there are no a priori restrictions on the set of allocations in a direct mechanism.

implemented via a so-called 'password' mechanism. In the 'password' mechanism the agent makes only one announcement about her valuation, after which she is asked to choose from a menu. The offered menu itself depends on the reported valuation. Thus, this mechanism combines the approach based on the revelation principle (reporting) with the approach based on the taxation principle (choosing from a menu)[3]. It is natural to use the term 'password' to describe this mechanism, because an agent's report can be seen as a password that opens access to different menus.

In the second class of environments that we study, only truthful messages about the agent's type ('state of the world') are costless, while the cost of a misrepresentation is finite and depends both on the true and announced types, as in Lacker and Weinberg (1989) and Maggi and Rodriguez-Clare (1995). We demonstrate that if the type space is finite, any allocation profile is implementable at zero communication cost. Thus, the existence of costless lies is necessary for there to be any impediment to implementation. The intuition for this result is straightforward: if the cost of a nontruthful report is bounded away from zero, then by requiring an agent to state her type sufficiently many times, the principal can make the cost of misrepresentation arbitrarily high. This result holds even if the type space is infinite, provided the marginal cost of misrepresentation is bounded away from zero.

When the marginal cost of misrepresenting one's type is not bounded away from zero, arbitrary allocations are no longer implementable, and the principal must elicit some nontruthful reports from the agent. Still, under mild regularity conditions, we establish an approximate implementation result: by eliciting a sufficiently large number of truthful and nontruthful reports, the principal can come arbitrarily close to any continuously differentiable allocation rule, and at the same time keep the communication cost arbitrarily small. Thus, the ability of the principal to request the agent to make sufficiently many announcements significantly expands the set of implementable allocations.

The rest of the paper is organized as follows. In section 2 we study environments with binary cost structures and establish our general implementation result. In section 3 we generalize the model with 'honest' and 'strategic' agents to a continuous type space. In section 4 we study environments without costless nontruthful messages. Section 5 concludes.

2 Binary Cost Structure.

We consider an environment with one principal and one agent. The set of decisions (allocations) controlled by the principal is denoted by X, and its typical element is denoted by x. The agent has utility function $u(x, \theta)$ with utility parameter θ, which is private information to the agent. Let Θ denote the space of possible values of the utility parameters. Further, let $\mathcal{C}$ denote the admissible communication set i.e., *the set of statements about the true state of the world, or messages, that can be made by an agent and understood by the principal*[4]. We assume that $\mathcal{C}$ is sufficiently large, so that claims of the form "my utility parameter is $\hat{\theta}$" belong to $\mathcal{C}$ for all $\hat{\theta} \in \Theta$. With a slight abuse of notation, the latter assumption can be

[3]We are grateful to Jacques Cremer for pointing our this interpretation.

[4]In the sequel, we use words "message," "claim," "statement," and "announcement" interchangeably. They all denote a minimal unit of communication i.e., an element of $\mathcal{C}$.

written as $\Theta \subset \mathcal{C}$. We also consider situations where $\Theta \equiv \mathcal{C}$.

The communication possibilities of a particular agent are characterized by her *feasible communication set* $\mathcal{M} \subset \mathcal{C}$. If an agent makes statement (sends message) m, she incurs zero cost if $m \in \mathcal{M}$ and an infinite (very large) cost if $m \notin \mathcal{M}$. Importantly, we assume that the cost of making any particular statement is independent of other statements that an agent also makes. Therefore, an agent with *feasible communication set* $\mathcal{M}$ can make a collection of statements (send a collection of messages) if and only if every element in this collection belongs to $\mathcal{M}$.

The set $\mathcal{M}$ is also agent's private information. Therefore, a full description of her type includes not only her utility parameter, but also her *feasible communication set*. Accordingly, we define an agent's type as the vector $t = (\theta, \mathcal{M})$. Let $T \subset \Theta \times \mathcal{P}(\mathcal{C})$ denote the set of all feasible types where $\mathcal{P}(\mathcal{C})$ denotes the 'power set' (set of all subsets) of $\mathcal{C}$. The probability distribution $F(.)$ over T is common knowledge. We do not assume any specific relation between θ and $\mathcal{M}$, except that an agent can always tell the truth i.e., $\theta \in \mathcal{M}$. Thus any environment is characterized by a tuple $\{X, \mathcal{C}, u(.,,), T, F(.)\}$.

Example 1 *Green and Laffont (1986): $\mathcal{C} = \Theta$. The preference parameter θ determines $\mathcal{M}$ uniquely, and the correspondence that maps θ to $\mathcal{M}$ is common knowledge. In this case, one can use notation $M(\theta)$ instead of $\mathcal{M}$. Defining the type as $t = (\theta, M(\theta))$ is redundant, as the utility parameter θ provides a sufficient description of the agent's private information.*

Example 2 *Alger and Ma (1998): $\mathcal{C} = \Theta = \{\theta_L, \theta_H\}$. With probability α the feasible communication set $\mathcal{M}$ of an agent with valuation θ_K, $K \in \{L, H\}$, is $\{\theta_K\}$ (the agent is 'honest'), and with probability $1 - \alpha$ it is $\{\theta_L, \theta_H\}$ (the agent is 'strategic'). In this environment the utility parameter θ does not determine the feasible communication set $\mathcal{M}$ uniquely. However, there is a correlation between θ and $\mathcal{M}$.*

Example 3 *The 'credible claims' model of Lipman and Seppi (1995). See also Bull and Watson (2001). An agent with utility parameter θ can make n_θ claims (statements) i.e., $\mathcal{M}_\theta = \{b_\theta^1, ..., b_\theta^{n_\theta}\}$ where b_θ^i is her i-th feasible claim, and cannot make any claim (statement) that is not in $\mathcal{M}_\theta$. As in Example 1, the correspondence mapping θ to $\mathcal{M}_\theta$ is common knowledge.*

Our main goal is to characterize the set of implementable allocation profiles, or social choice functions. As we focus on the implementability question, we will consider the largest possible set of mechanisms and avoid imposing any restrictions on it. In particular, we allow for mechanisms in the communication stage of which an agent may be requested to make a number of statements and send a number of messages about her preference parameter. This is the key point that distinguishes our approach to implementation. As we have noted above, in most contributions studying mechanisms with costly communication, it is assumed, sometimes implicitly, that an agent can make only one statement or send only one message.

Mechanisms in which an agent sends several messages are natural in the context of the 'credible claims' model where the claims can have various contents. However, in the case where $\mathcal{C} = \Theta$ further explanation of our approach is warranted.

Suppose that the agent's type is $t_0 = (\theta_0, \mathcal{M}_0)$. Then if the principal requests the agent to submit n announcements regarding her true utility parameter, the set of feasible reports includes all arrays $(\hat{\theta}_1, ..., \hat{\theta}_n)$ s.t. $\hat{\theta}_i \in \mathcal{M}_0$ for each $i \in \{1, ..., n\}$. However, it is not necessarily the case that $\hat{\theta}_1 = \hat{\theta}_1 = ... = \hat{\theta}_n$. That is, the agent can make different (i.e. conflicting) statements about her utility parameter. But since the cost of making any statement in independent of other statements made by the agent, no matter how many statements she makes, she can never announce a $\hat{\theta}$ which does not belong to her *feasible communication set* $\mathcal{M}$.

This can be interpreted in several ways. First, an agent may be requested to submit several documents and report her utility parameter in each of them. For example, income has to be reported on income tax forms, loan and mortgage applications, children's college financial aid forms. Moreover, questions requesting the same information can be stated in a different form in each document, yet at the same time be unambiguous. In particular, questions about income, wealth, education and experience can be worded differently in different documents. Then, the ability of an agent to answer these questions differently will depend on her communication abilities, or on her personal cost of sending messages providing conflicting information.

Second, the principal can hire several deputies each of whom requests a separate report from the agent. For example, different departments of the same organization often collect the same information about employees, clients, customers or suppliers.

As we will show below, mechanisms where agents send multiple messages (make a number of statements) allow to screen different types of agents to the largest possible extent by exploiting the costly communication structure. Therefore, it is necessary to consider such mechanisms when we wish to characterize the set of implementable outcomes. They provide a universal analytical tool that allows to achieve this goal. Below we will also investigate whether optimal allocation profiles can be implemented via mechanisms in which the number of messages that an agent is required to send is small. The relation between the former and the latter mechanisms is the same as the relation between direct mechanisms and indirect mechanisms (such as auctions) that implement the same outcomes.

Formally, let us fix a social choice function $f(\theta, \mathcal{M}) : T \mapsto X$ and consider mechanism $G : \mathcal{P}(\mathcal{C} \cup \phi) \mapsto X$ defined as follows : $G(\theta, M) = f(\theta, M \cup \{\theta\})$ if $(\theta, M \cup \{\theta\}) \in T$, and $G(\theta, M) = \underline{x}$ otherwise, where $\underline{x}$ is the 'worst outcome' that minimizes $u(x, \theta)$ $\forall \theta \in \Theta$. We assume that such $\underline{x}$ exists. In an environment with transferable utility the latter assumption is without loss of generality.

In mechanism $G(.)$ the agent is asked to make all the statements that she can make. By convention, the mechanism designer interprets the first statement as her true utility parameter. The subsequent statements are interpreted simply as elements of her feasible communication set $\mathcal{M}$.

We will say that an agent's report is $\mathcal{M}-$truthful if she announces her true utility parameter in her first statement, and also makes all statements/sends all messages that are feasible for her i.e., the agent of type $t = (\theta, \mathcal{M})$ announces $(\theta, m_1, ..., m_n)$ s.t. $\{m_1, ..., m_n\} = \mathcal{M} \setminus \theta$ [5]. In this case we will say that mechanism $G(.)$ $\mathcal{M}-$truthfully implements social choice function $f(.)$. The following is the central result of this section.

Theorem 1 *Any implementable social choice function is $\mathcal{M}-$truthfully implementable via mechanism $G(\cdot)$. Furthermore, there exists an environment $\{X, \mathcal{C}, u(.,.), T, F(.)\}$ and social*

[5] It is intuitive to call such reporting strategy 'truthful' because the agent fully reveals both her utility parameter and her *feasible communication set.*

choice function $f(.)$ such that $f(.)$ is not implementable via any mechanism with message space Θ, but is implementable via mechanism $G(\cdot) : \mathcal{P}(\mathcal{C} \cup \phi) \mapsto X$.

Proof: Fix an environment $\{X, \mathcal{C}, u(.,,), T, F(.)\}$, and suppose that the social choice function $f(.)$ is implementable via some mechanism $(S, g(.))$ where S is the strategy space, and $g: S \mapsto X$ is outcome function. Let $s(t)$ be an equilibrium strategy of type $t = (\theta, \mathcal{M})$ in this mechanism. Using the same technique as in the proof of the revelation principle, define outcome function $G(.)$ as follows: $G(t) = g(s(t))\ \forall t \in T$ and $G(t) = \underline{x}\ \forall t \notin T$.

Let us show that submitting an $\mathcal{M}-$truthful report is an optimal strategy for agent of type t in the mechanism with outcome function $G(t)$. Suppose to the contrary that a profitable deviation does exist in this mechanism, i.e. the agent of type $t \equiv (\theta, \mathcal{M})$ gets a higher payoff if she announces $t' \equiv (\theta', \mathcal{M}')$ and obtains the outcome $g(s(t'))$. Then $s(t')$ cannot be feasible for the agent of type t in mechanism $(S, g(.))$. Therefore, action $s(t')$ must involve sending a message m'' s.t. $m'' \in \mathcal{M}'$, but $m'' \notin \mathcal{M}$. However, this implies that t cannot announce $t' \equiv (\theta', \mathcal{M}')$ in mechanism $G(\cdot)$. This contradiction completes the proof of the first part of the theorem.

To prove that the mechanism $G(\cdot)$ permits to implement a larger set of social choice functions than would a mechanism with message space Θ in which an agent makes only one statement about her utility parameter, let us consider an example from Green and Laffont (1986): $X = \{x_1, x_2, x_3\}$, $\Theta = \{\theta_1, \theta_2, \theta_3\}$, and the relation between θ and $\mathcal{M}$ is characterized by the following correspondence $M(\theta)$ which is common knowledge: $M(\theta_1) = \{\theta_1, \theta_2\}$, $M(\theta_2) = \{\theta_2, \theta_3\}$, $M(\theta_3) = \{\theta_3\}$. We assume the following payoff structure:

$$u(x_1, \theta_i) < u(x_3, \theta_i) < u(x_2, \theta_i) \ \ \forall i \in \{1, 2, 3\}$$

Fix the social choice function $f(\theta_1, M(\theta_1)) = x_1$, $f(\theta_2, M(\theta_2)) = x_2$, $f(\theta_3, M(\theta_3)) = x_3$, and consider any mechanism in which the agent's message consists of a single element of Θ. Then $f(.)$ can be implemented only if all three agent types send different messages. Since type θ_3 can only send message "θ_3", it follows that type θ_2 must send message "θ_2" in equilibrium. However, then type θ_1 would imitate the message sent by type θ_2. Thus, $f(.)$ is not implementable.

On the other hand, $f(.)$ is $\mathcal{M}-$ truthfully implementable via the mechanism with the following outcome function $G(.)$: $G(\theta_1, \theta_2) = x_1$, $G(\theta_2, \theta_3) = x_2$, $G(\theta_3) = x_3$, and $G(\mathcal{S}) = x_1$, where $\mathcal{S}$ is any other report. *Q.E.D.*

The intuition behind theorem 1 is easy to understand. When two types have different *feasible communication sets*, one of them (say, type A) cannot make all the statements that are feasible for the other (say, type B). If this property is exploited in a mechanism, then type A cannot imitate type B in the communication stage. Therefore, an allocation profile implemented via such mechanism need not satisfy standard incentive constraint that type A gets a higher payoff from the allocation designed for her than from the allocation designed for type B. Obviously, the larger is the set of incentive constraints that are taken care of in this way without being imposed on the allocation profile, the less restricted is the allocation profile and the larger is the set of implementable social choice functions. Because mechanism

$G(.)$ requires the agent to make all feasible statements, it allows to take care of the maximum possible number of incentive constraints at the communication stage, without having to impose them on the allocation profile. Therefore the set of implementable social choice functions is maximal under $G(.)$.

Specifically, in mechanism $G(.)$ it is impossible for any type $(\theta, \mathcal{M})$ to obtain the allocation designed for type $(\theta', \mathcal{M}')$ if the former cannot fully mimic the latter i.e., if $\mathcal{M}' \backslash \mathcal{M} \neq \phi$. Hence we have:

Corollary 1 *A social choice function $f : T \to X$ is implementable if and only if for all $(\theta, \mathcal{M}) \in T$ the following incentive constraints are satisfied :*

$$u(f(\theta, \mathcal{M}), \theta) \geq u(f(\theta', \mathcal{M}'), \theta) \quad \forall (\theta', \mathcal{M}') \in T \text{ such that } \mathcal{M}' \subset \mathcal{M}.$$

Corollary 1 specifies which incentive constraints need to be imposed on the allocation profile. Together with theorem 1, it provides a method for characterizing the set of implementable social choice functions. For this reason, and also because the agent reports her type truthfully in mechanism $G(.)$, where truthfulness has to be broadly construed, theorem 1 can be viewed as an extension of the revelation principle to the costly communication stage.

Both Green and Laffont (1986) and Alger and Ma (1998) demonstrate the failure of the revelation principle when type is defined in a standard way as the utility parameter θ, and communication stage involves only one announcement by the agent. Our result is complementary, since we use a broader definition of type and consider mechanisms with a larger space of reporting strategies.

There are nevertheless some notable special cases in which expanding the set of reports beyond Θ does not enlarge the set of implementable social choice functions, and a single announcement of the utility parameter θ is sufficient. In particular, consider the following:

Nested Range Condition (NRC) (Green and Laffont 1986): *For any three distinct elements $\theta_1, \theta_2, \theta_3 \in \Theta$, if $\theta_2 \in M(\theta_1)$ and $\theta_3 \in M(\theta_2)$, then $\theta_3 \in M(\theta_1)$.*

Green and Laffont (1986) establish that in the context of their model (see Example 1), NRC is a sufficient condition for the revelation principle to hold in the traditional sense. So, when NRC holds, a social choice functions is implementable if and only if it is implementable via a mechanism in which the agent truthfully reports her preference parameter θ. They also demonstrate that the revelation principle fails when NRC does nor hold: in some environments there are social choice functions that are implementable but not truthfully implementable via a direct mechanism with message space Θ.

In the context of our model where *feasible communication set* $\mathcal{M}$ is not generally a deterministic function of the utility parameter θ and the communication space $\mathcal{C}$ may be larger than Θ, there is the following useful version of the Nested Range Condition:

Nested-Range Condition* (NRC*): *For any $(\theta, \mathcal{M})$ and $(\theta', \mathcal{M}') \in T$, if $\theta' \in \mathcal{M}$, then $\mathcal{M}' \subset \mathcal{M}$.*

It is easily seen that NRC* is equivalent to NRC. Indeed, suppose that NRC* holds, and consider $(\theta, \mathcal{M})$ and $(\theta, \mathcal{M}')$ that both belong to T. Since the agent always has the ability to tell the truth, we have $\theta \in \mathcal{M}$ and $\theta \in \mathcal{M}'$. NRC* then implies that $\mathcal{M} = \mathcal{M}'$, so that there is a one-to-one relationship between θ and $\mathcal{M}$, which we may denote by $M^c(\theta)$. As a consequence, the condition NRC* reduces to the requirement that if $\theta' \in M^c(\theta)$ then $M^c(\theta') \subset M^c(\theta)$, which is just NRC.

Under NRC* our model becomes essentially equivalent to the one studied by Green and Laffont (1986) even though we may have $\mathcal{C} \neq \Theta$. So it is easy to see, that in this case their Theorem 1 also applies, and all implementable social choice functions can be implemented via a mechanism where an agent is only asked to report her preference parameter θ once, and she reports truthfully.

However, in the context of our model condition NRC* is very restrictive. It is violated if there exists even a single pair of types that have the same utility parameter θ, but differ in their communication abilities. When NRC* does not hold, theorem 1 shows that a mechanism in which the agent sends several messages allows to implement a larger set of social choice functions than could be implemented via a mechanism in which the agent is asked to send only one message about her preference parameter. To see just how dramatic an effect of sending several messages can be, consider the following condition.

Non-Nested Range Condition (NNRC*): Consider any $(\theta, \mathcal{M}), (\theta', \mathcal{M}') \in T$ s.t. either $\theta \neq \theta'$ or $\mathcal{M} \neq \mathcal{M}'$. Then $\exists \theta''$ s.t. $\theta'' \in \mathcal{M}'$ and $\theta'' \notin \mathcal{M}$.

NNRC* can also be rewritten as follows. Consider any $(\theta, \mathcal{M})$, $(\theta', \mathcal{M}') \in T$ s.t. either $\theta \neq \theta'$ or $\mathcal{M} \neq \mathcal{M}'$. Then, $\mathcal{M} \bigtriangleup \mathcal{M}' \neq \phi$. It is easy to see that under NNRC*, an agent's communication abilities uniquely determine her utility parameter θ, i.e. the set of types T is such that the mapping from feasible communication sets $\mathcal{M}$ into θ is a function.

NNRC* is essentially the opposite of NRC*. Under NRC*, if an agent with preference parameter θ_1 can claim to have preference parameter θ_2, she can always fully mimic her by making all statements that are feasible for an agent with preference parameter θ_2. Under NNRC*, no type can make the same set of announcements as another type, and hence fully mimic this other type. We therefore have:

Theorem 2 *Suppose NNRC* holds. Then any social choice function $f : T \to X$ is implementable.*

Proof: By corollary 1 the set of incentive constraints that an implementable social choice function has to satisfy is empty. Therefore, any social choice function is implementable. *Q.E.D.*

Whenever condition NNRC* holds, perfect screening of agents is achieved by identifying their communication abilities. Condition NNRC* is quite strong, so typically we would expect the communication structure to be somewhere in between the extreme cases described by NRC* and NNRC*.

In such intermediate cases, all agent types can be divided into two categories. The first category includes only such types $(\theta, \mathcal{M})$ that for all $(\theta', \mathcal{M}') \in T$ satisfying $\theta' \in \mathcal{M}$ we have $\mathcal{M}' \setminus \mathcal{M} \neq \phi$. The second category includes such types $(\theta, \mathcal{M}) \in T$ that $\exists (\theta'', \mathcal{M}'') \in T$ for which $\mathcal{M}'' \subset \mathcal{M}$. Corollary 1 implies that no incentive constraints preventing types from the first category from imitating any other types need to be imposed on the implementable allocation profile. So, in the optimal mechanism maximizing the principal's profits all types from the first category earn zero surplus. To be more specific, consider the following example.

Example 4 *The principal (firm) with increasing convex cost function $c(q)$ sells quantity q of the good to an agent (consumer) with valuation $\theta v(q)$, where $v(q)$ is a concave non-negative function. The parameter θ is privately known by the agent and takes one of n possible values $0 < \theta_1 < ... < \theta_n$, where $n > 2$. The probability that $\theta = \theta_i$ is equal to $\pi_i > 0$. The principal's objective is to maximize the expected value of $p - c(q)$ where p is a transfer paid by the agent. The agent has limited ability to misrepresent the true value of θ: $M(\theta_i) = \{\theta_{i-1}, \theta_i\}$ for $i > 1$, and $M(\theta_1) = \theta_1$.*

According to Corollary 1, the only incentive constraint that an implementable allocation profile needs to satisfy is that the agent with valuation θ_2 not be willing to mimic the agent with valuation θ_1. Then, it is easy to see that the optimal mechanism will implement efficient quantities for all types except θ_1. Only an agent with valuation θ_2 will obtain a positive surplus, while the rest will be held to their reservation utility levels.

This is so because the agent with valuation θ_2 has a reporting advantage relative to all other types. Specifically, an agent with valuation θ_i where $1 < i < n$ can send a message θ_{i-1} which distinguishes her from an agent with valuation θ_{i+1}. Yet, there is no message that an agent with valuation θ_1 can send that is not available to the agent with valuation θ_2. If the former had such a message available, for example if she could report θ_n, then the agent with valuation θ_2 would not receive any surplus either.

This example and the discussion preceding it highlight the importance of agent's communication abilities as a determinant of the amount of surplus that she earns. When an agent's communication abilities are limited, she may not earn any surplus even if her privately known productivity is high, while less productive agents with better communication abilities will be able to obtain a positive surplus.

Note that the optimal allocation profile in example 4 can be implemented via a mechanism that requires the agent to send only one message. We will show how this can be done below, when we discuss the so-called 'password' mechanism.

Finally, let us apply our results to the model of Alger and Ma (1998) (see Example 2). To focus on the main issues, we restate their model as a two-party non-linear pricing model similar to that in Example 4. Specifically, a principal (a firm or an HMO, as in the original model) with an increasing convex cost function $c(q)$ sells quantity q of the good to an agent (consumer) with valuation $\theta v(q)$ where $v(q)$ is a concave non-negative function. The parameter θ can take 2 possible values: low θ_L with probability π and high θ_H with complementary probability. Further, with probability α independent of the realized valuation parameter the agent is 'strategic' so that $M(\theta_K) = \{\theta_L, \theta_H\}$ ($K \in \{L, H\}$). With complementary probability $1 - \alpha$ the agent is 'honest' so that $M(\theta_K) = \theta_K$ ($K \in \{L, H\}$).

Alger and Ma (1998) consider mechanisms in which an agent makes only one statement about her utility parameter. Thus, she reports either θ_L or θ_H, and therefore the principal can only implement allocation profiles that consist of at most two different quantity-transfer pairs. The results of Alger and Ma (1998) restated in the nonlinear pricing context are as follows.

When α is sufficiently small, i.e. the agent is very likely to be 'honest,' it is optimal for the principal to use a mechanism in which an agent who reports valuation θ_H is assigned a quantity q_H^* that is efficient when the agent's valuation is high i.e., satisfies $\theta_H v'(q_H^*) = c'(q_H^*)$, and pays a transfer $p_H = \theta_H v(q_H^*)$. An agent who reports a low valuation is assigned a quantity q_L^* that is efficient when the agent's valuation is low i.e., satisfies $\theta_L v'(q_L^*) = c'(q_L^*)$, and pays a transfer $p_L = \theta_L v(q_L^*)$. In equilibrium, an 'honest' agent with high valuation θ_H reports θ_H, and thus obtains the first allocation that leaves her with zero surplus. Both the 'honest' and 'strategic' agents with valuation θ_L and 'strategic' agent with valuation θ_H report θ_L, and obtain the second allocation. Thus, the only type who earns a positive surplus is the 'strategic' agent with high valuation.

When only a single report is allowed, this allocation profile is optimal, because when it is very likely that the agent is 'honest', the principal prefers to offer a mechanism that extracts as much surplus as possible from 'honest' types. The principal can achieve this goal by offering quantity-transfer pairs that are efficient when the agent reports her true valuation and leave no surplus to such agent. The drawback of this mechanism lies in the fact that it induces 'strategic' high-value agent to report a low valuation and consume an inefficient quantity.

However, the principal can implement a more efficient and more profitable allocation profile via mechanism $G(.)$ in which the agent has to make two statements regarding her valuation. In this case, agent's report can be denoted by $(\hat{\theta}_1, \hat{\theta}_2)$.

The inability of an 'honest' agent to lie implies that both her first and second messages in this mechanism are equal to her true valuation. Since without loss of generality the agent reports $\mathcal{M}-$truthfully (see theorem 1), we can focus on equilibria where 'strategic' high-valuation and low-valuation types report (θ_H, θ_L) and (θ_L, θ_H) respectively. Thus, a 'strategic' agent makes two different statements and is offered an allocation that depends on the order of announcements. Mechanism $G(.)$ therefore allows to implement allocation profiles that consist of up to four different quantity-transfer pairs. The only incentive constraints that have to be imposed on an allocation profile implementable via this mechanism are that a 'strategic' agent (with any valuation) gets a higher payoff from the allocation designed for her than from an allocation designed for any other type. By corollary 1, this is the minimal set of incentive constraints that an implementable allocation profile has to satisfy.

Imposing these incentive constraints on the principal's maximization problem, it is easy to derive the following optimal allocation profile. First, both 'honest' and 'strategic' high-value agents are assigned efficient quantity q_H^* s.t. $\theta_H v'(q_H^*) = c'(q_H^*)$. Both 'honest' and 'strategic' low-value agent are assigned quantity q_L which satisfies

$$\left(\theta_L - \frac{\alpha\pi(\theta_H - \theta_L)}{1-\pi}\right) v'(q_L) = c'(q_L)$$

We will assume that q_L is positive.

The 'honest' high-valuation agent pays transfer $\theta_H v(q_H^*)$, leaving her no surplus, while the 'strategic' high-valuation agent pays a lower transfer equal to $\theta_H v(q_H^*) - (\theta_H - \theta_L)v(q_L)$. The second term reflects the informational rent which this type could earn by imitating a low-value agent. Both low-valuation types pay transfer $\theta_L v(q_L)$ which holds them to their reservation level. Note that q_L is below the efficient quantity which reflects the usual tradeoff between efficiency and informational rent. However, the distortion is smaller than in the standard case without communication costs, because informational rent has to be paid only to a 'strategic' high-value agent, and not to her 'honest' counterpart.

Mechanism $G(.)$ is more profitable for the principal. It is also more socially efficient, as it generates smaller quantity distortion than any mechanism in which the agent makes only one statement about her valuation. Intuitively, this is achieved because mechanism $G(.)$ induces those types who are capable of lying to send conflicting messages: state once that her type is θ_H and state once that it is θ_L. As a result, it becomes possible to identify an 'honest' agent and her valuation for free, and at the same time to screen 'strategic' agents.

An interesting and possibly surprising implication of the results in this section is that individuals who make conflicting or contradictory statements should not be penalized for such behavior. In the optimal mechanism these individuals obtain higher payoffs than the individuals who do not make contradictory statements and are less suspect of lying. This is done to prevent individuals with low personal cost of lying from imitating someone else's behavior and to improve the overall efficiency of the mechanism.

Rewarding individuals who make contradictory statements rather than punishing them does not appear to be at odds with reality. For example, there is a wide range of institutions that collect the same information from individuals, but perform their functions separately from each other, without cross-checking information submitted to them. In many cases, sharing such information about individuals is prohibited by privacy laws. Privacy laws then appear to encourage individuals to make contradictory statements, and our results explain why such institutional design may be optimal.

A case in point is statements about income. An individual has to declare her income to the Internal Revenue Service for tax purposes, to the banks when applying for a loan, to colleges and Universities with regards to financial aid for the children. These institutions do not crosscheck information with each other. This leaves open the possibility that an individual might submit different reports to different institutions. Specifically, people who are more prone to misrepresentation or have weaker ethical norms could overstate their income when applying for a loan, and/or understate it on the financial aid form, but still report the truth to the IRS. From the social welfare point of view, this outcome can dominate the outcome which would arise if these institutions cross-checked individuals' reports and imposed penalties on the ones whose statements are contradictory. If this was the case, the consumers who are prone to misrepresentation may still misreport their income, but then they will be also be misrepresenting it for tax purposes, which conceivably could result in a higher social welfare loss. Furthermore, some individuals may decide to work less and earn less income because they can no longer understate their income to colleges. Thus, our results suggests that privacy laws prohibiting information sharing across institutions may be a part of a socially optimal design of institutions.

To complete this section, we address the issue of implementing optimal allocation profiles via mechanisms in which an agent sends a smaller number of messages than in mechanism $G(.)$. We do it for two reasons. First of all, mechanisms that rely on fewer messages are simpler and, therefore, easier to implement. Second, mechanisms in which agents send conflicting messages and are rewarded for this may be unacceptable in certain situations for political or ethical reasons. For example, this is likely to be the case in the taxation situation where an agent has to report her income to the tax authority.

In several environments that we have considered above, and in particular in the model with 'honest' and 'strategic' agents as well as in Example 4, mechanism $G(.)$ can be replaced with the following so-called 'password' mechanism. In the 'password' mechanism an agent is asked to report her utility parameter only once. After she makes her announcement, the agent is offered a choice from a menu of possible allocations. The offered menu itself depends on the reported utility parameter. Thus, the agent's message can be seen as a 'password' because its content determines which menu the agent gains access to.

In the non-linear pricing model with 'honest' and 'strategic' agents that we have studied above, the derived optimal allocation can be implemented via a 'password' mechanism in the following way. An agent is asked to report whether her valuation is high or low. If a high valuation is reported, the agent is assigned the allocation designed for the 'honest' high-valuation type. If a low valuation is reported, the agent is offered a menu consisting of two allocations: $(q_H^*, \theta_H v(q_H^*) - (\theta_H - \theta_L)v(q_L))$ which is designed for 'strategic' high-value type, and $(q_L, \theta_L v(q_L))$ which is designed for the low-value types. It is easy to see that this mechanism implements our optimal allocation profile. Only 'honest' high-value type will report a high valuation. All the other types will report a low valuation and then make the 'right' choices from the menu: pick the allocations that were designed for them.

Similarly, a 'password' mechanism can be used to implement the optimal allocation in Example 4. In this case, the agent is also asked to announce her valuation once. If valuation θ_i $(2 \leq 1 \leq n-1)$ is reported, the allocation designed for an agent with valuation θ_{i+1} is assigned. If valuation θ_1 is announced, the agent is offered a menu which consists of the allocations designed for an agent with valuation θ_1 and an agent with valuation θ_2.

The 'password' mechanism is essentially a 'hybrid' mechanism. It combines an implementation method based on the revelation principle (reporting the valuation) with an implementation method based on the taxation principle (choosing from a menu).

3 'Honest' and 'Strategic' Agents: A Generalization.

In this section we generalize the analysis of the model with 'honest' and 'strategic' agents to the case where the set of possible valuations is infinite, instead of a two-point one. Specifically, we consider the same non-linear pricing model as in the previous section. An agent (consumer) has valuation θq for quantity q of the good where θ is consumer's private information and can take any value in the interval $[0, 1]$. For simplicity, we assume that θ is distributed uniformly on this interval and that, independently of her valuation, the agent is 'honest' and cannot lie about her valuation with probability $1/2$. The agent's reservation utility level is 0.

The principal (firm) designs the mechanism to trade with the agent. Its cost of producing quantity q of the output is equal to $\frac{q^2}{2}$. These simplifying assumptions are made for technical reasons, as the mechanism design problem in this environment is non-trivial. We will focus on finding the optimal mechanism which maximizes the principal's expected payoff. This allows us to highlight the differences between the optimal mechanisms in this environment and in the standard non-linear pricing model where all agents are 'strategic' and can misrepresent their valuations.

Theorem 1 and Corollary 1 imply that the only incentive constraints that have to be imposed on the implementable allocation profile are the ones that ensure that a 'strategic' agent is unwilling to imitate either a 'strategic' agent with a different valuation or any 'honest' agent. An 'honest' agent's valuation can be identified at the communication stage of a mechanism.

Then, it is sufficient to consider a mechanism in which an agent is asked to make only two statements θ^1 and θ^2 about her valuation. Note that an 'honest' agent with valuation θ can only send messages θ^1 and θ^2 s.t. $\theta^1 = \theta^2 = \theta$. So the mechanism should assign an allocation designed for this type after receiving such pair of messages. Further, if an agent sends messages θ^1 and θ^2 s.t. $\theta^1 \neq \theta^2$ the mechanism should assign the allocation designed for 'strategic' agent with valuation θ^1. Clearly, in this mechanism an 'honest' agent is unable to imitate a 'strategic' agent, and the only incentive constraints that need to be imposed on the allocation profile for it to be implementable are that a 'strategic' agent with valuation θ gets a higher payoff from the allocation designed for her than from any other allocation. By Corollary 1 this is the minimal set of incentive constraints that has to be imposed on any implementable allocation profile.

Furthermore, any implementable allocation profile can also be implemented via the following 'password mechanism' in which an agent is asked to report her valuation only once. If an agent announces valuation $\theta > 0$, then the mechanism assigns the allocation designed for an 'honest' agent with valuation θ. If an agent reports $\theta = 0$, then she is offered a menu consisting of allocations designed for all 'strategic' types and the allocation designed for 'honest' agent with valuation 0. In this 'password' mechanism report $\theta = 0$ can be considered as a 'password' that an agent needs to send to access the menu of allocations designed for 'strategic' types. Since an 'honest' agent with valuation $\theta > 0$ is unable to send such message, she cannot access this menu. Again, the only incentive constraints that need to be imposed in this mechanism are those that prevent a 'strategic' agent from imitating any other agent type.

The optimal allocation profile is characterized in the following theorem.

Theorem 3 *The optimal allocation profile which maximizes the principal's expected profits and which can be implemented either via the mechanism with two announcements or via the 'password' mechanism is as follows:*

The quantity allocation $q(\theta)$ to the 'strategic' agent with valuation θ, and the transfer $t^s(\theta)$ paid by her, are given by:

$$q(\theta) = \begin{cases} \sqrt{3}^{\sqrt{3}}(\sqrt{3}-1)^{\sqrt{3}+1}\theta^{\sqrt{3}+1} & \text{if } \theta \in [0, \frac{1}{3-\sqrt{3}}] \\ 2\theta - 1 & \text{if } \theta \in [\frac{1}{3-\sqrt{3}}, 1] \end{cases}$$

$$t^s(\theta) = \theta q(\theta) - \int_0^1 q(s)ds$$

The quantity allocation $g(\theta)$ to the 'honest' agent with valuation θ, and the transfer $t^\tau(\theta)$ paid by her, are given by:

$$g(\theta) = \begin{cases} \sqrt{3}^{\sqrt{3}}\theta^{\sqrt{3}+1} \left(= q(\frac{\theta}{\sqrt{3}-1})\right) & \text{if } \theta \in [0, \frac{1}{\sqrt{3}}] \\ \theta & \text{if } \theta \in [\frac{1}{\sqrt{3}}, 1] \end{cases}$$

$$t^\tau(\theta) = \theta g(\theta)$$

Before proceeding with the proof of the theorem, let us describe the results in an intuitive way. The problem of designing an optimal mechanism is non-standard because two sets of incentive constraints have to hold for a 'strategic' agent: she should not be willing to imitate either a 'strategic' agent with a different valuation or an 'honest' agent. It is important to understand which of these constraints are binding in an optimal mechanism.

First, we establish that the standard 'downward' incentive constraints between strategic types are still binding, which implies that the surplus (informational rent) $U(\theta)$ that a 'strategic' agent with valuation θ earns is equal to $\int_0^\theta q(x)dx$. (It can be shown that $U(0) = 0$.)

In addition, there is another binding constraint for a 'strategic' agent with valuation θ in the 'lower' part of the valuation space $[0, \frac{1}{3-\sqrt{3}}]$: she is just indifferent between the allocation designed for her and the allocation designed for an 'honest' agent with a lower valuation $r(\theta) = (\sqrt{3} - 1)\theta$. Since all 'honest' agents are held at their reservation utility levels, this implies that $U(\theta) = g(r(\theta))(\theta - r(\theta))$.

To reduce informational rent $U(\theta)$ of a 'strategic' agent with valuation $\theta \in [0, \frac{1}{3-\sqrt{3}}]$ the principal has to decrease the quantities assigned to 'strategic' agents with valuations less than θ and the quantity $g(r(\theta))$ assigned to an 'honest' agent with valuation $r(\theta)$. Thus, reducing those rents imposes higher efficiency losses when 'honest' agents are present than in the standard situation where all agents are 'strategic.'

As a result, the well-known tradeoff between efficiency and informational rents is resolved in favor of higher efficiency i.e., the downward distortion in quantity allocations assigned to 'strategic' types with valuations on the interval $[0, \frac{1}{3-\sqrt{3}}]$ is less than the downward distortion in the quantity allocations $q^n(\theta)$ assigned to the agents with the same valuations in the standard non-linear pricing model. Note that $q^n(\theta) = \max\{0, 2\theta - 1\}$ under the simplifying assumptions made above, and the efficient quantity is $q^*(\theta) = \theta$. Then for $\theta \in (0, \frac{1}{3-\sqrt{3}})$ we have:

$$q^n(\theta) < q(\theta) < g(\theta) \leq q^*(\theta) = \theta.$$

The last inequality is strict when $\theta < r(\frac{1}{3-\sqrt{3}}) = \frac{1}{\sqrt{3}}$: some downward distortion in $g(\theta)$ is still optimal on this interval. The inequality $q(\theta) < g(\theta)$ holds because either $g(\theta) = q^*(\theta) = \theta$ or $\exists \theta' > \theta$ s.t. $U(\theta') = g(\theta)(\theta' - \theta)$. On the other hand, $U(\theta') = \int_0^{\theta'} q(x)dx > q(\theta)(\theta' - \theta)$. The last inequality follows because $q(\theta)$ is increasing.

On the 'upper' part $[\frac{1}{3-\sqrt{3}}, 1]$ of the valuation space, incentive constraints saying that 'strategic' agent is not willing to imitate an 'honest' agent are no longer binding. This happens because informational rent $U(\theta) = \int_0^\theta q(x)dx$ of a strategic agent becomes large enough to make imitating an 'honest' agent suboptimal. Therefore, as far as 'strategic' agents are concerned, the situation becomes identical to the standard non-linear pricing model. Hence $q(\theta) = 2\theta - 1$ on this interval. This also implies that the quantity allocation to an 'honest' agent with valuation above $r(\frac{1}{3-\sqrt{3}}) = \frac{1}{\sqrt{3}}$ is unrestricted by any binding incentive constraint from 'strategic' agents. Therefore, $g(\theta) = q^* = \theta$ for $\theta \in [\frac{1}{\sqrt{3}}, 1]$.

Note that because $q^n(\theta) < q(\theta)$ on $(0, \frac{1}{3-\sqrt{3}}]$, 'strategic' agents obtain a larger informational rent in the presence of 'honest' agents than in a standard situation where there are no 'honest' agents. 'Strategic' agents are paid more not to imitate the 'honest' ones.

A notable feature of the optimal allocation is absence of rationing: all agents whose valuations exceed 0 are assigned a positive quantity. This result stands in contrast with the standard case. In the non-linear pricing model with the same functional forms but without 'honest' types it is optimal to assign zero quantity to all types with valuations below 1/2. Furthermore, rationing is also generic in the multidimensional non-linear pricing model, as shown by Armstrong (1996).

Absence of rationing in our mechanism is due to the 'common cutoff' property established in lemma 4: if the principal rations 'strategic' agents by assigning zero quantity to those of them whose valuations are below some positive threshold level $\underline{\theta} > 0$, then she also has to assign zero quantity to 'honest' agents whose valuations are below $\underline{\theta}$. Yet, the principal can extract full surplus from 'honest' agents. This makes rationing suboptimal.

Proof of theorem 3: Corollary 1 implies that any implementable allocation profile $\{(q(\theta), t^s(\theta)), (g(\theta), t^\tau(\theta))\}$ must satisfy the following set of incentive constraints:

$$q(\theta)\theta - t^s(\theta) \geq q(\theta')\theta - t^s(\theta') \quad \forall \theta, \theta' \in [0,1] \tag{1}$$
$$q(\theta)\theta - t^s(\theta) \geq g(\theta')\theta - t^\tau(\theta') \quad \forall \theta, \theta' \in [0,1] \tag{2}$$

Mechanisms that implement allocation profiles that satisfy this set of incentive constraints have been described above. Also, the following individual rationality constraints have to be satisfied $\forall \theta \in [0,1]$:

$$q(\theta)\theta - t^s(\theta) \geq 0 \tag{3}$$
$$g(\theta)\theta - t^\tau(\theta) \geq 0 \tag{4}$$

Thus, optimal allocation profile solves the following problem:

$$\max_{(q(\theta),t^s(\theta)),(g(\theta),t^\tau(\theta))} 1/2 \int_0^1 \left(t^s(\theta) - \frac{q(\theta)^2}{2} \right) d\theta + 1/2 \int_0^1 \left(t^\tau(\theta) - \frac{g(\theta)^2}{2} \right) d\theta \tag{5}$$
$$\textit{subject to } (1),\ (2)\ (3),\ (4)$$

The individual rationality constraint (4) of an 'honest' agent with any valuation must be binding in an optimal mechanism. Otherwise, the principal can increase her profits by increasing the transfer required from such agent without affecting any incentive or individual rationality constraints. It is easy to see that when (4) are binding, incentive constraints (2) are not binding for all θ, θ' s.t. $\theta > \theta'$.

The presence of incentive constraints (2) does not allow to use the standard 'indirect utility' approach to solve this problem. Our method of solution involves a number of steps. First, the following lemma establishes sine basic properties of the solution.

Lemma 1 *Optimal quantity schedules $q(\theta)$ and $g(\theta)$ have the following properties:*

(i) $g(\theta) \leq \theta \ \forall \theta \in [0,1]$, and $g(\theta)$ is non-decreasing.

(ii) $q(\theta)$ is non-decreasing, and $q(1) = 1$. Furthermore, $q(\theta)$ has at most countably many points of discontinuity. Right-hand and left-hand limits exist at all discontinuity points of $q(\theta)$.

(iii) Let $U(\theta) \equiv \theta q(\theta) - t^s(\theta)$. Then $U(\theta)$ is uniformly continuous and a.e. (almost everywhere) differentiable, and $U(\theta) - U(\theta') = \int_{\theta'}^{\theta} q(s)ds$ a.e..

Proof: see the Appendix.

Lemma 1 shows that, as in a standard adverse selection problem, downwards incentive constraints between 'strategic' consumers are binding. Furthermore, the family of incentive constraints (1) holds if and only if $q(\theta)$ is nondecreasing and $U(\theta)-U(\theta')=\int_{\theta'}^{\theta} q(s)ds$. We can use this fact to replace (1) and substitute $t^s(\theta)$ out. Further, using the fact that (4) must be binding to substitute $t^\tau(\theta)$ out, and integrating $U(\theta)=U(0)+\int_0^\theta q(s)ds$ by parts, we conclude that problem (5) is equivalent to the following problem:

$$\max_{q(\theta)\geq 0, g(\theta)\geq 0, U(0)\geq 0} -U(0)+\int_0^1\left((2\theta-1)q(\theta)-\frac{q(\theta)^2}{2}\right)d\theta+\int_0^1\left(\theta g(\theta)-\frac{g(\theta)^2}{2}\right)d\theta \quad (6)$$

$$\textit{subject to}: \quad (i)\ q(\theta) \text{ is nondecreasing} \quad (7)$$

$$(ii)\ ICT(\theta,\theta'): \quad U(\theta)\equiv U(0)+\int_0^\theta q(s)ds \geq (\theta-\theta')g(\theta') \ \ \forall \theta,\theta' \in [0,1] \quad (8)$$

The following lemmas which are proved in the appendix establish several useful properties of the solution.

Lemma 2 *Optimal quantity schedule $q(\theta)$ is continuous on $(0,1)$.*

Lemma 3 *In the optimal mechanism $U(0)=0$.*

Lemma 4 ***Common cutoff**. Optimal quantity schedules are such that $\forall\theta\in[0,1)$ $q(\theta)=0$ if and only if $g(\theta)=0$.*

By lemmas (1)-(3), we can without loss of generality impose the following additional constraints on problem (6): (iii) $q(.)$ is continuous, (iv) $g(\theta)\leq 1=q(1)$, (v) $U(0)=0$. The next lemma shows that we can now replace the family of incentive constraints $ICT(\theta,\theta')$ in (8) with a simpler family of constraints.

Lemma 5 *Let $r(\theta)=\theta-\frac{U(\theta)}{q(\theta)}$ if $q(\theta)>0$ and $r(\theta)=\theta$ if $q(\theta)=0$. Suppose that $q(.)$ is nondecreasing, continuous, $g(\theta)\leq 1=q(1)$, and $U(0)=0$. Then:*
(i) $r(\theta)$ is nonnegative, nondecreasing, strictly increasing if $q(.)$ is strictly increasing at θ, and continuous. Furthermore, $r(0)=0$.
(ii) $U(\theta)\geq(\theta-\theta')g(\theta')$ $\forall\theta,\theta'\in[0,1]$ if and only if $q(\theta)\geq g(r(\theta))$ $\forall\theta\in[0,1]$.

Since $r(\theta)$ is continuous and nondecreasing and $r(0)=0$, $r(.)$ maps $[0,1]$ onto $[0,r(1)]$. The inverse image $r^{-1}(\hat\theta)$ from $[0,r(1)]$ is a singleton if $q(.)$ is strictly increasing at θ s.t. $r(\theta)=\hat\theta$. However, even if $r^{-1}(\hat\theta)$ is not a single point, it is easy to see that $q(r^{-1}(\hat\theta))$ is unique (see the reasoning in the proof of lemma 5(i). Then we have the following important result:

Lemma 6 *Fix a nondecreasing continuous quantity schedule $q(\theta)$ s.t. $q(1)=1$, and set $U(0)=0$. Then optimal quantity schedule $g(\theta)$ that maximizes (6) subject to (8) is given by:*

$$g(\theta)=\min\{\theta,q(r^{-1}(\theta))\} \ \ \textit{if} \ \ \theta\leq r(1)$$
$$g(\theta)=\theta \ \ \textit{if} \ \ \theta>r(1) \quad (9)$$

Proof: ¿From (6) and (8) it is easy to see that, under any schedule $q(.)$, it is optimal to set $g(\theta) \leq 1 \ \forall \theta \in [0,1]$. Therefore, we can impose $g(\theta) \leq 1$ as an additional constraint on (6) without loss of generality. Then by (ii) of lemma 5, the family $ICT(\theta, \theta')$ of incentive constraints in (8) can be replaced with the following family: $q(\theta) \geq g(r(\theta)) \ \forall \theta \in [0,1]$. Then the result follows by inspection of the principal's expected profit function (6). *Q.E.D.*

Lemma 6 allows us to reduce the dimensionality of the principal's maximization problem. Since $g(.)$ is completely determined by $q(.)$ according to (9), $q(.)$ remains the only choice variable. Imposing the additional constraints $(iii) - (v)$, which is without loss of generality, and using (9), we conclude that the problem of maximizing (6) subject to (7) and (8) is equivalent to the following problem:

$$\max_{q(\theta)\geq 0} \int_0^{r(1)} \left(\tilde{\theta} \min\{\tilde{\theta}, q(r^{-1}(\tilde{\theta}))\} - \frac{\left(\min\{\tilde{\theta}, q(r^{-1}(\tilde{\theta}))\}\right)^2}{2} \right) d\tilde{\theta} + \int_{r(1)}^1 \frac{s^2}{2} ds$$
$$+ \int_0^1 \left(\theta q(\theta) - \frac{q(\theta)^2}{2} - (1-\theta) q(\theta) \right) d\theta \qquad (10)$$
$$\textit{subject to}: \ q(.) \ \textit{is nondecreasing}, \ \textit{continuous}, \ q(1) = 1$$

To solve this problem, we will first relax the constraint that $q(.)$ is nondecreasing. Later we will check that the solution to the relaxed problem satisfies this constraint. We would like to reinterpret (10) as an optimal control problem and solve it using Pontryagin's Maximum Principle. This turns out to be possible if we use the derivative $q'(.)$ as the control variable. Then, to apply the Maximum Principle, we have to restrict $q'(.)$ to be piecewise continuous or, equivalently, to assume that $q(.)$ belongs to the space $C_p^1([0,1])$ of piecewise smooth (continuous and piecewise continuously differentiable) functions on $[0,1]$. Yet, we have only established above that $q(.)$ is a continuous function i.e., $q(.) \in C([0,1])$ and that $q'(.)$ exists almost everywhere. Nevertheless, the following lemma demonstrates that we can without loss of generality assume that $q(.) \in C_p^1([0,1])$.

Lemma 7 *Suppose that $q^*(\theta)$ is a solution to maximization problem (10) on the domain $C_p^1([0,1])$. Then $q^*(\theta)$ also maximizes (10) on the domain $C([0,1])$.*

Proof: see the Appendix.

Lemma 7 implies that if we find a solution $q^*(.)$ to (10) on the domain $C_p^1([0,1])$, then $q^*(.)$ will also be a solution to this problem on a larger space $C([0,1])$. So, let us assume that $q(.) \in C_p^1([0,1])$.

Note that $r(\theta)$ is differentiable whenever $q(\theta)$ is, and $r'(\theta) = \frac{q'(\theta)U(\theta)}{q(\theta)^2} = \frac{q'(\theta)(\theta - r(\theta))}{q(\theta)}$. Using this expression and making a change of variables in the first integral of (10), we can rewrite the principal's problem as follows:

$$\max_{q(.)\geq 0} \int_0^1 \left(r(\theta)\min\{r(\theta), q(\theta)\} - \frac{(\min\{r(\theta), q(\theta)\})^2}{2} \right) \frac{q'(\theta)(\theta - r(\theta))}{q(\theta)} d\theta$$

$$+ \int_{r(1)}^1 \frac{s^2}{2} ds + \int_0^1 (2\theta - 1)q(\theta) - \frac{q(\theta)^2}{2} d\theta \tag{11}$$

$$\text{subject to: } \quad r'(\theta) = \frac{q'(\theta)}{q(\theta)}(\theta - r(\theta)) \tag{12}$$

(11) and (12) constitute an optimal control problem with control variable $q'(\theta)$, state variables $q(\theta)$ and $r(\theta)$, and 'scrap value' equal to $\int_{r(1)}^1 \frac{s^2}{2} ds = \frac{1 - r(1)^3}{6}$ at $\theta = 1$, and constraints $r(0) = 0$ and $q(1) = 1$. We will omit the constraint $q(\theta) \geq 0$ for now and check that it is satisfied later.

The following is the Hamiltonian of this problem with costate variables $\lambda(\theta)$ and $\delta(\theta)$ associated with the laws of motion of $q(\theta)$ and $r(\theta)$ respectively:

$$H(q, r, \lambda, \delta, \theta) = \left(r\min\{r, q\} - \frac{(\min\{r, q\})^2}{2} \right) \frac{q'(\theta - r)}{q} + (2\theta - 1)q - \frac{q^2}{2} + \lambda q' + \delta \frac{q'(\theta - r)}{q} \tag{13}$$

The transversality conditions on the costate variables are: $\quad \lambda(0) = 0, \quad \delta(1) = -\frac{r(1)^2}{2}$.

The necessary first-order conditions on any given interval depend on whether on this interval $q(\theta) > r(\theta)$ (case 1) or $q(\theta) < r(\theta)$ (case 2) or $q(\theta) = r(\theta)$ (case 3).

There are no open intervals such that $q(\theta) = r(\theta)$ for all θ in the interval (case 3). Indeed, on such an interval we have $q'(\theta) = r'(\theta)$, which in combination with (12) implies that $r(\theta) = q(\theta) = \theta/2$. Then, the following first-order conditions have to be satisfied:

$$-\lambda'(\theta) = \frac{\partial H}{\partial q} = -\frac{q'(\theta - r)}{2} + 2\theta - 1 - q - \delta\frac{q'}{q}$$

$$-\delta'(\theta) = \frac{\partial H}{\partial r} = -q'\left(\theta - \frac{3r}{2} - \frac{\delta}{r}\right)$$

$$0 = \frac{\partial H}{\partial q'} = \frac{r^2}{2} + \delta + \lambda$$

It is easy to show that these first-order conditions cannot hold simultaneously if $r = q = \theta/2$.

Case 1: $q(\theta) > r(\theta)$. The necessary first-order conditions are:

$$-\lambda'(\theta) = \frac{\partial H}{\partial q} = -\frac{r^2}{2}\frac{q'(\theta - r)}{q^2} + (2\theta - 1) - q - \delta\frac{q'(\theta - r)}{q^2}$$

$$-\delta'(\theta) = \frac{\partial H}{\partial r} = r\frac{q'(\theta - r)}{q} - \left(\frac{r^2}{2} + \delta\right)\frac{q'}{q}$$

$$0 = \frac{\partial H}{\partial q'} = \left(\frac{r^2}{2} + \delta\right)\frac{\theta - r}{q} + \lambda$$

The unique solution to these equations is: $\delta(\theta) = -\frac{r^2(\theta)}{2}$, $\lambda(\theta) = 0$, $q = 2\theta - 1$, $r(\theta) = \frac{\theta^2+c}{2\theta-1}$ where c is a constant of integration which will be determined later.

Substituting the values of λ and δ into (13), it is easy to check that $H(q, r, \lambda, \delta, \theta)$ is concave. Hence, sufficient conditions for optimality are also satisfied.

Case 2: $q(\theta) < r(\theta)$. The first-order conditions are:

$$-\lambda'(\theta) = \frac{\partial H}{\partial q} = -\frac{q'(\theta - r)}{2} + (2\theta - 1) - q - \delta\frac{q'(\theta - r)}{q^2} \tag{14}$$

$$-\delta'(\theta) = \frac{\partial H}{\partial r} = q'(\theta + q/2 - 2r) - \delta\frac{q'}{q} \tag{15}$$

$$0 = \frac{\partial H}{\partial q'} = (r - q/2)(\theta - r) + \delta\frac{\theta - r}{q} + \lambda \tag{16}$$

To solve this system of the first-order conditions, combine (14) and (15) to get:

$$-\left(\lambda(\theta) + \delta(\theta)\frac{\theta - r}{q}\right)' = -\frac{q'(\theta - r)}{2} + 2\theta - 1 - q + q'(\theta + q/2 - 2r)\frac{\theta - r}{q} - \frac{\delta(\theta)}{q} \tag{17}$$

On the other hand, differentiating (16) we get:

$$-\left(\lambda(\theta) + \delta(\theta)\frac{\theta - r}{q}\right)' = -\frac{q'(\theta - r)}{2} + (r - q/2) + q'(\theta + q/2 - 2r)\frac{\theta - r}{q} \tag{18}$$

Equating the right-hand sides of (17) and (18) we get:

$$\delta = q(2\theta - 1 - q/2 - r) \tag{19}$$

Differentiating (19) we obtain:

$$\delta' = q'(2\theta - 1 - q/2 - r) + q\left(2 - q'/2 - \frac{q'(\theta - r)}{q}\right) = 2q + q'(\theta - 1 - q) \tag{20}$$

Finally, equate the right-hand sides of (15) and (20) and use (19) to obtain:

$$q'r = 2q \tag{21}$$

Thus, we have a system of differential equations consisting of (21) and 'the law of motion':

$$r' = \frac{q'(\theta - r)}{q} \tag{22}$$

Before attempting to solve this system, we need at first to characterize the regions where Cases 1 and 2 apply. This is done in the following lemma:

Lemma 8 *(i)* $\exists\overline{\theta}$ *such that Case 2 applies on* $[0, \overline{\theta})$*, and Case 1 applies on* $(\overline{\theta}, 1]$*.*
(ii) $q(\theta) > 0\ \forall\theta > 0$*.*

Proof: see the Appendix.

Let us now solve for $r(\theta)$ and $q(\theta)$ on $[0,\overline{\theta})$. Combining (21) with (22) we get:

$$r' = 2\frac{\theta - r}{r} = 2\frac{\theta}{r} - 2 \tag{23}$$

which can be rewritten as:

$$r'r + 2r = 2\theta \tag{24}$$

By lemmas 5 and 8, the appropriate initial condition for (24) is $r(0) = 0$. Under this initial condition, (24) has two solutions $r(\theta) = (\sqrt{3}-1)\theta$ and $r(\theta) = -(\sqrt{3}+1)\theta$. Only the first solution applies in our case, because $r(\theta)$ must be nonnegative. It remains to show that there are no other nonnegative solutions. This requires a proof because $(0,0)$ is a point of singularity of (24).

To argue by contradiction, suppose that (24) has two non-negative solutions $r_1(\theta)$ and $r_2(\theta)$. Let $s(.) = \frac{r^2(.)}{2}$. Then we can rewrite (24) as follows:

$$s' = 2\theta - \sqrt{8s} \tag{25}$$

The solutions $r_1(.)$ and $r_2(.)$ give rise to two different solutions $s_1(.)$ and $s_2(.)$ to (25) with the initial condition $s(0) = 0$. Therefore, $\exists \theta_0 \in (0,1)$ s.t. without loss of generality $s_1(\theta_0) > s_2(\theta_0)$.

Note that $s_1(\theta) > 0$ and $s_2(\theta) > 0$ $\forall \theta > 0$. At any point (s,θ) s.t. $s > 0$, $\theta > 0$ (25) is a regular ordinary differential equation, and its right-hand side $2\theta - \sqrt{8s}$ is continuous and Lipschitz in s on some open neighborhood of (s,θ). Therefore, only one solution can pass through any such point (s,θ) (see Coddington (1989), Theorem 1, p. 223), so that $s_1(\theta) > s_2(\theta)$ $\forall \theta \in (0,\theta_0]$. Then,

$$s_1(\theta_0) - s_2(\theta_0) = \int_0^{\theta_0} s_1'(\theta) - s_2'(\theta) d\theta = \int_0^{\theta_0} \sqrt{8s_2(\theta)} - \sqrt{8s_1(\theta)} d\theta < 0$$

This contradiction proves the uniqueness of solution to (24).

Using $r(\theta) = (\sqrt{3}-1)\theta$ to solve (21), we obtain $q = k\theta^{\sqrt{3}+1}$ where k is a constant of integration which will be determined later. Using (16) and (19) we get:
$\delta = k\theta^{\sqrt{3}+1}\left((3-\sqrt{3})\theta - \frac{k}{2}\theta^{\sqrt{3}+1} - 1\right)$ and $\lambda = \sqrt{3}\left(k\theta^{\sqrt{3}+1} + 1 - 2\theta\right)$. Substituting these expressions into the Hamiltonian, it is easy to check that the latter is concave. Hence, Arrow's sufficient conditions for optimality are satisfied.

Since $q(.)$ and $r(.)$ are continuous on $[0,1]$, and $r(\overline{\theta}) = q(\overline{\theta})$, we get: $q(\overline{\theta}) = 2\overline{\theta} - 1 = r(\overline{\theta}) = (\sqrt{3}-1)\overline{\theta}$. So, $\overline{\theta} = \frac{1}{3-\sqrt{3}}$. Finally, by pasting $r(.)$ and $q(.)$ at $\overline{\theta}$ continuously we compute k and c in the solutions for $q(\theta)$ and $r(\theta)$ in cases 2 and 1 respectively. From $q(\overline{\theta}) = k\theta^{\sqrt{3}+1} = 2\overline{\theta} - 1$ we obtain: $k = \sqrt{3}^{\sqrt{3}}(\sqrt{3}-1)^{\sqrt{3}+1}$, and from $r(\overline{\theta}) = q(\overline{\theta})$ we get $c = -1/6$. *Q.E.D.*

4 Non-Binary Costs and Multiple Messages.

Up to now, we have considered situations in which an agent's cost of sending a particular message could take only two possible values: zero (a feasible message) or infinity (an infeasible message). In this section, we consider more general cost structures. However, in contrast to the environments studied in the previous section, we assume that there are no costless nontruthful messages.

To simplify the model, we impose a restriction on the admissible communication set $\mathcal{C}$ defined above as *a set of statements about the true state of the world, or messages, that can be made by an agent and understood by the principal.* We assume that $\mathcal{C} = \Theta$ i.e., the only messages that are allowed in the mechanism are the agent's statements about her privately known preference parameter θ. We assume that an agent with true preference parameter θ who sends message $\hat{\theta}$ incurs cost $c(\hat{\theta}, \theta)$ which has the following properties. Stating the truth is costless i.e. $c(\theta, \theta) = 0$, but sending messages misrepresenting the truth is costly i.e., $c(\hat{\theta}, \theta) > 0 \ \forall \hat{\theta} \neq \theta$.

This framework is similar to the ones studied by Lacker and Weinberg (1989) and Maggi and Rodriguez-Clare (1995). These authors assume, albeit implicitly, that an agent can incur communication cost at most once. This view is consistent with an interpretation that the agent incurs a physical cost, such as hiding the crop, to distort the signal received by the principal.

However, an alternative scenario under which an agent incurs communication cost more than once is also plausible. An agent may have to support each message with a different piece of evidence. Then each nontruthful message would require a costly distortion of the corresponding piece of evidence.

Similarly, an agent may blush or suffer an emotional distress each time she has to tell a lie. Then, as we have discussed in the previous section, the principal may require the agent to report the same information several times, sometimes in different forms, or hire several deputies to request the same information from the agent. For example, a person normally has to report her income on tax forms, college financial aid forms, loan and mortgage applications.

So, in contrast to Lacker and Weinberg (1989) and Maggi and Rodriguez-Clare (1995), we consider an environment in which an agent incurs a separate communication cost each time she sends a message, irrespective of the other messages that she also sends and, thus, irrespective of whether the other messages are different or identical. Formally, this implies that the payoff structure is as follows. If an agent with utility parameter θ sends n messages $\hat{\theta}_1, ..., \hat{\theta}_n$ and the mechanism selects allocation $x \in X$, then the agent obtains the following payoff:

$$u(x, \theta) + \sum_{i=1}^{n} c(\hat{\theta}_i, \theta) \tag{26}$$

We assume that the set X is compact, $u(x, \theta)$ is continuous in x, and there exists a 'worst' outcome $\underline{x}$ which minimizes $u(., \theta) \ \forall \theta$[6].

Our first result demonstrates that the absence of costless nontruthful messages has surprisingly strong implication for the set of implementable allocations.

Theorem 4 *Suppose that the space of utility parameters Θ is finite. Then there exists $n < \infty$ such that if the principal can offer a mechanism in which the agent has to report her utility*

[6] In a more general model, communication costs may also depend upon a separate parameter t, so that agents with the same preference parameter θ but different t may have different communication costs.

parameter θ at least n times, then every allocation profile $x : \Theta \to X$ is implementable, at zero communication cost.

Proof: Let $\varepsilon = \min_{\hat{\theta} \neq \theta} c(\hat{\theta}, \theta)$, $H = \max_{\theta} \max_{x' \neq x} |u(x', \theta) - u(x, \theta)|$. Note that the continuity of $u(x, \theta)$ in x, the compactness of X, and the finiteness of Θ imply that H is finite. Let n be the smallest integer that is no less than $\frac{H}{\varepsilon}$. Fix an allocation profile $x(\theta)$, and consider a mechanism in which the agent is asked to report her true utility parameter n times. Given any vector of announcements $(\hat{\theta}_1, ..., \hat{\theta}_n)$, the mechanism assigns allocation $g(\hat{\theta}_1, ..., \hat{\theta}_n)$ s.t.:

$$g(\hat{\theta}_1, ..., \hat{\theta}_n) = \begin{cases} \underline{x}, & \text{if } \exists j \neq i \text{ such that } \hat{\theta}_i \neq \hat{\theta}_j \\ x(\hat{\theta}_1), & \text{otherwise.} \end{cases} \tag{27}$$

Clearly, the agent cannot improve her payoff by sending a vector of non-coincident messages. Furthermore, if type θ reports a vector containing $\hat{\theta} \neq \theta$ as each element, her payoff is $u(x(\hat{\theta}), \theta) - n \times c(\hat{\theta}, \theta)$. But note that

$$u(x(\hat{\theta}), \theta) - n \times c(\hat{\theta}, \theta) \leq u(x(\hat{\theta}), \theta) - n\varepsilon \leq u(x(\theta), \theta),$$

where the last inequality holds by the choice of n. We conclude that any allocation profile $x : \Theta \to X$ is implementable. Furthermore, since each agent type reports the truth n times, no communication costs are incurred. *Q.E.D.*

No matter how small the cost of sending a nontruthful message is, as long as this cost remains bounded away from zero, repeated sending of such messages makes nontruthful reporting very costly. Then by requiring the agent to send a sufficiently large number of messages, the mechanism makes the agent's cost of imitating any other type prohibitively large, regardless of the allocation to be implemented. Thus, theorem 4 implies that when the principal has the ability to elicit multiple messages, the only barrier to implementation is the existence of costless nontruthful messages.

Finiteness of the utility parameter space Θ played a crucial role in the above argument. Suppose now that space Θ has infinitely many elements. By requiring the agent to send a sufficiently large number of messages, it is easy to guarantee that nonlocal incentive constraints are satisfied i.e. the agent of type θ is not willing to imitate any other type that is sufficiently different. However, local incentive constraints preventing the agent from imitating agents whose utility parameters are 'close' to hers, could pose problems if the number of messages has to remain finite. Yet, our next theorem shows that all allocation profiles that satisfy a mild regularity condition are implementable.

We will henceforth assume that Θ is a compact subset of some finite-dimensional Euclidean space, and that $c : \Theta \times \Theta \to \mathbf{R}_+$ is continuous. We also make the following three assumptions:

Assumption 1 *There exists $L < \infty$ s.t. $\|x(\theta') - x(\theta)\| \leq L\|\theta' - \theta\|$, $\forall \theta', \theta \in \Theta$.*

Assumption 2 *There exists $\underline{c} > 0$ s.t. $c(\theta', \theta) \geq \underline{c}\|\theta' - \theta\|$, $\forall \theta', \theta \in \Theta$.*

Assumption 3 *There exists $K < \infty$ s.t. $|u(x', \theta) - u(x, \theta)| \leq K\|x' - x\|$, $\forall \theta \in \Theta$.*

Assumption 1 requires the allocation to be Lipschitz continuous with Lipschitz constant L. Assumption 2 puts a lower bound on the cost of lying. Finally, Assumption 3 strengthens the continuity requirement on the agent's utility function to uniform continuity in x. Since $X \times \Theta$ is compact, Assumption 3 will hold if $u(.,.)$ is C^1. Under these conditions, we have:

Theorem 5 *Suppose that Assumptions 2 and 3 hold. There exists $n < \infty$ such that if the principal can offer a mechanism in which the agent has to report her utility parameter at least n times, then any allocation profile $x(\theta)$ satisfying Assumption 1 is implementable with zero communication cost.*

Proof: Fix allocation $x(\theta)$ that satisfies Assumption 1. Let n be the smallest integer that is no less than $KL/\underline{c}$. Consider the same mechanism as in Theorem 4. This mechanism implements allocation profile $x(\theta)$ if the following incentive constraint is satisfied $\forall \theta, \hat{\theta}$: $u(x(\hat{\theta}), \theta) - nc(\hat{\theta}, \theta) \leq u(x(\theta), \theta)$. But this inequality holds, since
$\left| u(x(\hat{\theta}), \theta) - u(x(\theta), \theta) \right| \leq KL\|\hat{\theta} - \theta\| \leq n\underline{c}\|\hat{\theta} - \theta\| \leq c(\hat{\theta}, \theta)$. *Q.E.D.*

Intuitively, local incentive constraints hold because the marginal benefit of lying is finite, but the marginal cost of announcing the utility parameter nontruthfully is bounded from below by $\underline{c}$. Then requesting the agent to report her utility parameter sufficiently many times ensures the optimality of truth-telling.

The reasoning we just gave suggests that the set of implementable allocation profiles may be more restricted when the marginal cost of announcing utility parameter nontruthfully is zero at the true type, i.e. $\frac{\partial c}{\partial \hat{\theta}}(\hat{\theta}, \theta)|_{\hat{\theta}=\theta} = 0$. Indeed, implementation in this case will generally require that the agent send nontruthful messages. This raises the issue of constructing optimal mechanisms, especially their communication stages, and leaves open the possibility that communication costs may become substantial.

To gain insight into this problem, we will specialize our model to the case where the type space is one-dimensional and set $\Theta = [\bar{\theta}, \underline{\theta}]$. We partition the outcome x into a production assignment $q \in Q$, where Q is a compact subset of $\mathbf{R}^{k-1}$, and a transfer p. Thus, $x = (q, p)$. For simplicity, we also assume that the agent's utility function is quasilinear in transfer, i.e.:

$$u(x, \theta) = v(q, \theta) + p$$

We restrict attention to communication cost functions that depend only on the distance between the true and the announced utility parameters:

$$c(\hat{\theta}, \theta) = C(|\hat{\theta} - \theta|)$$

Finally, we make the following assumptions on $C(\cdot)$ and $v(\cdot, \cdot)$:

Assumption 4 *$C(\cdot)$ is twice continuously differentiable, with $C(0) = C'(0) = 0$, and $0 < \underline{\omega} \leq C''(x) \leq \bar{\omega} < \infty$.*

Assumption 5 *$v(q, \theta)$ is twice continuously differentiable.*

We then have the following "approximate implementation" theorem:

Theorem 6 *For every continuously differentiable allocation profile $\{q(\theta), p(\theta)\}$ and every $\varepsilon > 0$, there exist $n < \infty$ and transfer rule $p^{\varepsilon}(\theta)$ satisfying $|p^{\varepsilon}(\theta) - p(\theta)| < \varepsilon$ $\forall\theta$ such that the allocation profile $\{q(\theta), p^{\varepsilon}(\theta)\}$ is implementable via a mechanism in which the agent sends n messages. The total communication cost incurred by an agent in this mechanism does not exceed ε.*

Proof: see the Appendix.

In the mechanism that implements allocation profile $\{q(\theta), p^{\varepsilon}(\theta)\}$ an agent with utility parameter θ makes n_1 truthful and n_2 nontruthful announcements i.e., reports θ n_1 times, and also reports $\tilde{s}(\theta) \neq \theta$ n_2 times. Then the first-order condition necessary for local incentive compatibility is:

$$\frac{dp^{\varepsilon}(\theta)}{d\theta} = -\nabla_q v(q(\theta), \theta)q'(\theta) + n_2 sgn(\tilde{s}(\theta) - \theta)C'(|\tilde{s}(\theta) - \theta|)\tilde{s}'(\theta) \tag{28}$$

By choosing n_2 and $\tilde{s}(\theta)$ appropriately we can ensure that (28) holds for arbitrary $(q(\theta), p^{\varepsilon}(\theta))$.

Because $C'(0) = 0$, truthful announcements do not have any effect on (28). So, in the absence of costly messages the second term in (28) would be zero, in which case (28) would impose a restriction on the set of implementable allocations in the form of a link between $q(\theta)$ and $p^{\varepsilon}(\theta)$. Costly messages eliminate the need for any such link.

Further, exploiting the convexity of $C(.)$ we can choose n_2 sufficiently large that (28) holds, yet the agent's cost of communication $n_2 C(|\tilde{s}(\theta) - \theta|)$ is arbitrarily small.

Finally, a sufficiently large number n_1 of costless truthful messages guarantees that the agent's payoff is quasi-concave in her reported utility parameter $\hat{\theta}$ (see the proof for details). Thus, truthful messages allow the second-order conditions to be satisfied and ensure global incentive compatibility without requiring any restrictions on the set of implementable allocation profiles.

Let us now compare theorem 6 with the results in Maggi and Rodriguez-Clare (1995) (which will be referred to as MR below) who study a similar model under the restriction that the agent makes only one announcement regarding her type θ. Although MR focus on finding the optimal (profit-maximizing) mechanism, and we are concerned with implementablility, comparing the necessary and sufficient conditions for implementation is instructive, since it allows to appreciate the full effect of communication involving multiple messages.

The necessary first-order condition for implementation in MR is given by condition (N) in Lemma 1 (p. 679). It is equivalent to (28) if we put $n_2 = 1$ and $\tilde{s}(\theta) \geq \theta$. Thus, costly communication in MR also allows to weaken the link between production assignment $q(\theta)$ and the corresponding transfer, and hence to implement a larger set of social choice functions. But the degree to which this link is weakened is limited by the magnitude of the incurred communication costs, which can be quite large when $n_2 = 1$. In contrast, when n_2 is unrestricted, as in our model, co-dependence between $q(\theta)$ and $p(\theta)$ is eliminated at a very small cost.

Finally, consider the second-order conditions for implementation. To satisfy them, MR have to impose the following restrictions: $q'(\theta) \geq 0$ and $\tilde{s}'(\theta) \geq 0$. (p. 679). In contrast, as explained above, in our model second-order conditions hold because the agent also has to make a sufficiently large number of truthful announcements of her utility parameter θ.

Consequently, we are able to implement any continuously differentiable allocation profile at a small (communication) cost.

The results of this section have a number interesting implications for screening and signaling. In order to exhibit these applications, consider a standard job-market screening problem i.e., an employer hiring an employee of unknown ability. It many real-world situations employers ask job-candidates to undergo through a number of tests, such as interviews. Such tests, or interviews, can be regarded as multiple signals or messages send by the employee. Applying the results of this section, we conclude that the problem of asymmetric information regarding the employee's ability can be overcome, i.e. the employer can learn the employee's ability at almost zero cost and with small efficiency losses, if the tests/interviews that the job-candidate goes through can be designed to have the following properties: (i) An employee's performance on one test is independent, on her performance on the other tests (This condition can be relaxed. The results remain valid if the dependence is not too strong). (ii) Each test identifies the employee's ability accurately, if the employee does not attempt to manipulate the results of the test by expending effort. (iii) The employee incurs zero (very small) cost when she does not attempt to misrepresent her type, and the cost of effort is positive and increasing in the magnitude of desired misrepresentation on a test.

Thus, our analysis allows to explain why in many situation the employers do not offer menus of contracts to the employees in order to screen them according to their ability, as suggested by the mechanism design approach. Instead, the employers carefully design the interviewing process. Indeed, the interviewing process in many professional job-markets appears to be consistent with the idea of requesting multiple messages/signals from the agent, with each signal being somewhat different from the others. For example, in the context of a departmental visit on the academic job-market a prospective candidate meets with faculty members working in different fields. It is conceivable that each conversation provides an independent signal of the candidate'a ability, because different faculty members, especially if they work in different fields, assess the candidate from different perspectives. Our results imply that if a job candidate has to go through sufficiently many such interviews, or other tests, then even if the candidate is able and willing to exert effort to misrepresent his/her ability, doing so will be too costly. So, a Department will have a very accurate estimate of the candidate's ability. Then it would not be necessary to do further on-the-job screening by offering a menu of possible contracts.

In some environments where signals are sent in a sequence it is likely that the agent's cost of each particular signal is affected by the previous ones. For example, suppose that a signal is an agent's performance on a specific test (job interview), and that the agent can misrepresent her ability by exerting costly effort (studying). Then the amount of effort that an agent with ability θ needs to exert in the n-th test to perform at a level corresponding to ability θ'_n could depend on the efforts that she took to prepare for the previous tests. Thus, the cost of the n-th signal is given by $c_n(\theta, \theta'_1, ..., \theta'_n)$.

It is easy to show that our results still hold if the effect of the true ability θ on the cost (effort) required to send signal θ'_n does not go to zero "too quickly' in n. For example, they remain valid if $c_n(\theta, \theta'_1, ..., \theta'_n) \geq \frac{c(\theta, \theta'_n)}{n}$ where $c(.,.)$ is some function satisfying the original conditions.

5 Conclusions

We have demonstrated that in environments with communication costs the ability of the principal to offer mechanisms in which an agent sends several messages significantly expands the set of implementable outcomes.

The results of the paper have a number of interesting implications. In particular, we have shown that an individual's communication abilities can play an important role in determining her payoff. Further, we have also shown that it may not be optimal to punish individuals who make conflicting statements and, thus, can be easily accused of lying. By not punishing individuals who make such conflicting statements the mechanism prevents them from imitating others, which enhances the overall social efficiency of the outcome. We intend to explore this point further in our research.

6 Appendix

Proof of lemma 1:

(i) First note that individual rationality constraints (4) are binding for all θ in the optimal mechanism. Suppose that $g(\theta) > \theta$ for some $\theta \in [0, 1]$. Then the value of the integrand in (5) can be increased by setting $g(\theta) = \theta$ and reducing the transfer $t^\tau(\theta)$ appropriately to make (4) binding. It is easy to see that incentive constraints (2) still hold for (θ', θ) $\forall\theta' \in [0, 1]$.

Suppose that $g(\theta_2) > g(\theta_1)$ for some $\theta_2 < \theta_1$. Then $(\theta - \theta_2)g(\theta_2) > (\theta - \theta_1)g(\theta_1)$ $\forall\theta > \theta_2$. Therefore, incentive constraints (2) are slack for all $\theta > \theta_1$. In this case, it is easy to see from (5) that optimality implies that $g(\theta_1) = \theta_1$. But then $g(\theta_2) > \theta_2$ which contradicts the property that $g(\theta) \leq \theta$.

(ii) The proof that $q(\theta)$ that satisfies incentive constraints (1) must be non-decreasing is standard and is therefore omitted.

Suppose that $q(\theta) > 1$ for some $\theta \in [0, 1]$. Let $\hat{\theta} = \inf\{\theta | q(\theta) > 1\}$. Since $q(.)$ is non-decreasing, $q(\theta) > 1$ $\forall\theta \in (\hat{\theta}, 1]$. Consider modified quantity-transfer schedules $(\tilde{q}(\theta), \tilde{t}^s(\theta))$ s.t. $\tilde{q}(\theta) = q(\theta)$, $\tilde{t}^s(\theta) = t^s(\theta)$ for $\theta \in [0, \hat{\theta})$, and $\tilde{q}(\theta) = 1$, $\tilde{t}^s(\theta) = \hat{\theta}(1 - q(\hat{\theta})) + t^s(\hat{\theta})$ for $\theta \in [\hat{\theta}, 1]$. It is easy to see that $(\tilde{q}(\theta), \tilde{t}^s(\theta))$ satisfies all incentive constraints. The quantity allocation $\tilde{q}(\theta)$ is more efficient. Furthermore, it is easy to show that a 'strategic' agent with valuation above $\hat{\theta}$ now earns a lower payoff under the modified schedule. Therefore, by offering this schedule the principal obtains higher profits.

Since $q(\theta)$ is non-decreasing and bounded, it is Riemann integrable by Theorem 6.9, p.126 in Rudin (1976) and a.e. differentiable by Theorem 3, p.100 in Royden (1987).

By Theorem 4.30, p.96 in Rudin (1976) $q(\theta)$ has at most countably many points of discontinuity. Right-hand and left-hand limits exist at all discontinuity points of $q(\theta)$ by Theorem 4.29, p.95 in Rudin (1976).

Suppose that $q(1) = \mu < 1$. Since $q(\theta)$ is nondecreasing, $q(\theta) \leq \mu$ $\forall\theta \in [0, 1)$. Inspecting (5) it is easy to see that the principal gets a higher payoff by modifying the mechanism and setting $q(\theta) = \frac{1+\mu}{2}$, $t^s(\theta) = t^s(\frac{3+\mu}{4}) + \frac{3+\mu}{4}(\frac{1+\mu}{2} - q(\frac{3+\mu}{4}))$ for $\theta \in [\frac{3+\mu}{4}, 1]$.

(iii) The family of incentive constraints (1) implies that $U(\theta) \geq U(\theta') + (\theta - \theta')q(\theta')$ and

$U(\theta') \geq U(\theta) + (\theta' - \theta)q(\theta)\ \forall \theta, \theta'$. Combining these two inequalities we have:

$$q(\theta)(\theta - \theta') \geq U(\theta) - U(\theta') \geq q(\theta')(\theta - \theta').$$

Since $q(\theta)$ is bounded and Riemann integrable, the above implies that $U(\theta)$ is uniformly and absolutely continuous. Therefore, by Theorem 14, p.110 in Royden (1987), $U(\theta) - U(\theta') = \int_{\theta'}^{\theta} q(s)ds$ and $U'(\theta) = q(\theta)$ a.e. *Q.E.D.*

Proof of lemma 2: Suppose that optimal quantity schedule $q(\theta)$ is discontinuous at $x \in (0,1)$. Let $q(x-)$ and $q(x+)$ be left-hand and right-hand limits of $q(\theta)$ at x respectively. By lemma 1, $q(x-)$ and $q(x+)$ exist. Thus, we have $q(x-) = q(x+) - 2\delta$ for some $\delta > 0$.

We will show that the principal can increase her expected profit by offering a mechanism with modified quantity schedule $\hat{q}(\theta)$ s.t. for appropriately chosen $\epsilon > 0$: (i) $\hat{q}(\theta) = q(\theta)$ for $\theta \in (0, x-\epsilon) \cup (x+\epsilon, 1]$, (ii) $\hat{q}(\theta) = q(\theta) + \delta$ for $\theta \in [x-\epsilon, x)$, (ii) $\hat{q}(x) = q(x-) + \delta$, (iii) $\hat{q}(\theta) = q(\theta) - \delta$ for $\theta \in (x, x+\epsilon]$.

After this modification, the agent earns informational rent $\hat{U}(\theta) = U(0) + \int_0^{\theta} \hat{q}(s)ds$. It is easy to see that $\hat{U}(\theta) \geq U(\theta)$. Therefore, all incentive constraints in (8) still hold.

¿From (6), the change in the firm's expected profit after this modification is equal to:

$$\int_{x-\epsilon}^{x} (2\theta - 1)(q(\theta) + \delta) - \frac{(q(\theta) + \delta)^2}{2} d\theta + \int_{x}^{x+\epsilon} (2\theta - 1)(q(\theta) - \delta) - \frac{(q(\theta) - \delta)^2}{2} d\theta$$
$$- \int_{x-\epsilon}^{x} (2\theta - 1)q(\theta) - \frac{q(\theta)^2}{2} d\theta - \int_{x}^{x+\epsilon} (2\theta - 1)q(\theta) - \frac{q(\theta)^2}{2} d\theta$$
$$= \delta \left(\int_{x-\epsilon}^{x} 2\theta d\theta - \int_{x}^{x+\epsilon} 2\theta d\theta \right) - \delta^2 \epsilon + \delta \left(\int_{x}^{x+\epsilon} q(\theta)d\theta - \int_{x-\epsilon}^{x} q(\theta)d\theta \right) \geq -2\delta\epsilon^2 + \delta^2\epsilon$$

The last inequality holds because $q(\theta_2) - q(\theta_1) \geq 2\delta$ when $\theta_1 < x < \theta_2$. If $\epsilon < \frac{\delta}{2}$, then $-2\delta\epsilon^2 + \delta^2\epsilon > 0$, and hence the principal gets a higher expected payoff under the modified quantity schedule $\hat{q}(\theta)$. Thus, a discontinuous quantity schedule cannot be optimal. *Q.E.D.*

Proof of lemma 3:

Suppose that in the optimal mechanism $U(0) = \underline{u} > 0$. Consider set $Z \subset \Theta$ s.t. $\theta \in Z$ iff

$$\underline{u} + \int_0^{\theta} q(x)dx = \sup_{\theta'} (\theta - \theta')g(\theta') \tag{29}$$

Set Z is non-empty, because otherwise the principal could reduce $U(0)$ and hence increase her expected profits without violating any of the incentive constraints in (8). Let $\hat{\theta}$ be the minimal element of Z. $\hat{\theta}$ exists because both the left-hand side and the right-hand side of (29) are continuous in θ.

Define $U^{\tau}(\theta) \equiv \sup_{\theta'} (\theta - \theta')g(\theta')$. Clearly, $U^{\tau}(\theta)$ is continuous and strictly increasing in θ. Let $\lambda \in (0,1)$. Then:

$$(\lambda\theta_1 + (1-\lambda)\theta_2 - \theta')g(\theta') = \lambda(\theta_1 - \theta')g(\theta') + (1-\lambda)(\theta_2 - \theta')g(\theta') \leq \lambda U^{\tau}(\theta_1) + (1-\lambda)U^{\tau}(\theta_2)$$

Therefore, $U^{\tau}(\lambda\theta_1 + (1-\lambda)\theta_2) \leq \lambda U^{\tau}(\theta_1) + (1-\lambda)U^{\tau}(\theta_2)$ i.e., $U^{\tau}(\theta)$ is convex.

Since $g(\theta) \leq 1$, we have $|U^\tau(\theta_1) - U^\tau(\theta_2)| \leq |\theta_1 - \theta_2|$ i.e., $U^\tau(\theta)$ is absolutely continuous. Therefore it possesses a.e. a nonnegative bounded derivative $\tilde{v}(\theta)$ s.t. $U^\tau(\theta) = \int_0^\theta \tilde{v}(x)dx$. Obviously, $\tilde{v}(\theta) \leq 1$. Since $U^\tau(\theta)$ is convex, $\tilde{v}(\theta)$ is non-decreasing. Therefore, at all θ the left-hand and the right-hand limits of $\tilde{v}(\theta)$ exist and are finite.

Let $v(\theta)$ be an extension of $\tilde{v}(\theta)$ to all of $[0, 1]$ defined in the following way: $v(\theta) = \tilde{v}(\theta)$ if $\tilde{v}(.)$ is defined at θ, and $v(\theta)$ is equal to the left-hand limit of $\tilde{v}(.)$ if $\tilde{v}(.)$ is not defined at θ. Then $U^\tau(\theta) = \int_0^\theta v(x)dx$, $v(\theta)$ is non-decreasing and does not exceed θ anywhere.

Note that $q(\hat{\theta}) \geq v(\hat{\theta})$. Otherwise, there exists $\theta_2 > \hat{\theta}$ s.t. $\underline{u} + \int_0^{\theta_2} q(x)dx < U^\tau(\theta_2)$ which is impossible because all incentive constraints in (8) hold by assumption. On the other hand, the left-hand limit of $v(\theta)$ at $\hat{\theta}$ denoted by $v(\hat{\theta}-)$ is such that $v(\hat{\theta}-) \geq q(\hat{\theta})$ because otherwise for some $\theta_3 < \hat{\theta}$, $\underline{u} + \int_0^{\theta_3} q(x)dx < U^\tau(\theta_3)$ which is impossible because all incentive constraints in (8) hold by assumption. Therefore, $v(\hat{\theta}-) = q(\hat{\theta})$.

Let us construct a modified quantity schedule $\hat{q}(\theta)$. If $v(\theta) \geq q(\theta)$ a.e. on $[0, \hat{\theta})$, then let $\hat{q}(\theta) = v(\theta)$ on $[0, \hat{\theta})$ and $\hat{q}(\theta) = q(\theta)$ on $[\hat{\theta}, 1]$. Then, $\hat{q}(\theta)$ is non-decreasing because both $q(\theta)$ and $v(\theta)$ are non-decreasing and $v(\hat{\theta}-) = q(\hat{\theta})$. Observe that $\int_0^\theta \hat{q}(x) - v(x)dx = \int_0^{\hat{\theta}} v(x) - q(x)dx - \int_0^\theta v(x) - q(x)dx \geq 0 \; \forall \theta \in [0, 1]$.

Now suppose that $v(\theta) < q(\theta)$ on a subset of $[0, \hat{\theta}]$ with a positive measure. Let $\theta_0 < \hat{\theta}$ be the minimal θ s.t. $\int_0^\theta \max\{v(x), q(x)\} - q(x)dx = \underline{u}$. Such θ_0 exists by the continuity of the integral in its upper limit.

If $q(\theta_0) \geq v(\theta_0-)$, then let $\hat{q}(\theta) = \max\{v(\theta), q(\theta)\}$ on $[0, \theta_0)$ and $\hat{q}(\theta) = q(\theta)$ on $[\theta_0, 1]$. $\hat{q}(.)$ is non-decreasing because both $q(.)$ and $v(.)$ are non-decreasing and $v(\theta_0-) \leq q(\theta_0)$. Obviously, $\int_0^\theta \hat{q}(x) - v(x)dx \geq 0 \; \forall \theta \in [0, 1]$.

If $q(\theta_0) < v(\theta_0-)$, then $\exists \theta_1, \theta_2$ s.t. $\theta_1 < \theta_0 < \theta_2 \leq \hat{\theta}$, $q(\theta_1) \geq v(\theta_1-)$, $q(\theta_2) = v(\theta_2-)$, and $q(\theta) < v(\theta) \; \forall \theta \in (\theta_1, \theta_2)$. Note that $\int_0^{\theta_1} \max\{v(x), q(x)\} - q(x)dx < \underline{u}$, while $\underline{u} < \int_0^{\theta_2} \max\{v(x), q(x)\} - q(x)dx$. Therefore, $\exists \alpha \in (0, 1)$ s.t.

$$\int_0^{\theta_1} \max\{v(x), q(x)\} - q(x)dx + \int_{\theta_1}^{\theta_2} (\alpha v(x) + (1 - \alpha)q(x)) - q(x)dx = \underline{u} \tag{30}$$

Define $\hat{q}(\theta) = \max\{v(\theta), q(\theta)\}$ on $[0, \theta_1)$, $\hat{q}(\theta_1) = q(\theta_1)$, $\hat{q}(\theta) = \alpha v(\theta) + (1 - \alpha)q(\theta)$ on (θ_1, θ_2), $\hat{q}(\theta) = q(\theta)$ on $[\theta_2, 1]$. Then, $\hat{q}(.)$ is non-decreasing because both $q(.)$ and $v(.)$ are non-decreasing, $q(\theta_1) \geq v(\theta_1-)$, $v(\theta) > q(\theta)$ on (θ_1, θ_2), $q(\theta_2) = v(\theta_2-)$. Let us show that $\int_0^\theta \hat{q}(x) - v(x)dx \geq 0 \; \forall \theta \in [0, 1]$. This is obvious if $\theta \in [0, \theta_1] \cup [\theta_2, 1]$. Now fix an arbitrary $\theta \in (\theta_1, \theta_2)$. Suppose that $\int_0^\theta \hat{q}(x) - v(x)dx < 0$. Then, since $\hat{q}(x) \leq v(x)$ on (θ_1, θ_2) we have $\int_0^{\theta_2} \hat{q}(x) - v(x)dx < 0$. But this contradicts the fact that $\int_0^{\theta_2} \hat{q}(x) - q(x)dx = \underline{u} \geq \int_0^{\theta_2} v(x) - q(x)dx$.

Suppose that the principal offers schedules $\hat{q}(.), g(.)$ and sets $U(0) = 0$. In this case (8) is satisfied $\forall \theta, \theta'$ because, as we have shown $\int_0^\theta \hat{q}(x) - v(x)dx \geq 0 \; \forall \theta$. Then, the principal's expected profits change by:

$$\underline{u} + \int_0^1 (2\theta - 1)(\hat{q}(\theta) - q(\theta)) - \frac{\hat{q}^2(\theta)}{2} + \frac{q^2(\theta)}{2} d\theta =$$
$$\int_0^1 2\theta(\hat{q}(\theta) - q(\theta)) - (\hat{q}(\theta) - q(\theta)) \left(\frac{\hat{q}(\theta) + q(\theta)}{2} \right) d\theta > 0$$

The equality follows from the fact that $\int_0^1 \hat{q}(x) - q(x)dx = \underline{u}$. The inequality follows because $q(\theta) \leq \theta$ and $\hat{q}(\theta) \leq \max\{v(\theta), q(\theta)\} \leq \theta$. *Q.E.D.*

Proof of lemma 4:

Suppose that $\exists \theta$ s.t. $q(\theta) = 0$ and $g(\theta) > 0$. Note that $t^\tau(\theta) = \theta g(\theta)$. By continuity of $q(\theta)$, $\exists \theta' > \theta$ s.t. $q(\theta') < g(\theta)$. Since $q(\theta)$ is nondecreasing, $U(\theta') = \int_0^{\theta'} q(s)ds < (\theta' - \theta)g(\theta)$ i.e., $ICT(\theta', \theta)$ in (8) fails.

Next, suppose that $g(\theta) = 0$ but $q(\theta) > 0$. Then, without violating any incentive constraints in (8), the principal can increase its profits by setting $g(\theta) = \min\{\theta, q(\theta)\}$ and $t^\tau(\theta) = g(\theta)\theta$. *Q.E.D.*

Proof of lemma 5:

(i) Since $q(\theta)$ is nondecreasing, $U(\theta) = \int_0^\theta q(s)ds \leq q(\theta)\theta$, and so $r(\theta) \geq 0$.

Let $\theta_2 > \theta_1$ and $q(\theta_1) > 0$. Then

$$r(\theta_2) - r(\theta_1) = \theta_2 - \theta_1 - \frac{q(\theta_1) \int_0^{\theta_2} q(s)ds - q(\theta_2) \int_0^{\theta_1} q(s)ds}{q(\theta_2)q(\theta_1)}$$
$$= \theta_2 - \theta_1 - \frac{\int_{\theta_1}^{\theta_2} q(s)ds}{q(\theta_2)} + \frac{(q(\theta_2) - q(\theta_1)) \int_0^{\theta_1} q(s)ds}{q(\theta_2)q(\theta_1)} \geq \frac{(q(\theta_2) - q(\theta_1)) \int_0^{\theta_1} q(s)ds}{q(\theta_2)q(\theta_1)} \geq 0$$

It is easy to see that both inequalities above are strict iff $q(\theta_2) > q(\theta_1)$.

Since $q(\theta)$ is continuous, $r(\theta)$ is continuous $\forall \theta$ s.t. $q(\theta) > 0$. Now consider $\hat{\theta}$ s.t. $q(\hat{\theta}) = 0$. By definition, $r(\hat{\theta}) = \hat{\theta}$. If $\exists \theta'' > \hat{\theta}$ s.t. $q(\theta'') = 0$, then by definition $r(\theta) = \theta$ on $[0, \theta'']$. Now suppose that $\hat{\theta} = \max\{\theta | q(\theta) = 0\}$. Since $q(\theta)$ is nondecreasing and so $q(\theta') = 0$ $\forall \theta' \leq \hat{\theta}$, we have $\frac{U(\theta)}{q(\theta)} \leq \theta - \hat{\theta}$ $\forall \theta > \hat{\theta}$. This implies that $\lim_{\theta \to \hat{\theta}+0} r(\theta) = \hat{\theta}$. Thus $r(.)$ is continuous at $\hat{\theta}$. Similarly, we can show that $r(0) = 0$.

(ii) Suppose that $ICT(\theta, \theta')$ in (8) hold $\forall \theta, \theta' \in [0, 1]$.

If $U(\theta) = 0$, then $q(\theta) = 0$ because otherwise $q(.)$ cannot be continuous, and $r(\theta) = \theta$. To show that $g(\theta) = 0$, suppose otherwise. Then by continuity of $q(.)$, $\exists \theta' > \theta$ s.t. $q(\theta') < g(\theta)$. Since $q(\theta)$ is nondecreasing, $U(\theta') = \int_0^{\theta'} q(s)ds < (\theta' - \theta)g(\theta)$ i.e. $ICT(\theta', \theta)$ in (8) does not hold. Contradiction.

If $U(\theta) > 0$, then by definition of $r(\theta)$, $U(\theta) = (\theta - r(\theta))q(\theta)$. Combining this with incentive constraint $ICT(\theta, r(\theta))$: $U(\theta) \geq (\theta - r(\theta))g(r(\theta))$, we conclude that $q(\theta) \geq g(r(\theta))$.

Now suppose that $q(\theta) \geq g(r(\theta))$ $\forall \theta \in [0, 1]$. Fix any pair $(\theta, \hat{\theta})$. Since $r(\theta)$ is continuous, nondecreasing and $r(0) = 0$, either $\hat{\theta} = r(\tilde{\theta})$ for some $\tilde{\theta} \in [0, 1]$ or $\hat{\theta} > r(1)$.

Suppose that $\hat{\theta} = r(\tilde{\theta})$ for some $\tilde{\theta} \in [0, 1]$. Note that $\tilde{\theta}$ need not be unique. However, if $\hat{\theta} = r(\tilde{\theta}_1) = r(\tilde{\theta}_2)$, then by the argument given in part (i) $q(\tilde{\theta}_1) = q(\tilde{\theta}_2)$. So pick any such $\tilde{\theta}$.

If $\theta \geq \tilde{\theta}$, then $q(\theta) \geq q(\tilde{\theta})$, and we have

$$U(\theta) = U(\tilde{\theta}) + \int_{\tilde{\theta}}^{\theta} q(s)ds \geq (\tilde{\theta} - \hat{\theta})q(\tilde{\theta}) + (\theta - \tilde{\theta})q(\tilde{\theta}) \geq (\theta - \hat{\theta})g(\hat{\theta})$$

where the first inequality holds because $q(.)$ is nondecreasing, while the second inequality holds because $\hat{\theta} = r(\tilde{\theta})$. Thus, $ICT(\theta, \hat{\theta})$ holds.

If $\theta < \tilde{\theta}$, then $q(\theta) \leq q(\tilde{\theta})$. Then $ICT(\theta, \hat{\theta})$ holds because we have:

$$U(\theta) = U(\tilde{\theta}) - \int_{\theta}^{\tilde{\theta}} q(s)ds \geq (\tilde{\theta} - \hat{\theta}))q(\tilde{\theta}) - (\tilde{\theta} - \theta)q(\tilde{\theta}) \geq (\theta - \hat{\theta})g(\hat{\theta})$$

Finally, suppose that $\hat{\theta} > r(1)$. Since $g(\hat{\theta}) \leq q(1)$, we have:

$$U(\theta) = U(1) - \int_{\theta}^{1} q(s)ds \geq (1 - r(1))q(1) - (1 - \theta)q(1) \geq (\theta - r(1))q(1) > (\theta - \hat{\theta})g(\hat{\theta})$$

Thus, $ICT(\theta, \hat{\theta})$ also holds in this case. *Q.E.D.*

Proof of lemma 7:

If $q^*(\theta)$ solves maximization problem (10) on the domain $C^1_p([0,1])$, but not on the domain $C([0,1])$, then there exists function $\hat{q}(\theta) \in C([0,1]) \setminus C^1_p([0,1])$ s.t. the objective function in (10) takes a strictly higher value under $\hat{q}(\theta)$ than under $q^*(\theta)$.

By the Stone-Weierstrass theorem, the space of continuously differentiable functions, which is a subspace of $C^1_p([0,1])$, is dense in $C([0,1])$. Therefore, $C^1_p([0,1])$ is dense in $C([0,1])$, and there exists a sequence $\tilde{q}_n(\theta) \in C^1_p([0,1])$ converging to $\hat{q}(\theta)$ in the *sup*−norm. The objective function (10) is continuous in the *sup*-norm. Therefore, $\exists N > 0$ s.t. $\forall n \geq N$ the objective function in (10) takes a strictly higher value under $\tilde{q}_n(\theta)$ than under $q^*(\theta)$. This contradicts the assumption that $q^*(\theta)$ is a solution on $C^1_p([0,1])$. *Q.E.D.*

Proof of lemma 8:

First of all, let us establish the following property of the solution: for any interval where Case 2 applies ($r(\theta) > q(\theta)$) $\exists k_1 > 0$ s.t.

$$q(\theta) \geq k_1 \left(\theta(\sqrt{3}+1)\right)^{\sqrt{3}+1} \tag{31}$$

To demonstrate this, combine (21) with (22) to get:

$$r(\sqrt{3}+1+r') = \frac{2}{\sqrt{3}+1}(\theta(\sqrt{3}+1)+r)$$

Using (21) in the above equation, we get:

$$\frac{q'}{q} = \frac{(\sqrt{3}+1)(\sqrt{3}+1+r')}{\theta(\sqrt{3}+1)+r}$$

The solution to this equation is $q = k_1 \left(\theta(\sqrt{3}+1)+r\right)^{\sqrt{3}+1}$. The constant of integration k_1 must be strictly positive, because we are in the region where $q(\theta) > r(\theta) \geq 0$, so (31) follows.

Further, note that it is not optimal to set $q(\theta) = 0 \ \forall\theta \in [0,1]$. For suppose otherwise. Then the value of the problem in (10). However, it can be made strictly positive by setting $q(\theta) = g(\theta) = 1/2 \ \forall\theta \in [1/2, 1]$.

Since $q(.)$ is nondecreasing and continuous, either $q(\theta) > 0 \ \forall\theta \in (0,1]$ or $\exists\underline{\theta} \in (0,1)$ s.t. $q(\theta) > 0 \ \forall\theta > \underline{\theta}$ and $q(\theta) = 0 \ \forall\theta \leq \underline{\theta}$. Suppose that such $\underline{\theta} > 0$ exists. Since $r(\theta)$ is continuous and $r(\underline{\theta}) = \underline{\theta}$, $\exists\zeta > 0$ s.t. $r(\theta) > q(\theta)$ for $\theta \in [\underline{\theta}, \underline{\theta}+\zeta]$. So Case 2 applies on $[\underline{\theta}, \underline{\theta}+\zeta)$. However, (31) contradicts the fact that $q(\underline{\theta}) = 0$. Thus, $q(\theta) > 0 \ \forall\theta \in (0,1]$.

Since $2\theta - 1 < 0$ for $\theta < 1/2$, $\exists\theta_2 > 1/2$ s.t. Case 2 ($r(\theta) > q(\theta)$) applies on $[0, \theta_2)$.

Let $\overline{\theta} = \inf\{\theta : r(\theta) < q(\theta)\}$. We will show that $\overline{\theta} < 1$. Since $q(\theta) > 0 \ \forall\theta \in (0,1]$, $U(1/2) > 0$. By lemma 1, $\lim_{\theta\to 1} q(\theta) = 1$. Therefore, $r(\theta) \equiv \theta - \frac{U(\theta)}{q(\theta)} \leq \theta - U(\theta) < 1 - U(1/2) < q(\theta)$ when θ is close to 1.

Finally let us establish that $q(\theta) > r(\theta) \ \forall\theta \in (\overline{\theta}, 1]$ i.e., the solution cannot switch back to Case 2 again. By definition of $\overline{\theta}$, $q(\overline{\theta}) = r(\overline{\theta})$ and $q(\theta) > r(\theta)$ for $\theta \in (\overline{\theta}, \overline{\theta}+\epsilon)$ for some $\epsilon > 0$ (remember that there are no intervals on which $r(.) = q(.)$). Therefore, $q'_+(\overline{\theta}) \geq r'_+(\overline{\theta})$ (where the subscript + denotes the derivative from the right). This is equivalent to $\overline{\theta} \leq q(\overline{\theta}) + r(\overline{\theta})$ because $r' = \frac{q'(\theta-r)}{q}$.

On $(\overline{\theta}, \overline{\theta}+\epsilon)$ $q'(\theta) = 2$ since we are in Case 1. Also $r'(\theta) \geq 0$. Combining this with $\overline{\theta} \leq q(\overline{\theta}) + r(\overline{\theta})$, we conclude that $\theta < q(\theta) + r(\theta)$ and hence $q'(\theta) > r'(\theta) \ \forall\theta \in (\overline{\theta}, \overline{\theta}+\epsilon)$. Thus, $q(\theta) > r(\theta) \ \forall\theta \in (\overline{\theta}, 1]$. *Q.E.D.*

Proof of theorem 6:

Fix a continuously differentiable allocation profile $\{q(\theta), p(\theta)\}$ and some $\varepsilon > 0$. Let $U(\theta) = v(q(\theta), \theta) + p(\theta)$. By assumption $U(\cdot)$ is a C^1 function. Thus there exists $U^\varepsilon(\cdot) \in C^2$ s.t. $\forall\theta \ |U^\varepsilon(\theta) - U(\theta)| < \varepsilon/2$.

Let us define mechanism $\tilde{G}_{n_1,n_2}(.)$ as follows. Fix some integers n_1, n_2, constant $\underline{p}$, and functions $\tilde{s}(.) : \Theta \mapsto \Theta$ and $\tilde{p}(.) : \Theta \mapsto \mathbf{R}$. Let $n = n_1 + n_2$ and define $\tilde{S}_{n_1,n_2}(\theta) = (s_1(\theta), ..., s_n(\theta))$ to be the following vector:

$$s_1(\theta) = s_2(\theta) = ... = s_{n_1}(\theta) = \theta \ \text{ and } \ s_{n_1+1}(\theta) = ... = s_n(\theta) = \tilde{s}(\theta)$$

Then mechanism $\tilde{G}_{n_1,n_2}(.)$ maps messages S sent by the agent into outcomes according to the following outcome function $g(.)$:

$$g(S) = \begin{cases} (q(\theta), \tilde{p}(\theta)), & \text{if } S = \tilde{S}_{n_1,n_2}(\theta) \\ (0, \underline{p}), & \text{if } S \neq \tilde{S}_{n_1,n_2}(\theta) \ \forall\theta \end{cases}$$

We will construct $\tilde{G}_{n_1,n_2}(.)$ to implement an allocation profile that satisfies the conditions of the theorem. First, define $\tilde{s}(\theta)$ as the unique solution to:

$$n_2 sgn(\tilde{s}(\theta) - \theta) C'(|\tilde{s}(\theta) - \theta|) = \frac{dU^\varepsilon(\theta)}{d\theta} - \frac{\partial v}{\partial\theta}(q(\theta), \theta) \tag{32}$$

where sgn stands for the sign operator. Thus if $\frac{dU^\varepsilon(\theta)}{d\theta} - \frac{\partial v}{\partial\theta}(q(\theta), \theta) \geq (\leq) 0$, then $\tilde{s}(\theta) \geq (\leq) \theta$. Assumption 4 guarantees that there exists a unique $\tilde{s}(\theta)$ satisfying (32).

When an agent with utility parameter θ uses strategy $\tilde{S}_{n_1,n_2}(\theta)$, she incurs communication costs $n_2 C(|\tilde{s}(\theta)-\theta|)$. Let us select n_2 so that $n_2 C(|\tilde{s}(\theta)-\theta|) < \varepsilon/2$. By convexity of $C(\cdot)$ and (32), we have:

$$n_2 C(|\tilde{s}(\theta)-\theta|) \leq n_2 C'(|\tilde{s}(\theta)-\theta|)|\tilde{s}(\theta)-\theta| = \left(\frac{dU^\varepsilon(\theta)}{d\theta} - \frac{\partial v}{\partial \theta}(q(\theta),\theta)\right)|\tilde{s}(\theta)-\theta| \tag{33}$$

Let $E = \max_{\theta\in\Theta} |\frac{dU^\varepsilon(\theta)}{d\theta} - \frac{\partial v}{\partial \theta}(q(\theta),\theta)|$. Since $U^\varepsilon(.)$ and $v(.,.)$ are $C^2(.)$ functions, $E<\infty$. Select $n_2 \geq \frac{E}{C'(\varepsilon/2E)}$. Then using (32) we have: $E \geq n_2 C'(|\tilde{s}(\theta)-\theta|) \geq \frac{E}{C'(\varepsilon/2E)} C'(|\tilde{s}(\theta)-\theta|)$. This implies that $|\tilde{s}(\theta)-\theta| \leq \frac{\varepsilon}{2E}$. Hence, by (33) $n_2 C(|\tilde{s}(\theta)-\theta|) \leq E|\tilde{s}(\theta)-\theta| \leq \frac{\varepsilon}{2}$.

Next, let $\tilde{p}(\theta) = p^\varepsilon(\theta)$ where:

$$p^\varepsilon(\theta) = U^\varepsilon(\theta) - v(q(\theta),\theta) + n_2 C(|\tilde{s}(\theta)-\theta|) \tag{34}$$

Note that $|p^\varepsilon(\theta) - p(\theta)| < \varepsilon$ since $|U^\varepsilon(\theta) - U(\theta)| < \varepsilon/2$ and $n_2 C(|\tilde{s}(\theta)-\theta|) < \varepsilon/2$.

Let us establish that both $\tilde{s}(\theta)$ and $p^\varepsilon(\theta)$ are continuous and differentiable. Since $U^\varepsilon(\theta)$ and $v(\cdot,\cdot)$ are C^2 functions, $q(\cdot)$ is C^1, and Θ is compact, the right-hand side of (32) is a continuous function on Θ bounded from above by some $0 < \hat{V} < \infty$. Then, since $C(.)$ is C^2 function and $0 < \underline{\omega} \leq C''(\cdot) \leq \overline{\omega} < \infty$, it follows that $\tilde{s}'(\theta)$ exists and is a C^0 function satisfying the following equation obtained by differentiating (32):

$$n_2 C''(|\tilde{s}(\theta)-\theta|)(\tilde{s}'(\theta)-1) = \frac{d^2U^\varepsilon}{d^2\theta}(\theta) - \frac{\partial^2 v}{\partial\theta\partial q}(q(\theta),\theta)q'(\theta) - \frac{\partial^2 v}{\partial \theta^2}(q(\theta),\theta) < \hat{V} \tag{35}$$

Differentiating the right-hand side of (34) and using (32) and (35), we get:

$$\frac{dp^\varepsilon(\theta)}{d\theta} = -\nabla_q v(q(\theta),\theta)q'(\theta) + n_2 sgn(\tilde{s}(\theta)-\theta)C'(|\tilde{s}(\theta)-\theta|)\tilde{s}'(\theta) \tag{36}$$

Obviously, $\frac{dp^\varepsilon(\theta)}{d\theta}$ is continuous.

In the rest of the proof we establish that n_1 and $\underline{p}$ can be chosen in such a way that is optimal for an agent with utility parameter θ to use strategy $\tilde{S}_{n_1,n_2}(\theta)$. At first, we will show that this agent gets a higher payoff by using reporting strategy $\tilde{S}_{n_1,n_2}(\theta)$ rather than $\tilde{S}_{n_1,n_2}(\hat{\theta})$ for some $\hat{\theta} \neq \theta$ provided that n_1 is sufficiently large. If she uses reporting strategy $\tilde{S}_{n_1,n}(\hat{\theta})$, she gets the following payoff:

$$W^\varepsilon(\hat{\theta},\theta) = v(q(\hat{\theta}),\theta) + p^\varepsilon(\hat{\theta}) - n_1 C(|\hat{\theta}-\theta|) - n_2 C(|\tilde{s}(\hat{\theta})-\theta|) \tag{37}$$

Thus, we need to show that $U^\varepsilon(\theta) \equiv W^\varepsilon(\theta,\theta) \geq W^\varepsilon(\hat{\theta},\theta)\ \forall\theta$ and $\hat{\theta}\in\Theta$. Differentiating (37) and using (36) to substitute for $\frac{dp^\varepsilon(\hat{\theta})}{d\hat{\theta}}$ we get:

$$\begin{aligned}
\frac{\partial W^\varepsilon(\hat{\theta},\theta)}{\partial\hat{\theta}} &= \nabla_q v(q(\hat{\theta}),\theta)q'(\hat{\theta}) + \frac{dp^\varepsilon(\hat{\theta})}{d\hat{\theta}} - n_1 sgn(\hat{\theta}-\theta)C'(|\hat{\theta}-\theta|) \\
&- n_2 sgn(\tilde{s}(\hat{\theta})-\theta)C'(|\tilde{s}(\hat{\theta})-\theta|)\tilde{s}'(\hat{\theta}) \\
&= \left(\nabla_q v(q(\hat{\theta}),\theta) - \nabla_q v(q(\hat{\theta}),\hat{\theta})\right) q'(\hat{\theta}) - n_1 sgn(\hat{\theta}-\theta)C'(|\hat{\theta}-\theta|) \\
&- n_2\left(sgn(\tilde{s}(\hat{\theta})-\theta)C'(|\tilde{s}(\hat{\theta})-\theta|) - sgn(\tilde{s}(\hat{\theta})-\hat{\theta})C'(|\tilde{s}(\hat{\theta})-\hat{\theta}|)\right)\tilde{s}'(\hat{\theta})
\end{aligned} \tag{38}$$

At first, let us establish the bound on the first term in (38). Let $F = \max_i \max_{\theta\in\Theta} \frac{\partial q_i(\theta)}{\partial\theta}$ and $m_i = \max_{(q,\theta)\in Q\times\Theta} \frac{\partial^2 v(q(\theta),\theta)}{\partial q_i \partial\theta}$. Then,

$$\left|\left(\nabla_q v(q(\hat{\theta}),\theta) - \nabla_q v(q(\hat{\theta}),\hat{\theta})\right) q'(\hat{\theta})\right| \leq F\left|\sum_{i=1}^{k-1}\int_{\theta}^{\hat{\theta}} \frac{\partial^2 v(q(\hat{\theta}),z)}{\partial q_i \partial z}dz\right| \leq F(\sum_{i=1}^{k-1} m_i)|\hat{\theta}-\theta|$$

To bound the last term in (38), observe from (35) that

$$|\tilde{s}'(\theta)| \leq 1 + \frac{\hat{V}}{n_2 C''(\tilde{s}(\theta)-\theta|)} \leq 1 + \frac{\hat{V}}{n_2\bar{\omega}} \equiv \kappa_{n_2}$$

Let $\Omega = \left|-sgn(\tilde{s}(\hat{\theta})-\theta)C'(|\tilde{s}(\hat{\theta})-\theta|) + sgn(\tilde{s}(\hat{\theta})-\hat{\theta})C'(|\tilde{s}(\hat{\theta})-\hat{\theta}|)\right|$. Suppose first that $\hat{\theta} < \theta$. If $\tilde{s}(\hat{\theta}) \leq \hat{\theta} \leq \theta$, then

$$\Omega = C'(\theta - \tilde{s}(\hat{\theta})) - C'(\hat{\theta} - \tilde{s}(\hat{\theta})) = \int_{\hat{\theta}}^{\theta} C''(z - \tilde{s}(\hat{\theta}))dz \leq \bar{\omega}|\hat{\theta} - \theta|$$

If $\hat{\theta} < \tilde{s}(\hat{\theta}) < \theta$, then

$$\Omega = C'(\theta - \tilde{s}(\hat{\theta})) + C'(\tilde{s}(\hat{\theta}) - \hat{\theta})) \leq 2C'(\theta - \hat{\theta}) \leq 2\bar{\omega}|\hat{\theta} - \theta|$$

Finally, if $\hat{\theta} < \theta < \tilde{s}(\hat{\theta})$, then

$$\Omega = C'(\tilde{s}(\hat{\theta}) - \hat{\theta}) - C'(\tilde{s}(\hat{\theta}) - \theta) = \int_{\hat{\theta}}^{\theta} C''(\tilde{s}(\hat{\theta}) - z)dz \leq \bar{\omega}|\hat{\theta} - \theta|$$

Proceeding in an entirely parallel fashion in the case where $\hat{\theta} > \theta$, we can again show that $\Omega \leq 2\bar{\omega}|\hat{\theta}-\theta|$. Thus, the absolute value of the last term in (38) is bounded by $2n_2\bar{\omega}|\hat{\theta}-\theta|\kappa_{n_2}$.

Consider now the middle term in (38), $\Psi \equiv -n_1 sgn(\hat{\theta}-\theta)C'(|\hat{\theta}-\theta|)$. If $\hat{\theta} > \theta$, then $\Psi = -n_1C'(|\hat{\theta}-\theta|) \leq -n_1\underline{\omega}(\hat{\theta}-\theta)$. On the other hand, if $\hat{\theta} < \theta$ then $\Psi = n_1C'(\theta - \hat{\theta}) \geq n_1\underline{\omega}(\theta - \hat{\theta})$. It follows that if $\hat{\theta} > \theta$, then

$$\frac{\partial W^{\varepsilon}}{\partial\hat{\theta}}(\hat{\theta},\theta) \leq \left(F\sum_{i=1}^{k-1} m_i + 2n_2\bar{\omega}\kappa_{n_2} - n_1\underline{\omega}\right)(\hat{\theta}-\theta)$$

and that if $\hat{\theta} < \theta$, then

$$\frac{\partial W^{\varepsilon}}{\partial\hat{\theta}}(\hat{\theta},\theta) \geq \left(-F\sum_{i=1}^{k-1} m_i - 2n_2\bar{\omega}\kappa_{n_2} + n_1\underline{\omega}\right)(\theta-\hat{\theta})$$

By selecting $n_1 \geq \left(F\sum_{i=1}^{k-1} m_i + 2n_2\bar{\omega}\kappa_{n_2}\right)/\underline{\omega}$, we therefore guarantee that $\frac{\partial W^{\varepsilon}}{\partial\hat{\theta}}(\hat{\theta},\theta) \leq 0$ for all $\hat{\theta} > \theta$ and that $\frac{\partial W^{\varepsilon}}{\partial\hat{\theta}}(\hat{\theta},\theta) \geq 0$ for all $\hat{\theta} < \theta$, showing that incentive compatibility is satisfied.

Finally, select $\underline{p}$ sufficiently small so that deviating to an alternative reporting strategy $S \neq \tilde{S}_{n_1,n_2}(\theta)\ \forall\theta\in\Theta$ is not optimal for any agent. *Q.E.D.*

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