Abstract

This paper analyzes whether the ease and speed of entry of new firms can mitigate the anti-competitive effects of a merger, in a dynamic model of endogenous merger. In our model, if the new firms can enter quickly, it is more likely that merger is motivated by efficiency as opposed to increased market power. Thus, there is less of a reason to challenge the merger. On the other hand, if entry of new firms becomes less costly, firms may have a stronger incentive to monopolize the industry through horizontal merger. We also show that when the incumbent can engage in entry deterrence activities, anti-merger policy can decrease welfare.

Keywords: Horizontal Mergers, Entry Deterrence

JEL Classification: K21, L41
1. Introduction

Suppose that two firms in an industry decide to merge. Under what conditions should they be allowed to do so? This is a very old question, but neither the government regulators nor academic economists have a clear-cut answer. The basic trade-off seems to be clear: The potential cost of a merger is that it could increase the merged firms’ market power, which would allow them to restrict output and increase prices. This would hurt consumers and create a deadweight loss. On the other hand, the merged firms might be able to better utilize their combined resources, resulting in more efficient production and potentially lower prices for consumers, if the efficiency gains are large enough.

It turns out that the above trade-off is fairly hard to assess, both in practice and in theory. There are many complicating factors – the efficiencies that the firms attribute to the proposed merger may be possible to achieve through other means (say, joint venture), the merger may not result in higher prices if the firms are able to implicitly collude even without merging, the merger may invite new entry, and so on. In this paper, we concentrate on the effects of potential entry following merger. The 1992 Horizontal Merger Guidelines issued jointly by the Federal Trade Commission and the U.S. Department of Justice pay considerable attention to the effects of new entry. In fact, a whole section (Section 3) out of a total of five sections is devoted to entry analysis. Moreover, several recent lower court decisions make it clear that the courts take seriously the argument that easy entry makes a proposed merger less harmful.\footnote{In United States v. Waste Management, Inc., 743 F.2d 976 (2nd Cir. 1984), the Second Circuit approved a merger between two Dallas commercial waste haulers that created a firm controlling 48.8\% of the Dallas market (see Pitofsky, 1990). The court based its decision on the claim that the Dallas market could be relatively easily entered, either by new entrants or from haulers established in nearby Fort Worth. For other decisions where ease of entry played an important role see, e.g., United States v. Syufy Enterprises, 903 F.2d 659 (9th Cir. 1990), United States v. Calmar, Inc., 612 F. Supp. 1298 (D.N.J. 1985), Laidlaw Acquisition Corp. v. Mayflower Group, Inc., 636 F. Supp. 1513 (S.D. Ind. 1986), or Consolidated Gold Fields, PLC v. Anglo American Corp., 698 F. Supp. 487 (S.D.N.Y. 1988).}
create room in the market for another firm, and the entry of a new firm may mitigate any anti-
competitive effects of the merger. This possibility is explicitly mentioned in the Guidelines: “In
markets where entry is ... easy ..., the merger raises no antitrust concern and ordinarily requires
no further analysis.” This policy is clearly fashioned so as to treat easy and speedy entry as a
mitigating factor in a merger challenge.

Our goal in this paper is twofold. First, we want to take the policy espoused in the Guidelines
at its face value and evaluate the argument that when new entry is easy and speedy, there is less
of a need to challenge a horizontal merger. Simple economic intuition would seem to support this
policy – easy and speedy entry should quickly eliminate the exertion of monopoly power of the
merged firms. However, we show that this intuition may be flawed. Second, we argue that in order
to fully assess the effects of an anti-merger policy, one has to take into account the alternative ways
in which firms may be able to acquire or protect their monopoly power. In particular, we study
the effectiveness of an anti-merger policy when firms can engage in entry deterrence.

We build and analyze a dynamic model of merger and entry. We allow for the possibility that
entry takes time, in order to capture the considerations expressed in the Guidelines, which recognize
that it is important to assess if the entry “can achieve significant market impact within a timely
period.” In particular, the firms in our model are infinitely lived and entry can occur only at the
beginning of each period. In such a dynamic model, the incumbent firms face a trade-off when
considering a merger with the purpose of gaining monopoly power: Until new entry occurs, they
can exercise their increased monopoly power and reap higher profits. However, the merger may
invite new firms into the industry, which makes monopolization short lived. Whether merger is
optimal or not then depends upon how quickly can new firms enter and how competitive is the
industry in the absence of mergers.

Our analysis of this trade-off yields some interesting results. First, we show that if the moti-
vation behind the merger is solely to gain monopoly power (i.e., the merger does not generate any efficiencies in production), then the incentive to merge decreases the faster are the new firms able to enter the industry. In particular, if the new entrants can establish their operation sufficiently fast, then mergers are never observed in this industry. Thus, the Guidelines’ focus on the quickness of new entry is justified in our model, albeit for a slightly different reason than the one espoused in the Guidelines. In our model, if the new firms can enter quickly, it is unlikely that a merger proposed by the incumbents is motivated by the goal of increased market power. Rather, it probably has the efficiency motivation and, therefore, there is less of a reason to challenge it.

Second, it turns out, rather surprisingly, that if there is a cost of entry that must be incurred by new firms, then, for reasonable parameter values, a decrease in this cost can increase the incentive of incumbent firms to gain monopoly power through merger. This is because low entry cost means that the ‘free entry’ industry structure is relatively competitive, which makes the interim gain from monopolization large. In such an industry, if the FTC follows the Guidelines and takes the ease of entry as a mitigating factor, it is likely to approve mergers that have the purpose of monopolizing the market and challenge those that are motivated by efficiency gains.

Finally, our analysis demonstrates that an important consideration in the assessment of an anti-merger policy should be the ease with which the incumbents can prevent new entry through deterrence activities. It might seem that this consideration is captured by the Guidelines in their analysis of the entry effects, in particular in the discussion of ease of entry and entry costs. However, the Guidelines appear to treat the entry costs as fixed and exogenously given. What we have in mind here are endogenous entry barriers erected by the incumbents, such as advertisement expenditures, investment in cost reducing technology, exclusive contracts with suppliers, or any other activities identified in the rich literature on entry deterrence. Our point is that these endogenous barriers to entry can depend upon the anti-merger policy. In particular, entry deterrence and merger are
to some extent substitutes to firms that wish to preserve their monopoly power. Hence, if merger becomes more costly or less likely, firms may optimally spend more resources on entry deterrence. Because the resources spent on entry deterrence represent a waste, tough anti-merger policy may decrease overall welfare. We use this argument to demonstrate that if entry deterrence is feasible and costly, ease of entry in our model always represents a mitigating factor in merger. The reason is that if the industry is to be monopolized, it is less wasteful to monopolize it through merger rather than through entry deterrence.

The literature on the analysis of horizontal mergers is vast with most models characterized by a fixed number of firms, nonendogenous merger, and a static time frame. The key papers in this category are the contributions by Salant, Switzer and Reynolds (1983), Deneckere and Davidson (1985), Perry and Porter (1985), and Farrell and Shapiro (1990). These papers are generally concerned with the effects of merger on price and surplus levels and the appropriate public policy toward mergers. Saloner (1987) shows that pre-merger predatory pricing by the incumbent can favorably affect the incumbent’s terms of takeover and decrease the possibility of entry. In a model due to Reitzes and Levy (1995), the ability to price discriminate can lead to welfare reducing mergers, which do not promote entry. Rasmusen (1988) considers a static and exogenous model of merger through the buy-out of entrants and shows that the prospect of subsequent buy-out may encourage otherwise unprofitable and inefficient entry. Kamien and Zang (1990), (1991) and (1993) consider endogenous models of merger and discuss the limits to monopolization through acquisition, but they too retain the assumptions of a fixed time frame and a fixed number of firms. Horn and Persson (2001) use notions of coalition formation from cooperative game theory to formulate an endogenous model of the merger process. However, this analysis, like the work of Kamien and Zang, employs a static model.

The papers more directly related to our work are those by Werden and Froeb (1998), Cabral
(1999) and Spector (2001), who all study the question of whether the industry equilibrium price increases or decreases as a result of a merger, when there exists a possibility of subsequent new entry. In addition, in a model of product differentiation and price competition, Cabral (1999) derives the interesting result that consumer welfare may be lower if the cost efficiencies created by merger are greater.

While each of the above three models studies entry subsequent to merger, each is a one period model focused on the effects of exogenous merger on consumer welfare. However, as was persuasively argued by Gowrisankaran (1999), policy makers need dynamic models with endogenous mergers “to correctly answer even simple antitrust questions.” Gowrisankaran offers such a model and studies the occurrence of and the effects of mergers on endogenously determined entry, exit, and investment. The model is sufficiently complex that it cannot be analytically solved and requires the use of numerical methods.²

Our model, like Gowrisankaran’s ambitious model, is a dynamic endogenous model of merger. We are able to solve the model analytically, because we have kept it simple by focusing our analysis on mergers motivated purely by the desire to monopolize the industry (that is, our mergers generate no efficiency gains) and by adopting simplifying assumptions regarding the merger process. On the other hand, we enrich the model by allowing the incumbent firm to spend resources on entry deterrence. Our analytic solution allows us to study the comparative dynamics of the speed of entry and the cost of entry on the incentive to merge. Further, we are able to analyze the effects of anti-merger policy when entry deterrence activities are not completely prohibited.

The rest of the paper proceeds as follows. In the next section we describe the model and discuss our main assumptions. Section 3 provides the analysis of the case where entry deterrence is not possible. Section 4 allows entry deterrence and explores its effects. Section 5 concludes. All proofs

²The first dynamic models of merger were presented by Berry and Pakes (1993) and Cheong and Judd (1992). They, too, use numeric methods to analyze their models.
are provided in the Appendix.

2. The Model

The industry. Consider an infinitely lived industry that is currently served by one incumbent firm, $I^3$. Time is continuous, but new firms can only enter at the beginning of discrete time periods, $t$, of length $\Delta$. At the beginning of each period $t$, the incumbent $I$ faces a threat of (instantaneous) entry from many potential entrants, who decide sequentially$^4$ whether to enter in that period or not. If a firm $i$ decides to enter at the beginning of period $t$, it has to incur a one-time entry cost $F > 0$. We will adopt the tie breaking rule that a firm enters the industry only if the expected profit after entry is strictly greater than the cost of entry, $F$. All firms in this economy discount the future using a common interest rate, $r$.

Profits. Let $\tilde{M}$ be the instantaneous monopoly profit (i.e., the monopoly profit per unit of time) in this industry, so that if no entry ever occurs, then $I$’s present value of all future profits is $M(\infty) = \int_0^\infty \tilde{M}e^{-rt}d\tau = \tilde{M}/r$. If at a given point in time exactly $k$ firms operate in this industry, then each of them earns an instantaneous profit of $\tilde{\pi}_k$ (thus, $\tilde{\pi}_1 = \tilde{M}$). We will assume that $\tilde{\pi}_k$ decreases in $k$, i.e., $\tilde{\pi}_{k+1} < \tilde{\pi}_k$, for each $k$. In addition, the combined industry profit is at most equal to the monopoly profit: $\tilde{M} \geq k\tilde{\pi}_k$ for any $k$.$^5$

Note that as in Salant, Switzer and Reynolds (1983), merger in this framework is equivalent to shutting down all but one of the merging firms. In other words, all firms in our model are always

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$^3$Although most of our qualitative results (including Propositions 1, 3, and 4) would also obtain in a two-period model, such a model would not necessarily be more tractable.

$^4$It is well known that if entry is simultaneous, the model can exhibit both pure and mixed strategy equilibria. The only purpose of the assumption that entry is sequential is to eliminate the mixed strategy equilibria.

$^5$We are deliberately avoiding tying our analysis to a specific model of oligopolistic competition, like Cournot or Bertrand models. We would like to note though, that both of the above assumptions, $\tilde{\pi}_{k+1} < \tilde{\pi}_k$ and $\tilde{M} > k\tilde{\pi}_k$ for all $k$, are satisfied in a wide range of models. For example, it can easily be checked that the first property is met in the traditional Cournot model with linear demand and cost. The property that $\tilde{M} > k\tilde{\pi}_k$ for any $k$ is also met in the general Cournot model, under strictly concave profit functions for each firm and the assumption that each firm’s output is a strategic substitute for any other firm’s output. See, for example, Tirole (1989), pp. 218-219.
identical with respect to their profitability, independent of whether they were created through a merger or established as new entrants. This means that there are no efficiency benefits to mergers in our model, so that any merger must be motivated purely by the prospect of increased market power (in general, this requires that firms have constant marginal costs and their products are homogeneous). While we are aware that many real world mergers are driven by efficiency considerations, our goal here is to evaluate the argument that ease of entry should be a mitigating factor in challenging a given merger. In order to focus on this argument, we find it useful to abstract from other possible mitigating factors, like efficiency gains. Nevertheless, we are confident that the model could be extended to allow for many forms of efficiency gains from merger, without affecting our qualitative results.

**Entry deterrence.** At the beginning of each period $t$, that period’s incumbent(s) can engage in deterrence activities to discourage entry in period $t$. Since entry deterrence considerations do not affect our analysis until Section 4, we defer the description of the entry deterrence process to that section.

**Merger proposals.** At the beginning of any period $t$ in which the industry contains $k \geq 2$ firms (including $I$), the firms can negotiate a merger (as long as mergers are not prohibited by the government). The merger bargaining has at most two rounds in any given period. The bargaining starts by $I$ making a merger offer to all firms that it decides to buy out in that period.\(^6\) It is assumed that if the offer is rejected by any of the target firms (or if $I$ chooses not to make any merger offer), the first round of bargaining is unsuccessful and all $k$ firms have to wait for a length of time $\tau < \Delta$ before one of them can make a new offer.\(^7\) During that time, each of them receives

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\(^6\)We will assume that whenever $I$ merges with some other firm or firms, the resulting firm is firm $I$, even if $I$ did not make the merger offer (although such a merger never happens in equilibrium). This assumption is inconsequential—we only adopt it to be specific.

\(^7\)Thus, the merger offers are contingent on the acceptance by all targeted firms. By allowing contingent offers, we are following Stigler’s (1950) suggestion for resolving the free-rider problem in the merger bargaining problem he was
a stream of $\pi_k$ profits. After the period of time $\tau$, the second round of bargaining starts by Nature choosing randomly one of the $k$ firms to make a new contingent offer. This offer, too, has to be accepted by all targeted firms; if any of them rejects, the bargaining process is over for the rest of the period.\footnote{Alternatively, we could have assumed that the bargaining process continues until an offer is accepted by all targeted firms, rather than ending after two rounds of bargaining. This would not change our results, although it would complicate the analysis because it would be necessary to consider how the bargaining procedes if it is not concluded by the end of the period (an out of equilibrium event) and if new firms enter at the beginning of the next period, $t+1$.} At the beginning of period $t+1$, if the industry contains at least two firms (including new entrants, if any), the bargaining process starts anew, with $I$ making the first offer.

**Solution concept.** Let $K_t$ be the set of all firms (including new entrants) present in the industry at the beginning of period $t$. If a firm is chosen to make a merger proposal, its strategy is characterized by the set of firms, $J \subset K_t$, to which it extends the buy-out offers, and by the buy-out prices, $P_i \geq 0$, it offers each firm $i \in J$. (In our analysis below, we do not use the buy-out prices explicitly. Rather, we couch our discussion in terms of the shares of the merger surplus captured by merger participants.) The strategy set of potential entrants in any given period consists of only two actions – enter and do not enter. The solution concept we employ is that of Markov Perfect Equilibrium (MPE), where the state variable is the number of firms, $k$, that are present in the industry at a given time.

### 3. The merger decision when entry deterrence is not possible

Before we proceed with the analysis, it is useful to derive the expressions for the firms’ one-period profits, as well as for their (potential) total lifetime profits. Analogous to the case of a permanent monopoly, the present value of total profits of each of $k$ permanent oligopolists is first to identify. This free rider problem is caused by the fact that, if there are relatively many firms in an industry, some of them may have an incentive to reject a merger offer hoping to benefit more if other firms merge and increase the industry’s concentration and price. See Kamien and Zang (1990) and Gowrisankaran and Holmes (2000), among others, for more recent investigations of this free rider problem. As recognized by Stigler, this problem vanishes if firms can make contingent merger offers.
\[ \pi_k(\infty) = \int_0^\infty \tilde{\pi}_k e^{-r\tau} d\tau = \tilde{\pi}_k/r. \] Similarly, the one period profit of each of \( k \) oligopolists is

\[ \pi_k(\Delta) = \int_0^\Delta \tilde{\pi}_k e^{-r\tau} d\tau = \frac{\tilde{\pi}_k}{r} [1-e^{-r\Delta}] = \frac{\tilde{\pi}_k}{r} [1-\delta(\Delta)], \]

where \( \delta(\Delta) \equiv e^{-r\Delta} \) is the one period cumulative discount factor.\(^9\) Thus, \( \pi_k(\infty) = \frac{\pi_k(\Delta)}{1-\delta(\Delta)}. \)

In this section, we analyze the firms’ entry and merger incentives under the assumption that entry deterrence is not possible. We start with the following simple example, in which the intuition can be seen most easily.

### 3.1. An example

Suppose that the industry can accommodate at most 2 firms. Suppose also, for illustration purposes in this example, that \( I \) has all the bargaining power vis a vis the other firms (as will become clear later, this corresponds to a \( \tau \) close to \( \Delta \)). In a one period setting (i.e., the industry only exists for a single period of length \( \Delta \)), the buy out decision is very simple: The entrant sells out at the beginning of the period if and only if it is offered at least \( \pi_2(\Delta) \) and \( I \) buys the entrant out whenever \( M(\Delta) - \pi_2(\Delta) > \pi_2(\Delta) \), or \( \pi_2(\Delta) < \frac{M(\Delta)}{2} \). Because this inequality is frequently satisfied in standard oligopoly models, in a one-period framework, we typically expect a monopoly-preserving merger.

In a dynamic setting, the situation is more complicated. If the entrant does not sell out, then it expects to get \( \pi_2(\Delta) \) in every period, so that the total present value of its profits is \( \pi_2(\infty) = \pi_2(\Delta)/[1 - \delta(\Delta)] \). This is the minimum it will accept when \( I \) proposes a merger. If \( I \) wants to preserve its monopoly position, it has to buy out a new entrant at the beginning of every period. Merger will therefore be observed if and only if

\[
\frac{M(\Delta)}{1-\delta(\Delta)} - \frac{\pi_2(\Delta)}{[1-\delta(\Delta)]^2} > \frac{\pi_2(\Delta)}{1-\delta(\Delta)},
\]

or

\[
M(\Delta) - \frac{2 - \delta(\Delta)}{1-\delta(\Delta)} \pi_2(\Delta) > 0.
\]

The expression on the left hand side is the total surplus created by the merger in any given period,

\(^9\) Again, we will use \( M(\Delta) \) instead of \( \pi_1(\Delta) \).
and the condition says that the industry will be monopolized if and only if this surplus is positive. Clearly, when $\delta(\Delta)$ is relatively small (the interest rate is high or the period $\Delta$ is long), then the surplus is high and the above condition is easily satisfied. On the other hand, if the interest rates are low or if $I$ faces a frequent threat of entry, then it is better off accommodating the entrant, rather than trying to preserve its monopoly position through merger, period after period.

3.2. The general case

Suppose now, more generally, that after any new entry, the number of firms present in this industry at the beginning of a period $t$, but before the start of merger negotiations, is $k_t \geq 2$. For lack of a better term, we will call $k_t$ the "pre-merger number of firms". The equilibrium in this model is characterized by (i) the equilibrium pre-merger number of firms in every period, $k^*$, (ii) the post-merger number of firms, (iii) the time of merger (if any) in every period (that is, whether the merger occurs at the beginning of the period or after a delay of the length $\tau$), and (iv) the shares of the merger surplus captured by the firms participating in the merger. Let $V^I(k^*)$ and $V^E(k^*)$ be the equilibrium present values of total profits of the incumbent $I$ and of an entrant $E$ respectively.

In equilibrium, it must be that the pre-merger number of firms, $k^*$, is such that, given the insider firms' merger strategies and other firms' entry strategies, entry is profitable for $k^*$ firms, but not for $k^* + 1$ firms, i.e., $V^E(k^* + 1) \leq F < V^E(k^*)$. In addition, each firm's merger strategy maximizes this firm's present value of profits, conditional on the new firms' entry strategies and other firms' merger strategies. All the strategies are stationary in the sense that they can only depend upon the number of firms in the industry, but not on the firms' behavior in the past periods.

Thus, suppose that $I$ buys out $j \leq k_t - 1$ competitors in period $t$. (While the relevant proofs allow for the out of equilibrium possibility that $k_t - j > k^*$, in the following discussion we will assume that $k_t - j \leq k^*$, to streamline the exposition.) Then the sum of the present values of all
$j + 1$ firms that merged in period $t$ is given by

$$\pi_{kt-j}(\Delta) + \delta(\Delta)V^I(k^*).$$

That is, if the $j + 1$ firms merge into a single firm, $I$, this firm becomes one of $k_t - j$ firms for the rest of the period and then, at the beginning of the period $t + 1$, has the present value of $V^I(k^*)$. On the other hand, if there is no merger, the total value of the $j + 1$ firms is

$$(j + 1)\pi_{kt}(\Delta) + \delta(\Delta)[V^I(k^*) + jV^E(k^*)].$$

In this case, each firm competes in period $t$ as one of $k$ oligopolists and then, at the beginning of the next period, gets its equilibrium continuation value. (Recall that the firms not merging are, at the beginning of period $t + 1$, equivalent in terms of profitability to new entrants.) The difference between the above two expressions represents the surplus, $S_{kt}^j(\Delta)$, created by period-$t$ merger:

$$S_{kt}^j(\Delta) = \pi_{kt-j}(\Delta) - (j + 1)\pi_{kt}(\Delta) - \delta(\Delta)jV^E(k^*).$$

We now solve the period-$t$ bargaining game through backward induction and characterize (in lemmas 1 and 2 below) the firms’ optimal merger strategies conditional on the pre-merger number of firms, $k_t$. Suppose that the first round of bargaining failed and, after the period of time $\tau$, Nature chose a firm $i$ to make the final merger offer in this period. At this point, the remaining surplus from a merger involving $j + 1$ firms is given by\textsuperscript{10}

$$S_{kt}^j(\Delta - \tau) = \pi_{kt-j}(\Delta - \tau) - (j + 1)\pi_{kt}(\Delta - \tau) - \delta(\Delta - \tau)jV^E(k^*).$$

\textsuperscript{10}It can be checked that this expression captures the merger surplus even if the merger does not include firm $I$.  

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Note that if $S^j_{k_t}(\Delta - \tau) > 0$ it decreases in $\tau$ and that $S^j_{k_t}(\Delta - \tau) \to S^j_{k_t}(\Delta)$ as $\tau \to 0$. This corresponds to a situation in which $I$ has no special advantage over the other firms in merger bargaining. On the other hand, $S^j_{k_t}(\Delta - \tau) \to -\delta(\Delta - \tau)j V^E(k^*) < 0$ for all $j$ as $\tau \to \Delta$. This means that if the bargaining delay, $\tau$, is too long (longer than some $\tau^* < \Delta$), any benefits from merger get dissipated in the event of a disagreement in the first round. In this case, $I$ has all the bargaining power.

**Lemma 1.** Suppose that $k_t \leq k^*$ and $S^j_{k_t}(\Delta - \tau) \geq 0$ for a $j < k_t - 1$. Then $S^M_{k_t}(\Delta - \tau) \equiv S^{k_t-1}_{k_t}(\Delta - \tau) > S^j_{k_t}(\Delta - \tau)$.

Lemma 1 says that, if positive, the second round merger surplus is maximized when the industry is monopolized. Note that this result also holds for the first round surplus, $S^j_{k_t}(\Delta)$, because the lemma allows for $\tau = 0$.

Since the second round of bargaining is final, firm $i$ that was chosen by Nature to make the merger proposals in this round in effect makes a take-it-or-leave-it offer to its targets. Hence, it captures the whole surplus, $S^j_{k_t}(\Delta - \tau)$. Because, by Lemma 1, this surplus is maximized when all firms merge, firm $i$ monopolizes the whole market whenever $S^M_{k_t}(\Delta - \tau) \geq 0$, otherwise, it makes no merger offer. Now, every firm in the industry is equally likely to be chosen by Nature to make the second round offer. Hence, if the first round of bargaining fails, then in the second round each firm gets, in expectation, $\max\{0, \frac{S^M_{k_t}(\Delta - \tau)}{k_t}\}$. This discussion leads to the following result.

**Lemma 2.** Suppose that $k_t \leq k^*$.

(a) If $S^M_{k_t}(\Delta) > 0$, the industry will be monopolized in period $t$, otherwise, no merger will occur in that period.

(b) Suppose that $S^M_{k_t}(\Delta) > 0$, i.e., the industry is monopolized in period $t$. If $\tau \leq \tau^*$, each firm
\[ i \neq I \text{ captures the share } \beta_{k_t} = \frac{\delta(\tau) S^M_{kt}(\Delta-\tau)}{S^M_{kt}(\Delta)} \text{ of the merger surplus } S^M_{kt}(\Delta). \] The incumbent, I, captures the share \[ \alpha_{k_t} = 1 - (k_t - 1)\beta_{k_t}. \] If \( \tau > \tau^* \), then \( \alpha_{k_t} = 1 \) and \( \beta_{k_t} = 0. \)

(c) If \( S^M_{kt}(\Delta) > 0 \), then \( S^M_{kt}(\Delta) = \frac{1-\delta(\Delta)}{1-\delta(\Delta)\alpha_{k^*}} \left[ M - \frac{k^* - \delta(\Delta)}{1-\delta(\Delta)} \pi_{k^*}(\Delta) \right]. \)

According to Lemma 2, if in a given period the pre-merger number of firms is not higher than the equilibrium number \( k^* \), then any merger observed in this period will be a merger to monopoly. Moreover, the industry will be monopolized if and only if the merger surplus is positive. Note that this conclusion does not depend upon the firms’ relative bargaining powers. Because the merger bargaining process is efficient, the firms reach an agreement whenever merger creates a surplus.

We now proceed by investigating the decision problems of potential entrants. For this, we first need to introduce some new notation. When deciding whether to enter, the firms anticipate the outcome of the merger process, conditional on the pre-merger number of firms, \( k_t \). Define \( V^E_1(k_t) \) as the expected present value of profits (gross of the cost of entry, \( F \)) earned by a period-\( t \) entrant, given that there is merger to monopoly in period \( t \), the pre-merger number of firms in that period is \( k_t \), and \( k_s = k^* \) for all \( s \geq t + 1 \). Let \( V^E_2(k_t) \) be defined similarly, but assuming that there is no merger in period \( t \), and let \( \hat{V}^E(k_t) \equiv \max\{V^E_1(k_t), V^E_2(k_t)\} \). Thus, \( \hat{V}^E(k_t) \) is the value of entry in period \( t \) if the pre-merger number of firms ends up to be \( k_t \) in that period and the firms’ merger strategies are as described in Lemma 2. Next, let \( k^0 \) be the free entry number of firms in this industry if no mergers are possible, i.e., \( \pi_{k^0+1}(\Delta) \leq F[1-\delta(\Delta)] < \pi_{k^0}(\Delta) \). Analogously, define \( \hat{k} \) by \( \hat{V}^E(\hat{k}+1) \leq F[1-\delta(\Delta)] < \hat{V}^E(\hat{k}) \). Finally, let \( n_{t-1} \) denote the (post-merger) number of firms that is in the industry at the end of period \( t - 1 \). Using these definitions, the following lemma characterizes the optimal entry strategies of new firms.

**Lemma 3.** (a) If \( n_{t-1} < \hat{k} \), then exactly \( \hat{k} - n_{t-1} \) new firms enter in period \( t \). Otherwise, no new firms enter in that period.
(b) In every period, the pre-merger equilibrium number of firms is \( k^* = \hat{k} \geq k^0 \).

Lemma 3 is just a version of the standard result regarding the free entry number of firms in an industry: firms keep entering as long as the expected present value of total profits after entry can cover the fixed cost of entry, \( F \). The lemma also says that the equilibrium pre-merger number of firms is at least as high as when mergers are not feasible. This is because if merger is observed in equilibrium, it must weakly increase each firm’s lifetime profit.

Given the entry decisions of firms, described in Lemma 3, and given the firms’ merger behavior, described in Lemma 2, we are now in a position to identify the conditions under which merger will be observed in this industry and characterize the equilibrium market structure. In particular, Lemma 2 implies that the industry will be monopolized in equilibrium if and only if the equilibrium merger surplus \( S_{k^0}^{M}(\Delta) \) is positive, i.e.,

\[
M(\Delta) - \frac{k^* - \delta(\Delta)}{1 - \delta(\Delta)} \pi_{k^*}(\Delta) > 0. \tag{2}
\]

The following proposition recasts condition (2) in terms of the model’s more primitive parameters, namely, in terms of the free entry number of firms in the absence of mergers, \( k^0 \).

**Proposition 1.** Let \( D(k) \equiv M - \frac{k - \delta(\Delta)}{1 - \delta(\Delta)} \pi_k(\Delta) \), let \( k^0 \) be as defined in Lemma 3, and let \( S_{k^0+1}^{M}(\Delta) \) be as given in Lemma 2(c).

(i) In the unique equilibrium of this game, the incumbent either buys out all entrants in every period or accommodates all of them in the first period, after which there is no new entry and no merger in subsequent periods.

(ii) If \( D(k^0) > 0 \), the industry is monopolized in every period. If \( D(k^0) \leq 0 \), the industry is monopolized in every period if and only if

\[
\pi_{k^0+1}(\Delta) + \beta_{k^0+1} S_{k^0+1}^{M}(\Delta) > F[1 - \delta(\Delta)].
\]
The entry strategies of new firms described in Lemma 3 mean that if \( I \) wants to preserve its monopoly position, it has to buy out all of the entrants in every period. This attracts \( k^* - 1 \) new firms to enter in the subsequent period and so on. Part (i) in Proposition 1 shows that the incumbent either struggles every period to preserve her monopoly position or gives up completely. It is never optimal in our framework for \( I \) to buy out only some entrants. This result is driven by the fact that the total industry profit is higher under monopoly than under any other industry structure.

Remark 1. Because Proposition 1 assumes that there are no governmental restrictions regarding merger, the number of real world firms pursuing buy-out strategies consistent with this proposition must necessarily be limited. However, one can find supporting evidence from the late 1800's, before antitrust policy was instituted. For example, in the late 1800's, the practice of monopolization through acquisition of entrants was adopted by Standard Oil Company and American Tobacco. More recent examples include Microsoft and Intel, who during the 1990's through the present (with less than rigorous enforcement of antitrust) have both pursued a vigorous strategy of buying out new entrants of importance.

Remark 2. While, empirically, merger to monopoly does not appear to be the prevalent case, some of the most important merger cases where the ease of entry played a major role involved companies that post-merger unambiguously dominated their respective markets. For example, in United States v. Calmar, Inc. the district court, using the ease of entry argument, declined to enjoin the merger of Calmar and Realex, two producers of plastic sprayers, whose post-merger market share in the regular sprayer market would have been 83%, with HHI more than 7,100. In

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12 See www.microsoft.com/msft/invest.htm for a listing of Microsoft’s acquisitions since 1994. Also see www.netaction.org/msft/world/table/html for a table showing more specifics regarding these buyouts. For a description of Intel’s acquisitions, see the company’s acquisitions and capital portfolio web page at http://www.intel.com/intel/finance/acquisitions/.
Laidlaw Acquisition Corp. v. Mayflower Group, Inc. the FTC declined to challenge Laidlaw’s attempted takeover of Mayflower. In the Pacific Northwest, the takeover would have increased Laidlaw’s share of the market for private contract bus services to school children to 85.9%. Again, the FTC’s decision was based primarily on the ease of entry argument. As our final example, we mention the United States v. Syufy Enterprises case. In 1981, Syufy entered the first-run movie market in Las Vegas and through a series of acquisitions came to dominate 90% of the market (reduced to 75% by the time of the suit). Using the ease of entry argument, the Ninth Circuit concluded that there was no violation of section 7.\footnote{See, e.g., Pitofsky (1990) for further details.}

Condition (2) preceding Proposition 1 shows that in our dynamic setting the incumbent is less likely to use merger to preserve her monopoly position than in a static, one period setting. (In a one period setting, merger is optimal whenever $M(\Delta) \geq k^* \pi_{k^*}(\Delta)$, which is always true in our model.) The reason is that the side effect of a merger is that it invites new entry in the future. Thus, the merger can only guarantee a monopoly position for one period, until new entrants appear, but the buy out price for each entrant reflects the entrant’s potential lifetime profit.

**3.3. The effects of the speed of entry**

The first implication of Proposition 1 is that the incentives to monopolize an industry through merger depend upon how quickly can new firms establish their operation in this industry. This relationship is shown formally in the following proposition.

**Proposition 2.** There exists a $\Delta^* \in (0, \infty)$ such that the industry is monopolized in every period if and only if $\Delta > \Delta^*$.

According to Proposition 2, the industry is monopolized through merger only if new firms cannot enter fast enough (that is, if $\Delta > \Delta^*$). Otherwise, the incumbent is better off being one of $k^*$
oligopolists, rather than buying out new entrants every period. As mentioned in the Introduction, this result lends some support to the recommendation found in the 1992 Merger Guidelines, according to which the FTC should assess if entry “can achieve significant market impact within a timely period.” However, the reason given in the Guidelines is just the initial part of our argument. According to the Guidelines, timely entry mitigates the anti-competitive effects of the merger. In our analysis, the reasoning does not stop there: Because the anti-competitive effects of merger are quickly mitigated, the firms do not have an incentive to merge in the first place. Hence, if a merger is initiated in such an industry, it is most likely because it generates production efficiencies (which we do not model here) – it cannot be motivated by market power. However, it is worth noting that this does not necessarily mean that such a merger should always be approved, because, regardless of motivation, it does create monopoly power.

3.4. The effects of the cost of entry

Throughout the rest of the paper we will keep the time period length $\Delta$ fixed. Thus, in order to economize on notation, from now on we will drop the argument $\Delta$ and write $M$ instead of $M(\Delta)$, $\pi_k$ instead of $\pi_k(\Delta)$, and so on.

The second implication of Proposition 1 is that, as condition (2) shows, the incentives to merge for the purpose of creating a monopoly depend upon the resulting industry structure if the merger does not occur. This is reflected through profit $\pi_k^*$ that would be earned by each of the firms in every period in the absence of a merger. According to Lemma 3, this profit is in turn determined by the ease of entry, as captured by the fixed cost of entry $F$. At a first blush, one might think that the lower is the cost of entry, the lower is the equilibrium industry concentration that we should expect to prevail in this economy in the absence of any anti-merger policy. However, as the following proposition demonstrates, this intuition is incorrect in the dynamic setting of the present
model.

**Proposition 3.** Suppose that each firm’s profit declines sufficiently fast with the number of firms in the industry, i.e., \((k - \delta)\pi_k\) decreases in \(k \geq 2\). Then as the cost of entry, \(F\), decreases, the industry is more likely to be monopolized through merger in every period.

According to Proposition 3, if each individual firm’s profit decreases sufficiently fast with the number of firms in the industry, and if mergers are allowed, then the industry is more likely to be monopolized if the cost of entry is relatively low. Note that the condition of Proposition 3 is implied by the stronger condition that the total profit of entrants, \((k - 1)\pi_k\), is decreasing in \(k\), and it is equivalent to this condition if \(\delta\) is close to 1. The intuition for Proposition 3 is most easily seen for the case where \(I\) has all the bargaining power, i.e., \(\beta_{k^*} = 0\): If the cost of entry is low then, in the absence of merger, the industry would be fairly competitive. This has two effects on \(I\)’s incentives to merge. First, if the profit per firm decreases fast with the number of firms, the incumbent’s alternative to merger (namely, being one of \(k^*\) oligopolists) is fairly unattractive. Second, the fast decline in profits per firm makes it relatively cheap to buy out all entrants, because the total buy-out price is \(\frac{(k^* - 1)\pi_{k^*}}{1 - \delta}\) and the condition of the proposition slows any rate of increase in this buy-out price with respect to increases in \(k^*\).

The importance of the result in Proposition 3 can be assessed by contrasting it with the 1992 Merger Guidelines. According to the Guidelines, ease of entry should be a mitigating factor in the challenge of a given merger.\(^{14}\) The implication seems to be that the lower is the sunk cost of entry, the less the need to prohibit merger. Our result shows that by following this guideline, the FTC might be led to approve mergers that are, in spite of the fact that new entry is easy, motivated purely

\(^{14}\)The notion of ‘ease of entry’ as used in the Guidelines is somewhat vague, as it is primarily defined in terms of the effects the new entry has on the degree of industry competitiveness. However, one of the steps the Guidelines propose for gauging the effects of the new entry is the assessment of the profitability of new entry in the presence of sunk entry costs. The ease of entry in our model is measured by sunk entry costs.
by the intent to monopolize the industry (without any gains in efficiency). Similarly, if entry is relatively difficult, in our model this may not be enough to make a merger profitable, unless there are substantial efficiency gains. Hence, challenging a merger proposal in such an industry, just because entry is relatively difficult, would be misguided.

**Example.**

Clearly, the relevance of Proposition 3 depends on how likely it is that the condition on profits holds. To demonstrate that this condition can hold in non-trivial environments, we now show that it is satisfied for the case where the firms compete as Cournot oligopolists with linear demand and cost. Let market demand per period be given by \( p = a - b \left( \sum q_i \right) \), where \( q_i \) denotes the output of firm \( i \) in a given period. If each firm has cost \( sq_i \), then per firm profit \( \pi_k \) is given by \( \pi_k = \frac{h}{(k+1)^2} \), where \( h \equiv \frac{(a-s)^2}{b} \). Using these expressions, the condition \((k - \delta + 1)\pi_{k+1} < (k - \delta)\pi_k \) becomes \( k^2 + k(1 - 2\delta) \geq (1 + 3\delta) \). It can easily be checked that with any \( \delta \in [0, 1] \), the condition is met for all \( k \geq 3 \). The condition is satisfied for \( k = 2 \), if \( \delta < 5/7 \).

To illustrate the proposition, assume again that \( \beta_{k^*} = 0 \) and note that in this case it must always be that \( k^* = k^0 \), because the entrants do not expect to share in any potential merger surplus. Note also that, in this example, condition (2) holds if and only if

\[
(1 - \delta)k^*^2 - 2(1 + \delta)k^* + 3\delta + 1 > 0. \tag{3}
\]

If we suppose that the cost of entry is some \( F_1 \in \left[ \frac{h}{10(1-\delta)}, \frac{h}{9(1-\delta)} \right) \), then the equilibrium pre-merger number of firms is \( k^* = 2 \) and (3) holds if and only if \( \delta < 1/5 \). Next suppose that the cost of entry decreases to some \( F_2 \in \left[ \frac{h}{25(1-\delta)}, \frac{h}{10(1-\delta)} \right) \). In this case \( k^* = 3 \), so that (3) holds if and only if \( \delta < 1/3 \). Thus, for discount factors \( \delta \in \left( \frac{1}{5}, \frac{1}{3} \right) \), the industry is not monopolized when the cost of entry is \( F_1 \), but it becomes monopolized when the cost of entry decreases to \( F_2 \).
We now add entry deterrence as an alternative to merger and turn our attention to \( I \)'s choice of the method for preserving her monopoly position.

4. Merger or entry deterrence?

In reality, merger is not the only tool that firms can use to preserve or gain monopoly power. If merger is likely to be challenged, then the incumbents may instead focus on driving their existing rivals out of the industry or preventing new firms from entering. Since the logic is the same in both of these cases, we will couch our analysis in terms of entry deterrence. We show here that the optimal anti-merger policy should take into account the possibility that the incumbents will step up their entry deterrence activities if they believe merger is not a feasible alternative, in which case a tough anti-merger policy can decrease overall welfare.

There are many possible activities a monopolist can engage in to try to prevent entry. This may include writing exclusive contracts with input suppliers, brand proliferation, limit pricing, vertical integration with a supplier, and so on. We will not model the specific nature of these activities – enough has been written about these by other authors. We will simply assume that these activities are costly to the incumbents and can represent a waste of resources. Examples of wasteful entry-deterrence activities include holding extra production capacity or investing excessively in cost reducing technology and R&D. We formalize entry deterrence in a simple way: We assume that in any given period, the incumbents can completely deter entry by spending the total amount of \( K > 0 \) on deterrence activities and that any amount smaller than \( K \) will not discourage a potential entrant.\(^{15}\) If at the beginning of a given period there are \( n \geq 1 \) incumbents, they share the amount \( K \) equally, i.e., each has to pay \( K/n \). Formally, this would be equivalent to a technology that allows the incumbents to increase the new firms’ cost of entry, \( F \), to a level at which no new entry

\(^{15}\)This formalization of entry deterrence is similar to that used, for example, in Bernheim (1984).
is profitable, i.e., \( k^* = 1 \). If entry is not deterred in a given period, the industry can still be monopolized in that period, through merger.

The timing of events that occur at the beginning of each period is summarized as follows:

1. The incumbents decide whether to spend \( K \) on deterrence activities.
2. If \( K \) is spent, then no entry occurs in that period. If less than \( K \) is spent, then new firms can enter as long as they find the entry profitable.
3. If mergers are not prohibited, industry insiders enter merger negotiations.

The next lemma demonstrates that our analysis of the merger behavior derived in the previous section continues to hold in the current framework.

**Lemma 4.** Suppose that entry is not deterred in equilibrium. Then the incumbent either buys out all entrants in every period or accommodates all of them in the first period, after which there is no new entry and no merger in subsequent periods.

In order to be able to assess the welfare effects of \( I \)'s trade-off between merger and entry deterrence, we first need to introduce the social cost of entry deterrence. When \( I \) spends an amount \( K \) on entry deterrence activities, this amount represents \( I \)'s dissipated profit, but not all of that is a deadweight loss. Some entry deterrence activities, such as limit pricing, result in consumer gains. To capture this situation, we define the social cost of entry deterrence as the difference \( K - \Delta CS \), where the term \( \Delta CS \geq 0 \) represents a gain in consumer surplus induced through entry deterrence. The case \( \Delta CS = 0 \) represents a situation where the deterrence activity is completely wasteful.

Due to the stationary nature of the game, if entry deterrence is an optimal strategy in some period, then it is optimal in every period. Hence, we need to consider two cases – one in which \( I \) deters entry in every period and one in which \( I \) never deters entry.

**Case 1:** \( I \) does not deter entry
Since this is the same problem as the one analyzed in the previous section, Proposition 1 immediately applies. That is, \( I \) buys out all entrants or otherwise it buys out none of them. In this case, \( I \)'s total discounted profit if entry is not deterred and merger is optimal is given by

\[
V_{ND}^I(k^*) = \frac{\pi_{k^*} + \alpha_{k^*} S_{k^*}^M}{1 - \delta},
\]

where the subscript \( ND \) stands for “no deterrence”. That is, in every period, \( I \) gets the oligopoly profit \( \pi_{k^*} \), plus its share of the merger surplus, \( S_{k^*}^M \).

**Case 2: \( I \) deters entry**

If \( I \) decides to deter entry, its total profits are given by

\[
V_{ED}^I = \frac{M - K}{1 - \delta}.
\]

Here, the subscript \( ED \) stands for “entry deterrence”.

First, entry deterrence can only be profitable if \( V_{ED}^I \geq \frac{\pi_{k^*}}{1 - \delta} \), that is, \( K < M - \pi_{k^*} \), because otherwise \( I \) would be better off allowing \( k^* - 1 \) firms to enter and accommodating them. Second, comparing \( V_{ND}^I(k^*) \) with \( V_{ED}^I \) we find that monopolization through entry deterrence is more profitable than monopolization through merger if \( K \leq K_1 \equiv M - \pi_{k^*} - \alpha_{k^*} S_{k^*}^M = \frac{(1 - \alpha_{k^*}) M - (1 - \alpha_{k^*} k^*) \pi_{k^*}}{1 - \delta \alpha_{k^*}} \).

Using this preliminary analysis, we obtain the following result.

**Proposition 4.** Let \( K_2 \equiv (k^* - 1) F + \Delta CS \). Suppose that (2) holds and entry deterrence cannot be prevented by the government.

(a) If \( K_1 < K < K_2 \), then antitrust laws that prohibit mergers increase social welfare.

(b) If \( K_{\max} \equiv \max \{ K_1, K_2 \} < M - \pi_{k^*} \) and \( K \in (K_{\max}, M - \pi_{k^*}) \), then antitrust laws that prohibit mergers decrease social welfare.
(c) For all other $K$, antitrust laws that prohibit mergers have no effect on social welfare.

While sometimes anti-merger laws can be socially beneficial, part (b) in Proposition 4 shows that, if entry deterrence cannot be prevented, prohibiting mergers can decrease social welfare. When entry deterrence cost is relatively high (above the threshold $K^{\text{max}}$), entry deterrence becomes an inefficient substitute for merger.\footnote{A related question is how successful are the anti-merger laws in creating a relatively competitive environment in the economy. The implicit assumption behind the FTC Guidelines seems to be that the application of anti-merger laws always results in a less concentrated industry structure. Is this assumption justified? Due to space considerations, we do not address this question in the present article. However, in an earlier version of this paper we demonstrate that for a range of parameter values, anti-merger policy can lead to a more concentrated industry than a laissez-faire policy.}

To relate the results of Proposition 4 to our previous analysis and to the role of the ease of entry in anti-merger policy, fix some $K'$ and assume that it is smaller than $M - \pi_{k^*}$, so that entry deterrence can be profitable. We are interested in expressing part (b) in the proposition in terms of the ease of entry, $F$. This part applies when $K_1$ and $K_2$ are both smaller than $K'$. It is clear that $K_2 = (k^* - 1)F + \Delta CS$ is small when $F$ is small. Now suppose that the incumbent has all the bargaining power, $i.e., \alpha_{k^*} = 1$. Then $K_1 = \frac{(k^* - 1)\pi_{k^*}}{1 - \delta}$, which tends to be small for small $F$ if the entrants’ combined profits, $(k^* - 1)\pi_{k^*}$, decline fast with the number of firms. On the other hand, if the entrants’ combined profits do not decrease fast enough with $k^*$ or if the incumbent’s bargaining power is relatively small, then small $F$ implies small $K_2$ but large $K_1$.\footnote{To see this, suppose that $I$’s bargaining power is the smallest possible, i.e., the same as the bargaining power of entrants (recall that this case corresponds to $\tau = 0$). Then $K_1 \equiv M - \pi_{k^*} - \beta_k S_{k^*} M = M - (1 - \delta)V_1^E(k^*)$, which increases in $k^*$, because, as was shown in the proof of Proposition 1, $V_1^E(.)$ is a decreasing function. Therefore, in this case $K_1$ is large when $F$ is small.} In this case, Proposition 3 says that anti-merger policy has no effect.

Hence, when wasteful entry deterrence is not prohibitively costly and the cost of entry is relatively low, antitrust laws that preclude mergers tend to decrease social welfare (or have no effect) in our model. This logic is very different from that in the Guidelines. The Guidelines consider easy entry as a mitigating factor in merger because it is presumed that the resulting monopoly power
would be partially offset due to likely subsequent entry. As we have shown in Proposition 3, this is argument is incomplete, because when the costs of entry are low, firms inside the industry may have a stronger incentive to buy out new entrants than when the entry costs are high. However, when firms can engage in entry deterrence activities, it is optimal in our framework to permit mergers if the cost of entry is low, in order to avoid a waste of resources spent by the incumbents on entry deterrence activities.

The policy implications of our model can then be summarized as follows. If it is hard to prevent the incumbents from engaging in entry deterrence activities (or in activities aimed at driving existing firms out of the industry), easy entry should always be considered as a mitigating factor in merger. However, if entry deterrence is not very wasteful or can easily be prevented, then Proposition 3 applies and low entry costs are not an unambiguous mitigating factor. In particular, the ease of entry defence should be discounted if the profit per firm decreases fast with the number of firms in the industry.

5. Conclusion

In this paper we take a closer look at the effects of entry on firms’ incentives to monopolize an industry through horizontal merger. The importance of entry of new firms (or expansion by existing rivals) in assessing the effects of a horizontal merger has been recognized both in academic literature (e.g., Werden and Froeb, 1998, Cabral, 1999, Spector, 2000) and in the 1992 Merger Guidelines issued jointly by the U.S. Justice Department and FTC. We show that the Guidelines’ recommendation that if new entry is relatively fast there is less need to challenge a merger, makes sense in our dynamic model, although our reasoning is somewhat different from the one found in the Guidelines. The Guidelines suggest that if the new entry is fast, it can quickly mitigate any anti-competitive effect of the merger. We point out that if this is true, then the incumbents cannot
gain by merging unless there are substantial synergies created by the merger. Hence, if a merger is
initiated in an industry where new entry would follow relatively quickly, then it is probably because
the merger would enhance the merged firms’ efficiency – if it did not, it would not be attempted in
the first place. This point is similar to the one made independently by Spector (2001). However,
Spector’s argument is based on a one period model and focuses on entry costs rather than the time
dimension of entry (speed of entry).\footnote{Spector’s result is a generalization of the idea found in Werden and Froeb (1998).}

Our second result is less supportive of the conclusions reached in the Guidelines. We demon-
strate that a decrease in exogenous entry costs can increase the motivation of the incumbent firms
to monopolize the market through horizontal merger. The alternative would be to operate in a
highly competitive industry implied by low entry costs, which makes merger look more attrac-
tive. In such an industry, following the Guidelines can lead the FTC to approve mergers aimed at
monopolizing the market and challenge those that generate efficiency gains.

The last point that we make is that a merger challenge decision should not be made without
considering the incumbents’ alternative means of preserving or gaining market power. Merger may
well be the most efficient of all these alternatives, and prohibiting it without preventing the firm
from channelling its monopolization efforts elsewhere could be counter-productive.
Appendix: Proofs

**Proof of Lemma 1:** We have \( S_{kt}^M(\Delta - \tau) = M(\Delta - \tau) - k_t \pi_{kt}(\Delta - \tau) - \delta(\Delta - \tau)(k_t - 1)V^E(k^*) > \pi_{kt-j}(\Delta - \tau) - (j+1)\pi_{kt}(\Delta - \tau) - \delta(\Delta - \tau)jV^E(k^*) = S_{kt}^j(\Delta - \tau) \) if \((k_t-j-1)\pi_{kt-j}(\Delta - \tau)-(k_t-j-1)\pi_{kt}(\Delta - \tau) - \delta(\Delta - \tau)(k_t-j-1)V^E(k^*) > 0\), where we have used \( M(\Delta - \tau) \geq (k_t-j)\pi_{kt-j}(\Delta - \tau) \).

Since \( k_t-j-1 > 0 \), the inequality in the lemma holds because \( \pi_{kt-j}(\Delta - \tau) - \pi_{kt}(\Delta - \tau) - \delta(\Delta - \tau)V^E(k^*) > S_{kt}^j(\Delta - \tau) \geq 0 \).

**Proof of Lemma 2:** Parts (a) and (b). If \( S_{kt}^M(\Delta) < 0 \), then, by Lemma 1, \( S_{kt}^j(\Delta) < 0 \) for all \( j \).

Therefore, no mergers are initiated, because at least one participating firm would have to be made worse off if the merger goes through.

Thus, suppose that \( S_{kt}^M(\Delta) > 0 \). Assume first that \( \tau < \tau^* \). Then also \( S_{kt}^M(\Delta - \tau) > 0 \), and, by the argument preceding the lemma, if the bargaining fails in the first round, the firm that gets to make the merger offers in the second round captures the whole remaining surplus, \( S_{kt}^M(\Delta - \tau) \).

Because every firm has the same chance of making the second round offer, and because the second round comes only after the period of time \( \tau \), the expected present value of getting into the second round is \( \delta(\tau) \frac{S_{kt}^M(\Delta - \tau)}{k_t} \) for every firm. Thus, \( I \) has to offer at least this amount to every firm it decides to buy out (being a profit maximizer, \( I \) offers exactly this amount). Therefore, \( I \)'s value if it buys out \( j \) competitors is \( \pi_{kt}(\Delta) + \left[ S_{kt}^j(\Delta) - j\delta(\tau) \frac{S_{kt}^M(\Delta - \tau)}{k_t} \right] + \delta(\Delta)V^I(k^*) \), where the term in the brackets represents the part of the merger surplus captured by \( I \). This term is maximized at \( j = k_t - 1 \), that is, when the industry is monopolized, because

\[
S_{kt}^M(\Delta) - (k_t - 1)\delta(\tau) \frac{S_{kt}^M(\Delta - \tau)}{k_t} > S_{kt}^j(\Delta) - j\delta(\tau) \frac{S_{kt}^M(\Delta - \tau)}{k_t}
\]

for any \( j < k_t - 1 \). To see this, rewrite the inequality as \( S_{kt}^M(\Delta) - S_{kt}^j(\Delta) > (k_t - j - 1)\delta(\tau) \frac{S_{kt}^M(\Delta - \tau)}{k_t} \).
which holds if
\[(j + 1)S_{kt}^M(\Delta) - k_t S_{kt}^j(\Delta) > 0,\]
because \(\delta(\tau) \leq 1\) and \(S_{kt}^M(\Delta - \tau) \leq S_{kt}^M(\Delta)\). Substituting for \(S_{kt}^M(\Delta)\) and \(S_{kt}^j(\Delta)\), we obtain
\[(j + 1)S_{kt}^M(\Delta) - k_t S_{kt}^j(\Delta) = (j + 1)[M(\Delta) - k_t \pi_{kt}(\Delta) - \delta(\Delta)(k_t - 1)V^E(k^*)]\]
\[-k_t[\pi_{kt-j}(\Delta) - (j + 1)\pi_{kt}(\Delta) - \delta(\Delta)jV^E(k^*)]\]
\[= (j + 1)M(\Delta) - k_t \pi_{kt-j}(\Delta) - (k_t - j - 1)\delta(\Delta)V^E(k^*)\]
\[\geq (k_t - j - 1)[\pi_{kt-j}(\Delta) - \delta(\Delta)V^E(k^*)] > (k_t - j - 1)S_{kt}^j(\Delta).\]

Hence, (*) holds if \(S_{kt}^j(\Delta) \geq 0\), because \(j < k_t - 1\). If \(S_{kt}^j(\Delta) < 0\), then the right hand side of (*) is negative, while the left hand side is positive (due to \(S_{kt}^M(\Delta) > 0\), so that (*) again holds.

Thus, in this case, \(I\)'s profit maximizing strategy in the first round of bargaining is to offer \(\delta(\tau)\frac{S_{kt}^j(\Delta - \tau)}{k_t}\) to every firm in the industry. Given that no firm can do better by rejecting the offer, each firm accepts in the first round. This proves the claim that if \(S_{kt}^M(\Delta) > 0\) and \(\tau < \tau^*\), the industry is monopolized, each firm \(i \neq I\) gets \(\delta(\tau)\frac{S_{kt}^i(\Delta - \tau)}{S_{kt}^i(\Delta)} S_{kt}^M(\Delta) = \beta_{kt} S_{kt}^M(\Delta)\) in addition to its disagreement value, and the incumbent receives \(S_{kt}^M(\Delta) - \delta(\tau)\frac{(k_t-1)S_{kt}^M(\Delta - \tau)}{k_t} = [1 - (k_t - 1)\beta_{kt}]S_{kt}^M(\Delta) = \alpha_{kt} S_{kt}^M(\Delta)\).

Finally, suppose that \(S_{kt}^M(\Delta) > 0\) and \(\tau \geq \tau^*\). Then \(S_{kt}^M(\Delta - \tau) < 0\), which means that if the bargaining fails in the first round, it is not worth resuming it in the second round. Hence, every firm approached by \(I\) in the first round accepts any buyout price higher or equal to its disagreement value, \(\pi_{kt}(\Delta) + \delta(\Delta)V^E(k^*)\). A profit-maximizing incumbent will therefore offer exactly this amount in the first round and each firm will accept. Thus, \(I\) captures the whole merger surplus \(S_{kt}^j(\Delta)\).

Because, \(S_{kt}^j(\Delta)\) is maximized at \(j = k_t - 1\) (Lemma 1), \(I\) again buys out all of its competitors.

Part (c). Suppose that \(S_{kt}^M(\Delta) > 0\), i.e., merger is optimal if the equilibrium pre-merger number
of firms is $k^*$. Then in every period, $k^* - 1$ new firms enter, to be subsequently bought out by $I$. Each entrant’s discounted present value is $V^E(k^*)$, while $I$’s value is $V^I(k^*)$. Because the total value of discounted profits in this industry, if monopolized, is $\frac{M}{1-\delta(\Delta)}$, it must be that

$$\frac{M}{1-\delta(\Delta)} = V^I(k^*) + (k^* - 1) V^E(k^*) + \delta(\Delta)(k^* - 1) V^E(k^*) + \delta(\Delta)^2 (k^* - 1) V^E(k^*) + \ldots$$

$$= V^I(k^*) + \frac{(k^* - 1)}{1-\delta(\Delta)} V^E(k^*).$$

Substituting for $V^I(k^*) = \frac{\pi_{k^*}(\Delta) + \alpha_{k^*} S_{k^*}(\Delta)}{1-\delta(\Delta)}$ and $V^E(k^*) = \frac{\pi_{k^*}(\Delta) + \beta_{k^*} S_{k^*}(\Delta)}{1-\delta(\Delta)}$, and using $\alpha_{k^*} + (k^* - 1) \beta_{k^*} = 1$, we obtain $S_{k^*}^M(\Delta) = \frac{1-\delta(\Delta)}{1-\delta(\Delta) \alpha_{k^*}} \left[ M - \frac{k^* - \delta(\Delta)}{1-\delta(\Delta)} \pi_{k^*}(\Delta) \right]$. \hfill \blacksquare

**Proof of Lemma 3:** (a) If $k_t \leq k^*$ and $S_{k_t}^M(\Delta) = M(\Delta) - k_t \pi_{k_t}(\Delta) - \delta(\Delta)(k_t - 1) V^E(k^*) < 0$, there is merger to monopoly (Lemma 2) and the entrants share the resulting surplus, $S_{k_t}^M(\Delta)$, according to their bargaining powers, $\beta_{k_t}$. In this case, the value after entry is $V_{1}^E(k_t)$, given by

$$V_{1}^E(k_t) = \pi_{k_t}(\Delta) + \delta(\Delta) V^E(k^*) + \beta_{k_t} S_{k_t}^M(\Delta).$$

When $S_{k_t}^M(\Delta) \leq 0$, no merger is expected and the value after entry is $V_{2}^E(k_t)$, given by

$$V_{2}^E(k_t) = \pi_{k_t}(\Delta) + \delta(\Delta) V^E(k^*).$$

If $k_t > k^*$, there are three possibilities. Let $\hat{S}_{k_t}^j(\Delta)$ denote the merger surplus if $k_t > k^*$ and $I$ buys out $j$ firms in period $t$, and let $\hat{\beta}_{k_t}$ be the share of this surplus that goes to a firm $i \neq I$ involved in the merger. The first possibility is that no merger creates a positive surplus, i.e., $\hat{S}_{k_t}^j(\Delta) \leq 0$ for all $j$. In this case, the value of entry is $V_{3}^E(k_t)$, given by

$$V_{3}^E(k_t) = \frac{\pi_{k_t}(\Delta)}{1-\delta(\Delta)}.$$
because if merger is not profitable in period \( t \), it is not profitable in any subsequent period. Moreover, no new firms enter in subsequent periods, because the number of firms in the industry is already greater than the equilibrium number, \( k^* \) (and \( k^* \) is unique, as will be shown in part (b)).

Now, due to the possibility that the entrant captures a share of the merger surplus (if positive), it must be that \( V^E(k^*) \geq \frac{\pi_{k^*}(\Delta)}{1-\delta(\Delta)} \). Hence, \( V^E(k_t) \geq \pi_{k_t}(\Delta) + \delta(\Delta) \frac{\pi_{k^*}(\Delta)}{1-\delta(\Delta)} \). For \( k_t > k^* \), this implies \( V^E(k_t) > \pi_{k_t}(\Delta) + \delta(\Delta) \frac{\pi_{k^*}(\Delta)}{1-\delta(\Delta)} = V^E(k_t) \). Hence, \( V^E(k_t) < F \) for \( k_t > k^* \).

The second possibility is that \( \hat{S}_{k_t}^j(\Delta) \) is maximized and positive for some \( j \geq k_t - k^* \). In this case, the same proof as in Lemma 1 shows that \( S^M_{k_t}(\Delta) > S^I_{k_t}(\Delta) \), which means that the value of entry is \( V^E_4(k_t) \). The final possibility is that \( \hat{S}_{k_t}^j(\Delta) \) is maximized and positive for some \( j^* < k_t - k^* \). In this case, no new mergers will occur, because if another merger could create positive surplus, then \( \hat{S}_{k_t}^j(\Delta) \) could not have been maximized at \( j^* \).

The value of entry in this case is therefore given by

\[
V^E_4(k_t) = \pi_{k_t}(\Delta) + \hat{\beta}_{k_t} \hat{S}_{k_t}^{j^*}(\Delta) + \delta(\Delta) \frac{\pi_{k_t-j^*}(\Delta)}{1-\delta(\Delta)},
\]

where \( \hat{S}_{k_t}^{j^*}(\Delta) = \pi_{k_t-j^*}(\Delta) - (j^*+1)\pi_{k_t}(\Delta) \). That is, in this case the benefit of the time-\( t \) merger is given by the extra profits the \( j^* + 1 \) firms earn in period \( t \) as a result of the merger – if they did not merge in period \( t \) they would earn \( \pi_{k_t}(\Delta) \) each and then merge in period \( t + 1 \). But \( \pi_{k_t}(\Delta) + \beta_{k_t} \hat{S}_{k_t}^{j^*}(\Delta) = \hat{\beta}_{k_t} \pi_{k_t-j^*}(\Delta) + [1 - (j^*+1)] \hat{\beta}_{k_t} \pi_{k_t}(\Delta) < \hat{\beta}_{k_t} \pi_{k_t-j^*}(\Delta) + (1 - \hat{\beta}_{k_t}) \pi_{k_t}(\Delta) \leq \pi_{k_t-j^*}(\Delta) \), which means that \( V^E_4(k_t) < \frac{\pi_{k_t-j^*}(\Delta)}{1-\delta(\Delta)} = V^E_3(k_t - j^*) \).

Now, let \( k'_t \) be the largest \( k_t \) such that \( V^E_4(k_t) > F \) and \( k''_t \) the largest \( k_t \) such that \( V^E_2(k_t) > F \) and let \( k_t^{\text{MAX}} = \max\{k'_t, k''_t\} \). Backward induction implies that the pre-merger number of firms
in period $t$ will be at least $k^\text{max}_t$.\footnote{That is, suppose that the industry contains $k^\text{max}_t$ firms. Then at least one more firm has an incentive to enter, because by the definition of $k^\text{max}_t$, the value of entry for that firm is $V^E(k^\text{max}_t) \equiv \max\{V^E(k^\text{max}_t), V^E(k^\text{max}_t - 1)\} > 0$ . Next suppose that the industry contains $k^\text{max}_t - 1$ firms. Then the entry value of firm $k^\text{max}_t - 1$ is also $V^E(k^\text{max}_t - 1) > 0$, because it expects that if it enters then, subsequently, firm $k^\text{max}_t$ will enter, too. Moving backwards in this fashion, one can show that if the industry contains $n < k^\text{max}_t$ firms, exactly $k^\text{max}_t - n$ firms have an incentive to enter, because for each of them, the anticipated value of entry is $V^E(k^\text{max}_t) > 0$.} At the same time, by construction of $k^\text{max}_t$, it must be that $V^E_1(k_t) < F$ and $V^E_2(k_t) < F$ for all $k_t > k^\text{max}_t$. Moreover, $V^E_3(k_t) < F$ and $V^E_4(k_t) < F$ because for $k_t > k^*$, $V^E_3(k_t) < V^E_2(k_t) < F$, as shown above, and $V^E_4(k_t) < V^E_3(k_t - j^*) < F$, where the last inequality follows from $k_t - j^* > k^*$. Thus, consider a $k_t > k^\text{max}_t$. If $k_t \leq k^*$, then the value of entry is either $V^E_1(k_t)$ or $V^E_2(k_t)$ which are both less than $F$, for $k_t < k^\text{max}_t$. If, on the other hand, $k_t > k^*$, then the value of entry is either $V^E_3(k_t)$ or $V^E_4(k_t)$, which were shown to be less than $F$, so entry is again not profitable. Therefore, once the industry contains at least $k^\text{max}_t$ firms, no additional firms will enter in period $t$. Moreover, exactly $k^\text{max}_t - n_{t-1}$ firms enter if $n_{t-1} < k^\text{max}_t$. Finally, by construction, $k^\text{max}_t$ is given by $\hat{V}^E(k^\text{max}_t + 1) \leq F [1 - \delta(\Delta)] < \hat{V}^E(k^\text{max}_t)$, where $\hat{V}^E(k_t) = \max\{V^E_1(k_t), V^E_2(k_t)\}$. This means that $k^\text{max}_t = \hat{k}$ (which is time independent, because the function $\hat{V}^E(\cdot)$ is the same in every period).

(b) By the arguments above, in equilibrium it must be $k_t = k^\text{max}_t = \hat{k}$ in every period. Hence, $k^* = \hat{k}$ in every period. Now, using $k_t = k^*$ for all $t$, we get $V^E_1(k^*) = \frac{\pi_{k^*}(\Delta) + \beta_{k^*}S^M_0(\Delta)}{1 - \delta(\Delta)}$ and $V^E_2(k^*) = \frac{\pi_{k^*}(\Delta)}{1 - \delta(\Delta)}$. Hence, $\hat{V}^E(k^*) \geq V^E_2(k^*)$ for all possible $k^*$. Because $k^* = \hat{k}$, $k^*$ is given by $\hat{V}^E(k^* + 1) \leq F [1 - \delta(\Delta)] < \hat{V}^E(k^*)$, while $k^0$ is defined by $V^E_2(k^0 + 1) \leq F [1 - \delta(\Delta)] < V^E_2(k^0)$. From $\hat{V}^E(k^*) \geq V^E_2(k^*)$, for all $k^*$, and because both $\hat{V}^E(k^*)$ and $V^E_2(k^*)$ are decreasing in $k^*$, it must be that $k^* \geq k^0$. \hfill $\blacksquare$

**Proof of Proposition 1:** Start with part (ii). If no merger occurs in equilibrium, then the equilibrium pre-merger number of firms is $k^* = k^0$ and the value of entry is

\[
V^E_2(k^0) = \frac{\pi_{k^0}(\Delta)}{1 - \delta(\Delta)}.
\]
On the other hand, if there is merger in equilibrium, then the value of entry is

\[ V_1^E(k^*) = \frac{\pi_{k^*}(\Delta) + \beta_{k^*}S_M^M(\Delta)}{1 - \delta(\Delta)} = \frac{\pi_{k^*}(\Delta) + \frac{\delta(\tau)}{k^*} S_{M}^{\hat{M}}(\Delta - \tau)}{1 - \delta(\Delta)}. \]

We will first show that \( V_1^E(k^*) \) decreases in \( k^* \). To see this, substitute for \( S_{M}^{\hat{M}}(\Delta - \tau) \) from (1) to obtain

\[ V_1^E(k^*)[1 - \delta(\Delta)] = \pi_{k^*}(\Delta) + \frac{\delta(\tau)}{k^*} [M(\Delta - \tau) - k^* \pi_{k^*}(\Delta - \tau) - \delta(\Delta - \tau)(k^* - 1)V_1^E(k^*)], \]

where we have used the fact that in this case \( V^E(k^*) = V_1^E(k^*) \). Rearranging and using \( \pi_{k^*}(\Delta) = \hat{\pi}_{k^*}[1 - \delta(\Delta)] \) and \( \delta(\tau)\delta(\Delta - \tau) = \delta(\Delta) \), we have

\[ V_1^E(k^*)[1 - \frac{\delta(\Delta)}{k^*}] = \frac{\delta(\tau)}{k^*} M(\Delta - \tau) + \pi_{k^*}(\tau). \]

Since the right hand side decreases in \( k^* \) and \( 1 - \frac{\delta(\Delta)}{k^*} \) increases in \( k^* \), it must be that \( V_1^E(\cdot) \) is a decreasing function of \( k^* \).

Next, notice that \( k^* > k^0 \) if and only if \( V_1^E(k^0 + 1) > F \). To see this, note that if \( V_1^E(k^0 + 1) \leq F \), then \( V_1^E(k) \leq F \) for all \( k \geq k^0 + 1 \) because \( V_1^E(\cdot) \) is a decreasing function. This means that \( \hat{V}^E(k) \leq F \) for all \( k > k^0 \), where \( \hat{V}^E(k) \) is the value function defined in Lemma 3. Therefore, it cannot be that \( k^* > k^0 \). On the other hand, if \( V_1^E(k^0 + 1) > F \), then \( \hat{V}^E(k^0 + 1) > F \), which implies that \( k^* > k^0 \).

Now, let \( k^* = k^0 \), and note that \( V_1^E(k^0) > V_2^E(k^0) \) if and only if \( S_{k^0}^M(\Delta) > 0 \), that is, if and only if \( D(k^0) > 0 \), because according to Lemma 2(c), the equilibrium merger surplus is given by

\[ S_{k^*}^M(\Delta) = \frac{1 - \delta(\Delta)}{1 - \delta(\Delta)\alpha_{k^*}} [M - \frac{k^* - \delta(\Delta)}{1 - \delta(\Delta)} \pi_{k^*}(\Delta)] = \frac{1 - \delta(\Delta)}{1 - \delta(\Delta)\alpha_{k^*}} D(k^*). \]

Suppose first that \( D(k^0) > 0 \) and assume that there is no merger in equilibrium. Because \( D(k^0) > 0 \), it must be that \( S_{k^0}^M(\Delta) > 0 \),
which means that the firms have an incentive to merge if the equilibrium number of them enter. This contradicts the assumption that there is no merger in equilibrium; hence, if $D(k^0) > 0$, the equilibrium includes merger.

Next assume that $D(k^0) \leq 0$. First, suppose that $\pi_{k^0+1}(\Delta) + \beta_{k^0+1} S_{k^0+1}^M(\Delta) > F[1 - \delta(\Delta)]$. Then $V_1^E(k^0 + 1) > F$, which, as shown above, means that $k^* > k^0$. This in turn implies that there must be merger in equilibrium, because otherwise it would have to be that $\hat{V}_1^E(k^0) = V_2^E(k^0)$, which would imply $k^* = k^0$. Finally, suppose that $\pi_{k^0+1}(\Delta) + \beta_{k^0+1} S_{k^0+1}^M(\Delta) \leq F[1 - \delta(\Delta)]$. This means that $V_1^E(k^0 + 1) \leq F$, which, from above, implies that $k^* = k^0$. But $D(k^0) \leq 0$ means that also $S_{k^0}^M(\Delta) = S_{k^*}^M(\Delta) \leq 0$, so that there is no merger in equilibrium.

Part (i). The part of this claim regarding $I$’s equilibrium buy-out strategy follows directly from part (a) in Lemma 2. To see that the equilibrium is unique (up to the identity of entrants), note that $k_{\text{max}}$ in the proof of Lemma 3 is unique by construction. Because $k^* = k_{\text{max}}$, the equilibrium number of firms, $k^*$, is also unique. 

**Proof of Proposition 2:** The industry is monopolized if and only if condition (2) preceding Proposition 1 holds. We will first show that, holding all other parameters fixed, an increase in $\Delta$ makes this condition more likely to hold, i.e., the left hand side of (2) ($\text{LHS}(2)$) weakly increases in $\Delta$. Using $\tilde{\pi}_k/r = \frac{\pi_{k+1}(\Delta)}{1 - \delta(\Delta)}$, condition (2) can be rewritten as

$$\tilde{\pi}_k [k^* - \delta(\Delta)] - \tilde{M}[1 - \delta(\Delta)] < 0.$$  

(2')

Note also that $k^0$ is independent of $\Delta$, because $\pi_{k^0+1}(\Delta) \leq F[1 - \delta(\Delta)] < \pi_{k^0}(\Delta)$, which defines $k^0$, is equivalent to $\tilde{\pi}_{k^0+1} \leq Fr < \tilde{\pi}_{k^0}$. We want to show that the $\text{LHS}(2')$ weakly decreases in $\Delta$.

Suppose first that an increase in $\Delta$ does not change $k^*$. Then differentiating the $\text{LHS}(2')$ with respect to $\Delta$ yields $\frac{\partial \text{LHS}(2')}{\partial \Delta} = \delta'(\Delta)[\tilde{M} - \tilde{\pi}_{k^*}] < 0$ for all $\Delta$, where the inequality follows because
\[ \delta'(\Delta) < 0. \]  
Next assume that \( k^* \) changes with \( \Delta \). Then there are two possibilities. First, \( k^* \) was equal to \( k^0 \) before \( \Delta \) increased. Since it must be that \( k^* \geq k^0 \) and \( k^0 \) does not change with \( \Delta \), \( k^* \) must have increased, so that now \( k^* > k^0 \). But this can only hold if the industry is monopolized (see the proof of Proposition 1). Hence, in this case \((2')\) must be satisfied after an increase in \( \Delta \), whereas it may have or may not have been satisfied before. Since this argument holds for any \( \Delta \), this implies that, in this case, \( LHS(2') \) weakly decreases in \( \Delta \).

The second possibility is that \( k^* > k^0 \) before the increase in \( \Delta \). In this case, it must be that \( \dot{V}^E(k^*) = V_1^E(k^*) = \frac{\pi_{k^*} M + \beta_{k^*} S_{k^*}^{M}(\Delta)}{1 - \delta(\Delta)} > F \). But the function \( V_1^E(k^*) \) is continuous in \( \Delta \). Hence the inequality \( V_1^E(k^*) > F \) is preserved if \( \Delta \) increases slightly. But this means that \( k^* \) does not change with a slight increase in \( \Delta \), which is the case we already considered. This concludes the proof of the claim that \( LHS(2') \) weakly decreases in \( \Delta \).

Now, setting \( \Delta = 0 \) yields \( \delta(\Delta) = 1 \), so that \( LHS(2') = \bar{\pi}_{k^*}[k^* - \delta(\Delta)] > 0 \) for any \( k^* \), which means that \((2')\) cannot hold. On the other hand, when \( \Delta \to \infty \), then \( \delta(\Delta) \to 0 \) so that \( LHS(2') \to \left( \bar{\pi}_{k^*} k^* - M \right) < 0 \), in which case \((2')\) is satisfied for all \( k^* \). Hence, there must exist a \( \Delta^* \in (0, \infty) \) such that \((2')\) holds if and only if \( \Delta > \Delta^* \).

**Proof of Proposition 3:** To prove the claim, it is enough to show that \((2)\) is more likely to hold as \( F \) decreases. First, note that if \((k - \delta)\pi_k\) decreases in \( k \) then \( M - k^* \delta + \frac{k^* + 1 - \delta}{1 - \delta} \pi_{k^* + 1} < M - k^* \delta + \frac{k^* + 1 - \delta}{1 - \delta} \pi_{k^*} \). That is, \( LHS(2) \) increases in \( k^* \). Therefore, the proposition holds if \( k^* \) increases as \( F \) decreases.

But this relationship holds, because both \( V_1^E(k^*) = \frac{\pi_{k^*} + \beta_{k^*} S_{k^*}}{1 - \delta} \) and \( V_2^E(k^*) = \frac{\bar{\pi}_{k^*}}{1 - \delta} \) are (weakly) decreasing functions of \( k^* \). For \( V_2^E(k^*) \), this claim is obvious; for \( V_1^E(k^*) \) it was proved in the proof of Proposition 1. Hence, the entry value function \( \dot{V}^E(\cdot) = \max\{V_1^E(\cdot), V_2^E(\cdot)\} \) (defined in Lemma 3) is a (weakly) decreasing function, which means that \( k^* \) (weakly) decreases in \( F \).

**Proof of Lemma 4:** Consider a period \( t \) in which \( I \) starts out as a monopoly. Suppose that in this period exactly \( k^* - n \) firms merge, where \( 0 < n < k^* \). Then at the beginning of period \( t + 1 \)
the industry starts with \( n \geq 2 \) incumbents. Because we know from Lemma 2 and Proposition 1 that, in the absence of entry deterrence, \( I \) either buys out all entrants or there is no merger, the only reason why, in the current setting, \( I \) would accommodate the \( n - 1 \) entrants is that these firms would share some of the future entry deterrence costs. That is, if entry deterrence in the future is never profitable, we are back to the scenario of the previous section, where Proposition 1 applies. On the other hand, if entry deterrence is optimal, then the profit to \( I \) when it is one of \( n \) incumbents must be bigger than when \( I \) is the sole incumbent, because accommodating the \( n - 1 \) firms at time \( t \) is costly (according to Proposition 1, \( I \)'s period-\( t \) profit would be higher if either all or none of the entrants were bought out).

Now, with \( n \) incumbents sharing \( K \) equally in period \( t + 1 \), that period’s profit to each of them when they deter entry is \( \pi_n - \frac{K}{n} \) (because there is no subsequent merger). But \( \pi_n - \frac{K}{n} < n\pi_n - K < M - K \), where the last inequality follows from \( n\pi_n \leq M \). Thus, whenever entry deterrence is profitable, \( I \) would be strictly better off as a sole incumbent at time \( t + 1 \), deterring entry to defend its monopoly position, rather than sharing the costs of entry deterrence in preserving an oligopoly with \( n \geq 2 \) firms. It therefore cannot be that, in equilibrium, fewer than \( k^* \) firms merge in a given period and the claim follows.

**Proof of Proposition 4:** First, note that if the monopoly is preserved through entry deterrence, the deadweight loss in that period is equal to \( K - \Delta CS \) (in addition to the deadweight loss from monopoly, which does not depend upon the method of monopoly preservation). If \( I \) uses merger, then the deadweight loss is equal to that period’s total costs of entry, \((k^* - 1)F\). Hence, entry deterrence is more efficient than merger if and only if \( K \leq K_2 \equiv (k^* - 1)F + \Delta CS \). We thus have the following result: Suppose (2) holds, \( K < M - \pi_{k^*} \), and there is no government regulation. (i) If \( K \leq \min\{K_1, K_2\} \), then \( I \) efficiently uses entry deterrence to preserve monopoly. (ii) If \( K_1 < K < K_2 \), then \( I \) uses merger to preserve monopoly, even though it would be more efficient.
to use entry deterrence. (iii) If $K_2 < K < K_1$, then $I$ uses entry deterrence to preserve monopoly, even though it would be more efficient to use merger. (iv) If $K \geq K^{\text{max}} \equiv \max\{K_1, K_2\}$, then $I$ efficiently uses merger to preserve monopoly.

Part (a) in the proposition follows from (ii) above, because prohibiting mergers under these parameter values induces $I$ to switch from using merger to using entry deterrence, which increases efficiency. Part (b) follows because if mergers are not feasible, $I$ uses entry deterrence whenever $K < M - \pi_k^*$. The result is then obtained immediately from (iv) above, because for $K > K^{\text{max}}$ it would be more efficient to use merger to preserve monopoly, rather than using entry deterrence.

For part (c), note that the rest of the parameter values fall either under (i) or (iii) above. In these cases, $I$ prefers using entry deterrence to merger, so prohibiting merger has no effect on its behavior. □
References


